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A FRACTURE CRITERION FOR SNOW

By R. L. Brown
(Department of Civil Engineering and Engineering Mechanics, Montana State University, Bozeman, Montana 59715, U.S.A.)

Abstract. A fracture criterion is developed for fine-grained granular snow and is shown to predict accurately the conditions required for fracture under multiaxial states of stress. The criterion is applied to two specific types of deformation histories. The first deformation path involves an initial high strain-rate to produce fracture from an unstressed rest configuration, and the second path involves multiple strain-rate paths. The criterion expresses the critical value of deviatoric free energy in terms of the volumetric free energy and internal energy dissipation, and it was found to predict fracture accurately for tension, compression, and shear.

Résumé. *Un critère de rupture pour la neige.* On décrit un critère de la rupture pour de la neige à grains fins et on montre qu'il prédit bien les conditions requises pour la rupture dans des champs de contraintes multiaxiaux. Le critère est appliqué à deux types spécifiques de processus de déformation. Le premier type de déformation comprend au début une grande vitesse de déformation pour produire une rupture à partir d'une configuration au repos sans contraintes, le second type comprend des efforts dans des directions multiples. Le critère exprime les valeurs critiques de l'énergie libre de déviation en fonction de l'énergie libre par volume et de la dissipation d'énergie interne; on a trouvé que ce critère prévoyait avec précision les ruptures à la tension, à la compression et au cisaillement.

Zusammenfassung. *Ein Bruch-Kriterium für Schnee.* Für feinkörnigen Schnee wird ein Bruck-Kriterium entwickelt, von dem sich zeigen lässt, dass es genau jene Verhältnisse voraussagt, die zum Bruch unter vielachsigen Spannungszuständen führen. Das Kriterium wird auf zwei spezifische Typen von Deformationsabläufen angewandt. Der erste geht von einer hohen Anfangsverformungsrate aus, um den Bruch aus einer unverspannten Restkonfiguration zu erzielen; der zweite bezieht sich auf vielfache Wege der Spannungsrate. Das Kriterium stellt den kritischen Wert der deviatorischen freien Energie in Abhängigkeit von der volumetrischen freien Energie und dem inneren Energieschwund dar. Es erwies sich als geeignet zur genauen Voraussage der Dehnung, Kompression und Scherung.

Introduction

There has been considerable interest in the fracture characteristics of snow for some time, since some basic understanding of this topic is needed in order to evaluate properly the conditions necessary for avalanche initiation. The earlier studies were of an observational nature in the field, where detailed studies were made of the fracture line in the crown area and the portions of the flank which were not destroyed by the avalanche.

Determination of the triggering mechanism invariably becomes a difficult task, since in a snow slope there exists a wide distribution of snow density and snow types, with complicated boundary conditions and a complex state of stress within the pack. In addition, it is difficult to ascertain just where the pack failure was initiated, i.e. in the crown, the base or the toe region.

It therefore seems quite reasonable that a respectable number of theories of failure have been put forward to explain just how an avalanche starts under natural or artificial triggering conditions, and it also appears reasonable to assume that there will continue to be considerable disagreement among avalanche researchers concerning the plausibility of the various theories.

One approach to the problem has been to look at snow simply as a material reacting to a stress or deformation situation and then to attempt to formulate a fracture criterion for the material. If successful, this criterion can then be of some use in studying the avalanche problem, since the fracture criterion can indicate the stress conditions under which snow can most readily fracture. However, this is not in itself an easy task. As a material snow is extremely difficult to characterize. In addition to the fact that it is non-linear and highly compressible, the material properties of snow also depend on temperature and strain history. The actual strength of snow (fracture stress) is dependent upon density, strain-rate, strain history, and snow type. Consequently, a general fracture criterion is an exceedingly difficult result to achieve, and quite probably the complexity of the criterion might preclude its use in engineering analysis.

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Previous studies by Salm (1971), Brown and Lang (1973, [1975]) and Brown and others (1973) have considered the characterization of the fracture properties of snow. Salm (1971) utilized a non-linear mechanical model to formulate a constitutive theory for snow in order to study the stress power and conserved energy levels at fracture under uniaxial compression. Brown and others (1973) attempted to formulate a fracture criterion which would express the fracture condition in terms of the histories of volumetric and deviatoric stress power. While this criterion did prove to yield a good correlation with experiment, it was quite involved and not readily applicable to any engineering analysis. Consequently, Brown and Lang [1975]) took a different approach and studied the levels of free energy and energy dissipation associated with fracture for a variety of deformation paths. It was observed in that paper that a criterion based on the free energy ψ and dissipation σ could yield a criterion which was physically meaningful and yet would be mathematically somewhat simpler than earlier criteria.

This paper reports the results of an effort to formulate a fracture criterion which is based upon energy concepts and hence may be put in forms which are usable in engineering analysis. The intent here is the formulation of a criterion which is general enough to encompass three-dimensional states of stress and variable deformation histories. Such generality necessarily makes a criterion extremely complicated, but simplification to special states of deformation will be considered.

First, the thermodynamic theory on which the fracture criterion is based will briefly be discussed. This has already been developed by Brown and Lang [(1975]) and will not be dealt with in detail here. Then the form of the criterion based on experimental evidence will be formulated, compared to test results, and discussed.

Thermodynamic formulation

The constitutive equation for snow is based upon the assumption that there exists a functional ψ, the Helmholtz free energy, which measures the recoverable strain energy in the body under isothermal conditions. By making use of the first and second laws of thermodynamics, one can show that the Piola stress, $\mathbf{T}$, and specific internal energy dissipation rate σ are directly expressible as Frechet (′) derivatives of ψ. This result therefore makes ψ a central state functional for this type of material. The kinematic variables describing deformation and strain are

$$\mathbf{F} = x\nabla_x, \tag{1}$$

$$\mathbf{E} = \tfrac{1}{2}(\mathbf{F}\cdot\mathbf{F}-\mathbf{1}), \tag{2}$$

$$\mathbf{G} = \mathbf{F}\cdot\mathbf{F}, \tag{3}$$

where $\mathbf{F}$ is the deformation gradient, x is the deformed coordinates, X is the original co-ordinates, $\mathbf{E}$ is the Lagrangian strain tensor, $\mathbf{1}$ is the unity tensor, and $\mathbf{G}$ is defined as the Green deformation tensor. Here ∇_x is the gradient operator with the Cartesian components $\partial/\partial X_i$. The form for the Helmholtz free energy finally arrived at by Brown (in press) is expressible as a functional on the strain history and is expanded into a series as follows:

$$\psi(t) = \sum_{I=2}^{5} \psi_I \tag{4}$$

where ψ_I are ith order convolutions on the history of the strain tensor. The thermodynamic state of snow is deformation-history dependent, and a convenient way of expressing this is by means of convolutions. The terms ψ_I have the forms:

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$$\psi_2 = \int_{-\infty}^{t} \int_{-\infty}^{t} \{\Psi_1(t-\tau_1, t-\tau_2) \operatorname{tr} [\dot{\mathbf{E}}(\tau_1)\dot{\mathbf{E}}(\tau_2)] + \Psi_2(t-\tau_1, t-\tau_2) \operatorname{tr} (\dot{\mathbf{E}}(\tau_1)) \operatorname{tr} (\dot{\mathbf{E}}(\tau_2))\} \, d\tau_1 \, d\tau_2, \tag{5a}$$

$$\psi_3 = \int_{-\infty}^{t} \int_{-\infty}^{t} \int_{-\infty}^{t} \Psi_3(t-\tau_1, t-\tau_2, t-\tau_3) \operatorname{tr} \dot{\mathbf{E}}_1 \operatorname{tr} \dot{\mathbf{E}}_2 \operatorname{tr} \dot{\mathbf{E}}_3 \, d\tau_1 \, d\tau_2 \, d\tau_3 \tag{5b}$$

$$\psi_4 = \int_{-\infty}^{t} \int_{-\infty}^{t} \int_{-\infty}^{t} \int_{-\infty}^{t} \{2\Psi_4 \operatorname{tr} (\dot{\mathbf{E}}_1\dot{\mathbf{E}}_2\dot{\mathbf{E}}_3\dot{\mathbf{E}}_4) + \Psi_5 \operatorname{tr} (\dot{\mathbf{E}}_1\dot{\mathbf{E}}_2) \operatorname{tr} (\dot{\mathbf{E}}_3\dot{\mathbf{E}}_4) + \Psi_6 \operatorname{tr} (\dot{\mathbf{E}}_1) \operatorname{tr} (\dot{\mathbf{E}}_2) \operatorname{tr} (\dot{\mathbf{E}}_3\dot{\mathbf{E}}_4) + \Psi_7 \operatorname{tr} (\dot{\mathbf{E}}_1) \operatorname{tr} (\dot{\mathbf{E}}_2) \operatorname{tr} (\dot{\mathbf{E}}_3) \operatorname{tr} (\dot{\mathbf{E}}_4)\} \, d\tau_1 \ldots d\tau_4, \tag{5c}$$

$$\psi_5 = \int_{-\infty}^{t} \ldots \int_{-\infty}^{t} \Psi_8 \operatorname{tr} (\dot{\mathbf{E}}_1) \ldots \operatorname{tr} (\dot{\mathbf{E}}_5) \, d\tau_1 \ldots d\tau_5. \tag{5d}$$

The term tr $(\cdot)$ is the trace of the tensor quantity in the parentheses, i.e. the sum of the diagonal terms in the matrix of tensor components. In an earlier paper, Brown (1976) truncated the series to a fourth-order expansion, but later the fifth-order fit given here was found to give better results and consequently is used in this study. The above expansion involves a total of eight relaxation-memory functions, Ψ_1 to Ψ_8.

The Piola stress $\mathbf{T}$ consistent with this form for ψ is

$$\begin{aligned}
\frac{1}{\rho_0} \mathbf{T} = & \int_{-\infty}^{t} \{\Phi_1(t-\tau)\dot{\mathbf{E}}(\tau) + \Phi_2(t-\tau) \operatorname{tr} (\dot{\mathbf{E}}(\tau))\} \, d\tau + \\
& + \int_{-\infty}^{t} \int_{-\infty}^{t} \Phi_3 \operatorname{tr} (\dot{\mathbf{E}}_1) \operatorname{tr} (\dot{\mathbf{E}}_2) \, d\tau_1 \, d\tau_2 + \\
& + \int_{-\infty}^{t} \int_{-\infty}^{t} \int_{-\infty}^{t} \{\Phi_4\dot{\mathbf{E}}_1\dot{\mathbf{E}}_2\dot{\mathbf{E}}_3 + \Phi_5\dot{\mathbf{E}}_1 \operatorname{tr} (\dot{\mathbf{E}}_2\dot{\mathbf{E}}_3) + \\
& + \Phi_6 \operatorname{tr} (\dot{\mathbf{E}}_1) \operatorname{tr} (\dot{\mathbf{E}}_2\dot{\mathbf{E}}_3)\mathbf{1} + \Phi_6' \operatorname{tr} (\dot{\mathbf{E}}_1) \operatorname{tr} (\dot{\mathbf{E}}_2)\dot{\mathbf{E}}_3 + \\
& + \Phi_7 \operatorname{tr} (\dot{\mathbf{E}}_1) \operatorname{tr} (\dot{\mathbf{E}}_2) \operatorname{tr} (\dot{\mathbf{E}}_3)\mathbf{1}\} \, d\tau_1 \, d\tau_2 \, d\tau_3 + \\
& + \int_{-\infty}^{t} \int_{-\infty}^{t} \int_{-\infty}^{t} \int_{-\infty}^{t} \Phi_8 \operatorname{tr} (\dot{\mathbf{E}}_1) \operatorname{tr} (\dot{\mathbf{E}}_2) \operatorname{tr} (\dot{\mathbf{E}}_3) \operatorname{tr} (\dot{\mathbf{E}}_4) \, d\tau_1 \ldots d\tau_4
\end{aligned} \tag{6}$$

where the functions Ψ_i and Φ_i are related on a one-to-one basis. If Ψ_6 is assumed to be completely symmetric in its arguments, then $\Phi_6 = \Phi_6'$, and there are then only eight memory functions in the constitutive equation. To evaluate the memory functions experimentally, complete symmetry in the arguments of all the memory functions Ψ_i was actually assumed. The forms finally chosen were

$$\left.\begin{aligned}
\Psi_i(t_1, t_2) &= \psi_i(t_1)\psi_i(t_2), & i &= 1, 2 \\
\Psi_3(t_1, t_2) &= \psi_3(t_1)\psi_3(t_2)\psi_3(t_3), & & \\
\Psi_i(t_1, t_2, t_3) &= \psi_i(t_1)\psi_i(t_2)\psi_i(t_3)\psi_i(t_4), & i &= 4, \ldots, 7 \\
\Psi_8(t_1, t_2, t_3, t_4) &= \psi_8(t_1) \ldots \psi_8(t_5), & &
\end{aligned}\right\} \tag{7a–d}$$

and the relationship between the functions Φ_i and Ψ_i are

$$\left.\begin{aligned} \Phi_i &= C_i\psi_i, && i = 1, 2 \\ \Phi_3 &= C_3\psi_3\psi_3, && \\ \Phi_i &= C_i\psi_i\psi_i\psi_i, && i = 4, \dots, 7 \\ \Phi_8 &= C_8\psi_8\psi_8\psi_8\psi_8, && \end{aligned}\right\} \quad (8a\text{–}d)$$

where the coefficients C_i are

$$\left.\begin{aligned} C_i &= 2[\tfrac{1}{2}\psi_i(0)]^{\frac{1}{2}}, && i = 1, 2 \\ C_3 &= 3[\tfrac{1}{3}\psi_i^2(0)]^{\frac{1}{3}}, && \\ C_i &= 4[\tfrac{1}{4}\psi_i^3(0)]^{\frac{1}{4}}, && i = 4, 5, 7 \\ C_6 &= 2[\tfrac{1}{2}\psi_6^3(0)]^{\frac{1}{4}}, && \\ C_8 &= 5[\tfrac{1}{5}\psi_8^4(0)]^{\frac{1}{5}}. && \end{aligned}\right\} \quad (8a\text{–}e)$$

For a more detailed discussion of the above development, the reader is referred to Brown (1976).

A Prony series expansion is used to approximate the memory functions

$$\Psi_i(t) = \sum_{j=1}^{3} B_{ij} \exp(-\gamma_{ij}t) \quad (9)$$

and the coefficients evaluated experimentally are given in Table I for fine-grained granular snow with a density of 330 ± 20 kg/m³. Since this snow was well sintered and exhibited good strength, the stress response of this material as indicated in Figure 1 was significantly greater than the gradient metamorphosed snow studied by Brown (1976).

TABLE I. MATERIAL COEFFICIENTS FOR FINE-GRAINED GRANULAR SNOW
$\rho = 330$ kg/m³, T = −10°C

Function No.	B_{i1}	B_{i2}	B_{i3}	γ_{i1}	γ_{i2}	γ_{i3}
1	3.887×10^5	3.142×10^5	6.150×10^4	3.00	0.40	0.010
2	5.314×10^3	2.420×10^5	-2.384×10^5	1.40	0.001	0.000 6
3	2.130×10^4	-1.354×10^4	1.077×10^3	0.30	0.220	0.080
4	9.273×10^2	1.851×10^2	2.947×10	0.60	0.200	0.001
5	8.424×10^3	-7.380×10^3	5.366×10^2	0.30	0.260	0.120
6	1.361×10^4	-1.190×10^4	7.05	0.30	0.280	0.000 8
7	4.771×10^3	-2.091×10^3	-8.868×10	0.45	0.310	0.06
8	1.340×10^3	5.552×10^2	1.297×10	12.00	0.500	0.001

For a given deformation process, Equations (4) and (6) can be used to calculate the stress response and the variation of the Helmholtz free energy. The energy dissipation under isothermal conditions is given by the relation

$$\sigma = \frac{1}{\rho_0}\,\mathrm{tr}\,(\mathbf{T}\dot{\mathbf{E}}) - \dot{\psi} \quad (10)$$

and may readily be calculated once $\mathbf{T}$ and ψ are found.

As was mentioned earlier, the intent of this paper was to formulate a fracture criterion in terms of the volumetric and deviatoric free energies. ψ gives the recoverable energy for the total deformation, which involves both volumetric and deviatoric contributions. To find the deviatoric portion ψ_D of the free energy, consider the term

$$\alpha = |\mathcal{J}|^{-\frac{1}{3}} \quad (11)$$

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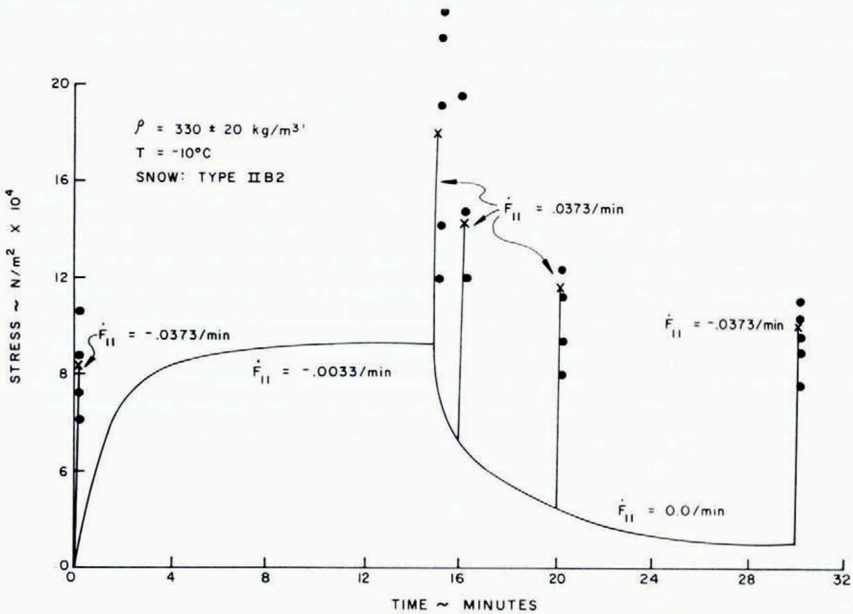

Fig. 1. Comparison of fracture criterion with experimental results for unconfined compression.

where $\mathcal{J} = \det|\mathbf{F}|$, the Jacobian of the deformation, determines the relative volume change by the relation

$$\frac{\rho_0}{\rho} = |\mathcal{J}|. \tag{12}$$

Now consider the deformation αX which has the deformation gradient

$$\mathbf{F}' = \alpha\mathbf{F} \tag{13}$$

where $\mathbf{F}$ is the true deformation gradient. One can readily show that this deformation is isochoric. If we define $\mathbf{E}'$ by

$$\mathbf{E}' = \tfrac{1}{2}[\mathbf{F}'\cdot\mathbf{F}'-\mathbf{1}], \tag{14}$$

one can show that

$$\mathbf{E} = \mathbf{E}'-\mathbf{K} \tag{15}$$

where

$$\mathbf{K} = \tfrac{1}{2}\mathbf{G}(\alpha^2-1) \tag{16}$$

and $\mathbf{E}'$ is the portion of the strain $\mathbf{E}$ which is isochoric. The functional for the free energy then can be put in the modified form

$$\begin{aligned}\psi &= \mathop{\Psi}_{s=0}^{\infty}[\mathbf{E}^t(s)]\\ &= \mathop{\Psi}_{s=0}^{\infty}[\mathbf{E}'^t(s)-\mathbf{K}^t(s)]\\ &= \Psi[\mathbf{E}'^t(s)]+\Psi_{\mathrm{V}}[\mathbf{K}^t(s)]+\Psi_{\mathrm{VD}}[\mathbf{E}'^t(s),\ \mathbf{K}^t(s)]\end{aligned} \tag{17}$$

so that the free energy is expressible as the sum of a purely deviatoric part

$$\psi_{\mathrm{D}} = \Psi[\mathbf{E}'^t(s)] \tag{18}$$

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and a part which contains the purely volumetric part and a remainder term containing coupling between the volumetric and deviatoric contributions of deformation. The difference $\psi - \psi_D$ we will refer to as the volumetric free energy ψ_V, and the relationship between ψ_D, ψ_V and σ at fracture will then be analyzed in the following section.

Fracture criterion

The fracture strength of snow is dependent upon the strain history, as has been demonstrated by Salm (1971) and Brown and Lang ([1975]). It was shown by Brown and Lang that for only uniaxial compression the history dependence of the fracture condition could probably be expressed in terms of ψ and σ, which respectively provide a measure of the elastic state and viscous state of the material, both history-dependent quantities. However, under multiaxial states of stress, it would appear more feasible to consider a criterion which is expressible in terms of ψ_D, ψ_V, and σ, since the mechanical properties are strongly dependent upon the nature of the volumetric deformation and the deviatoric deformation. Since the condition depends on the history of the deformation, a functional representation would be appropriate, such as

$$\psi_D = \mathop{\mathbf{F}}_{s=0}^{\infty} [\sigma^t(s), \psi_V], \tag{19}$$

i.e. the critical value of ψ_D depends on the instantaneous value of ψ_V and the history of the internal energy dissipation. Here $\sigma^t(s) = \sigma(t-s)$, where s represents the length of time since a point in the past. In this study we are interested in developing simple forms of the above general criterion applicable to special types of deformation histories. The first case is a situation where the material is initially in an unstressed rest configuration and is suddenly subjected to a high strain-rate which initiates fracture almost immediately. The second case involves a high rate perturbation of a material which is already in a stressed state. These forms of strain histories are relevant, since they are characteristic of many situations leading to avalanche initiation. Such examples would include the effect of explosives, skiers, sonic boom, earthquakes, wind gusts, or stress waves, among others.

In the first case, since the material is unstressed, the value of the dissipation at the onset of the deformation is zero, and a form of the fracture criterion for this special case could be:

$$\psi_D = F(\psi_V) \tag{20}$$

In the second case, if σ_0 is the value of the dissipation at the onset of a high strain-rate, then a first-order approximation of the criterion could be

$$\psi_D = F(\psi_V, \sigma_0)$$

where the initial dissipation σ_0 is assumed to be the predominant quantity defining the dissipative state within a small time period after the high strain-rate was applied to the material. Mathematically, the above approximation amounts to assuming a Taylor series expansion of the history $\sigma^t(s)$ about the instant when the perturbation is applied to the material and then retaining only the first term in the expansion. Since ψ and σ are smooth functions of time, this approximation is valid under the restrictions indicated.

In order to determine specific forms for the fracture criterion empirically, a complete set of fracture tests were run under conditions given in Table II. In all, about 23 deformation paths were followed in tension, compression, and shear. Data from some of the paths, such as paths No. 12 and 20, were not usable, since no specimens of the correct snow type or density were tested for those experiments.

The first nine deformation paths correspond to the first case described above (called brittle fracture). Figures 1, 2, 3 and 4 show the comparison of the fracture criterion for brittle fracture. For each test, the values of ψ_V and ψ_D were calculated at the instant fracture

TABLE II. DEFORMATION PATHS USED IN EXPERIMENTAL PROGRAM

Test No.	*Mode*	*Strain-rate* min^{-1}	*Time* min
1	Tension	0.026 0	0–F*
2	Tension	0.019 4	0–F
3	Tension	0.013 2	0–F
4	Tension	0.006 5	0–F
5	Compression	−0.037 3	0–F
6	Compression	−0.026 6	0–F
7	Compression	−0.019 8	0–F
8	Compression	−0.014 46	0–F
9	Shear	0.018 60	0–F
10	Shear	0.014 9	0–F
11	Compression	−0.003 33	0–15
	Compression	−0.014 46	15–F
12	Compression	−0.003 33	0–15
		−0.006 66	15–F
13	Compression	−0.003 33	0–15
		−0.037 33	15–F
14	Compression	−0.003 33	0–15
		0	15–16
		−0.037 3	16–F
15	Compression	−0.003 33	0–15
		0	15–20
		−0.037 3	20–F
16	Compression	−0.003 30	0–15
		0	15–30
		−0.037 30	30–F
17	Shear	0.001 85	0–15
		0.018 6	15–F
18	Shear	0.000 95	0–15
		0.018 6	15–F
19	Shear	0.003 70	0–15
		0	15–30
		0.018 6	30–F
20	Shear	0.003 70	0–15
		0	0–20
		0.018 6	30–F
21	Shear	0.000 95	0–15
		0	15–30
		0.018 6	30–F
22	Tension	0.001 33	0–15
		0.026 0	15–F
23	Tension	0.001 33	0–15
		0	15–30
		0.026 0	30–F

* F indicates time of fracture.

occurred, and these results are represented graphically in Figure 4 which shows both average results and the scatter incurred. In Figures 1–3 a comparison is shown between the experimental results for stress and the value predicted by the fracture criterion.

The results in Figure 4 suggest that a linear relationship such as

$$\psi_D = \alpha - \beta\,[\mathrm{sgn}\,(p)]\psi_v \tag{21}$$

would be a very accurate approximation to the average fracture conditions for brittle fracture.

Figure 5 and Figures 1–3 illustrate the results for the second case, ductile fracture. In Figure 5, the deviation from Equation (21) is plotted against the energy dissipation σ_0. Again we see a linear variation, so that the fracture condition may be approximated by

$$\psi_D = \alpha - \beta\,[\mathrm{sgn}\,(p)]\,\psi_v + \gamma\sigma_0 \tag{22}$$

where $\gamma = 1.60$. The data points in this figure represent the average experimental results for each deformation path. The discrepancy between theory and experiment is here larger

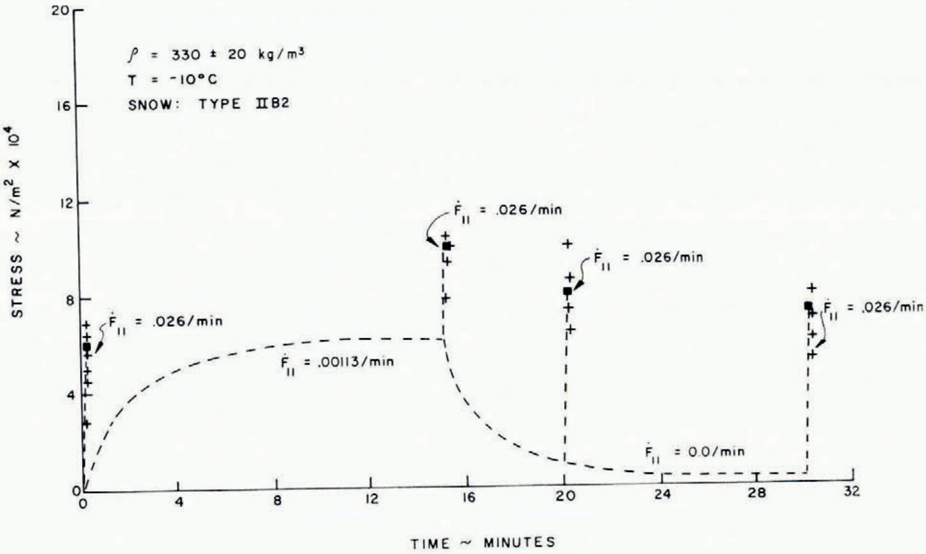

Fig. 2. Comparison of fracture criterion with experimental results for tension.

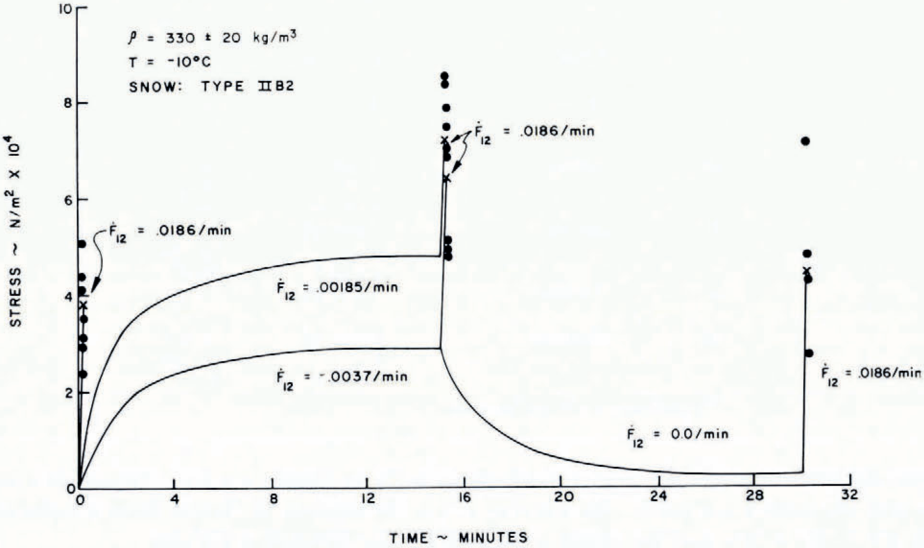

Fig. 3. Comparison of fracture criterion with experimental results for shear.

than for brittle fracture, although not intolerable except for deformation path No. 13. In this particular path, the material was in a very dissipative state, and the approximations used for the functional in Equation (19) probably lead to a significant error for cases involving such large values of dissipation. Figures 1–3 illustrate how well the criterion approximates the experimental values of fracture stress for compression, tension, and shear.

A word should be said about the experimental scatter encountered in this project. As indicated in the figures, the scatter is quite low when compared to the scatter usually encountered in fracture tests of snow. A great amount of care was exercised in cutting and

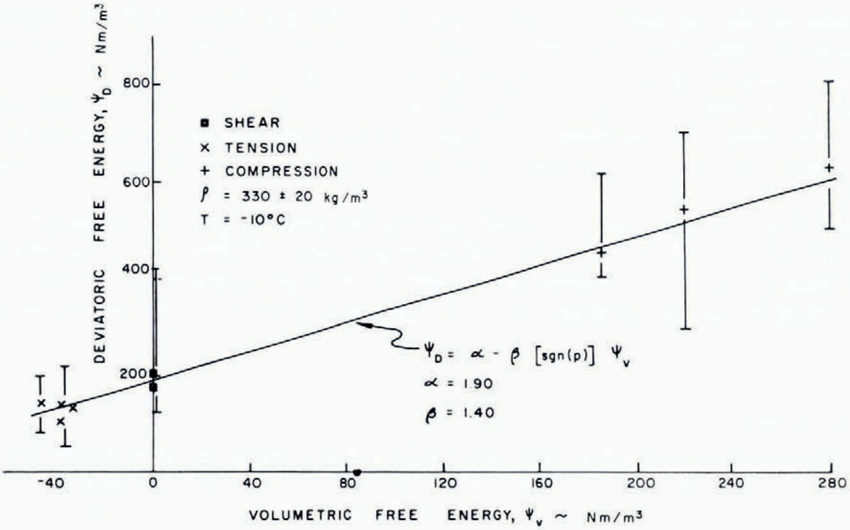

Fig. 4. Comparison of fracture criterion with experimental results for brittle fracture conditions.

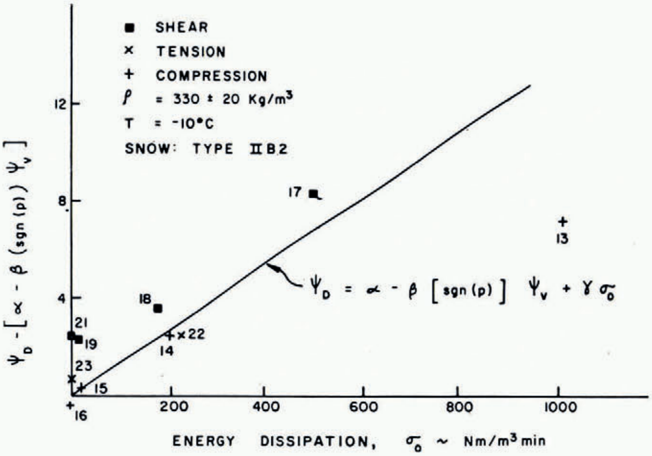

Fig. 5. Comparison of fracture criterion with experimental results for ductile fracture conditions.

handling the specimens, and they were also given several days curing time after cutting to allow any damage to be partially mended. Also, only snow within a narrow density range (330 kg/m³) were used, since density strongly influences strength, and care was taken to use snow which was fine-grained and granular.

Discussion and conclusions

The results of this paper do show that a fracture criterion can be developed to characterize the strength of snow under multiaxial states of stress. For the two special forms considered here, the criterion takes on the simple forms given in Equations (21) and (22). These then determine the critical value of ψ_D required for fracture. Therefore, when a strain perturbation is applied to snow, by calculating the variation of ψ_V and ψ_D, Equation (21) or Equation (22) will determine when the critical value of ψ_D is reached.

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The form of this fracture criterion is similar to the Coulomb–Mohr criterion since it relates volumetric and deviatoric effects, but in snow the dissipation rate plays a very central role in determining strength and consequently must be taken into account.

The specimens here were quite small (about 1×10^{-3} m³) and consequently size effects should be considered. Sommerfeld (1974) has shown that the fracture strength of snow varies with the sample size, and by utilizing Weibull's method, size effects may systematically be accounted for.

It should also be noted that these results are for fine-grained granular snow with a density of 330 kg/m³, which makes it a fairly high-strength snow. Other results for lower-density snow gave significantly lower values of fracture stress. For instance, at a density of about 200 kg/m³, typical fracture stresses were only about a quarter of the values indicated in Figures 1–3.

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DISCUSSION

H. Gubler: Salm's investigation on the rheological behaviour of snow (1971) show that the dissipative terms in a fracture criterion are negligible for strain-rates above about 1 s⁻¹. This agrees well with your experiments.

R. L. Brown: That is encouraging to hear. I have not read Bruno Salm's pioneering work for quite some time, so I must admit I am not well acquainted with the numerical values in his paper.

J. W. Glen: The quantity σ_0 in your theory is the energy dissipation taking place in the material at the instant the final strain-rate is applied. You have used the formula for stress-relaxation conditions, when stress is presumably decreasing at constant strain. If strain is constant, surely strain-rate is zero, so how do you define energy dissipation as non-zero, and how is it determined?

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Brown: When the strain-rate is terminated after a period of deformation, the material begins to relax and the stress decreases in approximately an exponential manner, so that after an extended period of relaxation, the stress may be only, say, 10% of the stress level attained before relaxation began. Likewise the internal energy dissipation decreases to very small values, but the dissipation does not disappear the instant the deformation stops, since the material is still under a load, and internal dissipation mechanisms are still active. As the stress relaxes, the dissipation also decreases in a uniform manner.

R. A. Sommerfeld: How do you get a negative hydrostatic pressure in a solid medium?

Brown: A negative pressure here results when the volumetric stress, which is just the trace of the stress tensor, becomes positive. In the tension test, the volumetric stress is possitive, and therefore for this case we have a negative hydrostatic pressure.

A. Dyunin: Have you attempted to derive a dimensionless criterion?

Brown: No, I have not.

M. Mellor: Did you make load-cycling tests and hysteresis measurements to obtain estimates of dissipation? If so, did you use the results to differentiate between brittle and ductile fracture modes?

Brown: No, we did not, and I think we really should have. These tests could have yielded some valuable information. To perform these experiments properly, an electrohydraulic testing machine such as the one at the U.S. Army Cold Regions Research and Engineering Laboratory would be very desirable, since a unit such as this has the capability and flexibility accurately to control the deformation cycle in almost any manner desired.

Gubler: The strain-rate resulting from skiers, explosives, etc., are very different. For example the strain-rates associated with explosives (outside the crater range) are 10^{-3} s^{-1} whereas for skiers they are ≈ 1 s^{-1}. From this it follows that shear waves originating from explosives will not initiate brittle fracture whereas skiers may initiate brittle fracture. Stability of a slope to brittle fracture and to ductile fracture differ widely.

Brown: I was not aware of this, and admittedly the statement in my paper which implied that explosives cause brittle fracture was based on intuition and not fact. Consequently I am surprised by this new information, which goes a long way towards explaining some of the puzzling questions about the effectiveness of explosives in snow.

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