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WATER WAVE SCATTERING BY A VERTICAL POROUS BARRIER WITH TWO GAPS

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Abstract

Explicit solutions are rarely available for water wave scattering problems. An analytical procedure is presented here to solve the boundary value problem associated with wave scattering by a complete vertical porous barrier with two gaps in it. The original problem is decomposed into four problems involving vertical solid barriers. The decomposed problems are solved analytically by using a weakly singular integral equation. Explicit expressions are obtained for the scattering amplitudes and numerical results are presented. The results obtained can be used as a benchmark for other wave scattering problems involving complex geometrical structures.

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1. Introduction

Thin vertical barriers have been widely used as breakwaters in ocean engineering practices in order to create tranquil zones. The boundary value problems that are associated with linearized deep water wave scattering by thin vertical barriers have been of interest to many researchers due to the mathematical difficulty involved in their analytical or semi-analytical methods of solution. The first nontrivial closed form solution was obtained by Havelock [4]. Then Dean [3] solved the scattering problem involving a submerged barrier by complex function theory. With the use of Havelock's formula, Ursell [16] handled the scattering problem of a surface-piercing finite-length barrier. Porter [13] solved the problem of a complete vertical barrier with a gap of finite length by a complex variable technique. More general problems with many finite barriers and finite gaps in the barriers were handled by Lewin [6], Mei [12] and Porter [14] with the help of complex function theory. Various mathematical procedures

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that had been devised in the theory of linear water waves were well catalogued by Mandal and Chakrabarti [11].

Chakrabarti et al. [1] reconsidered the scattering problem of a barrier with a single gap and solved it analytically by the aid of a weakly singular integral equation. The scattering problem involving a complete vertical barrier with many gaps of finite length can be routinely worked out by making use of the bounded solution of a logarithmic singular integral equation over multiple intervals of finite length [7]. Manam and Kaligatla [8] extended the method to the study of scattering of membrane-coupled gravity waves by a vertical solid barrier with a single gap.

Vertical porous barriers are known for effective dissipating characteristics. They are used, in practice, mainly due to their advantage in reducing wave loads. Ever since Chwang [2] derived a boundary condition on a thin vertical porous barrier based on the Darcy law, there has been a quest for the development of a mathematical method to analytically treat the problem of deep water wave scattering by vertical porous barriers. The boundary condition was later modified by Yu and Chwang [17] so as to include the inertial effects of the porous barrier.

Recently, Manam and Sivanesan [9] devised a decomposition method to solve the scattering problem that involves thin vertical partial porous barriers. The solution method is motivated by the work of Porter and Evans [15]. It establishes a connection between the solution potentials that describe wave scattering by a complementary arrangement of partial vertical solid barriers. Moreover, by introducing a modified integral relation between the wave potentials pertaining to scattering problems involving vertical barriers of either a solid or a porous type with a certain configuration, Manam and Sivanesan [10] explicitly solved the scattering problem involving porous barriers.

The purpose of this paper is two-fold. First, the weakly singular integral equation method of solution to the scattering problem involving a vertical solid barrier with two gaps of finite length in it is demonstrated. Second, we intend to work out the explicit method of solution for the scattering problem that involves a vertical porous barrier with two gaps of finite length. This is done by decomposing the problem into two problems that are solved explicitly by the aid of a weakly singular integral equation. In Section 2, the scattering problem involving a solid or a porous barrier with two gaps in it is formulated based on the linearized theory of water waves. The integral connection between the solid and the porous wave potentials, decomposition of the original problem and the explicit solutions for the decomposed problems are given in Section 3. Numerical results for the scattering quantities and concluding remarks are provided in Sections 4 and 5, respectively.

2. Formulation of the scattering problem

A Cartesian coordinate system (x, y) with the y-axis increasing vertically downwards is used as a reference frame in an inviscid and incompressible fluid under gravity, where $y = 0$ is the position of the mean free surface. The barrier occupies the position $x = 0, \quad y \in B = (0, \infty) \setminus G$ as shown in Figure 1, where $G = (a_1, b_1) \cup (a_2, b_2)$.

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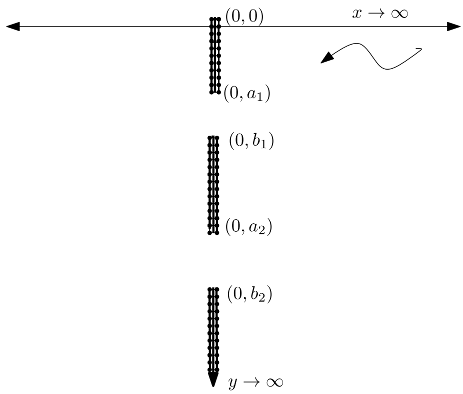

FIGURE 1. Schematic diagram.

The two-dimensional fluid motion is considered to be time harmonic and irrotational, and is described by the velocity potential $\Phi_1(x, y, t) = \mathrm{Re}[\hat{\phi}_1(x, y)\, e^{-i\omega t}]$, where ω is the angular frequency and $i = \sqrt{-1}$. Then, for wave scattering by an incident wave $\phi^0(-x, y) = e^{-iKx-Ky}$ from the positive x-axis, the spatial potential $\hat{\phi}_1(x, y)$ satisfies the following set of equations:

$$\hat{\phi}_{1xx} + \hat{\phi}_{1yy} = 0 \quad \text{in } x \in \mathbb{R}, \quad y > 0, \tag{2.1}$$

$$K\hat{\phi}_1(x, 0) + \hat{\phi}_{1y}(x, 0) = 0 \quad \text{on } x \neq 0, \tag{2.2}$$

where $K = \omega/g$ with g being the acceleration due to gravity,

$$|\nabla\hat{\phi}_1(x, y)| \to 0 \quad \text{as } y \to \infty, \tag{2.3}$$

$$|\nabla\hat{\phi}_1(x, y)| \sim O(r^{-1/2}) \quad \text{as } r = \sqrt{x^2 + (y - t)^2} \to 0, \tag{2.4}$$

where $(0, t)$ can be any of the barrier edges $(0, a_1)$, $(0, b_1)$, $(0, a_2)$ and $(0, b_2)$ in the fluid. Now consider the equations

$$\begin{aligned} \hat{\phi}_{1x}(0^\pm, y) &= -iK\Gamma(\hat{\phi}_1(0^+, y) - \hat{\phi}_1(0^-, y)), \quad y \in B, \\ \hat{\phi}_1(0^+, y) &= \hat{\phi}_1(0^-, y), \quad y \in G, \end{aligned} \tag{2.5}$$

where $\Gamma = \gamma(s + if)/[Kd(s^2 + f^2)]$ is the nondimensional porous effect parameter (see the paper by Yu and Chwang [17]), in which γ is the porosity constant, d is the plate thickness, s is the resistance force coefficient and f is the inertial force coefficient,

$$\hat{\phi}_1(x, y) \sim \begin{cases} \phi^0(-x, y) + R_p\, \phi^0(x, y) & \text{as } x \to \infty, \\ T_p\, \phi^0(-x, y) & \text{as } x \to -\infty. \end{cases}$$

Due to the barrier symmetry, the scattering problem may also be considered by allowing incident waves from the negative x-axis. Then the resulting velocity potential $\Phi_2(x, y, t) = \mathrm{Re}[\hat{\phi}_2(x, y)e^{i\omega t}]$ and the potential $\Phi_1(x, y, t)$ differ by a phase difference. Now the spatial potential $\hat{\phi}_2(x, y)$ satisfies (2.1)–(2.5), the boundary condition

$$\hat{\phi}_{2x}(0^\pm, y) = -iK\Gamma(\hat{\phi}_2(0^-, y) - \hat{\phi}_2(0^+, y)), \quad y \in B$$

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and the radiation condition

$$\hat{\phi}_2(x,y) \sim \begin{cases} \phi^0(-x,y) + R_p\,\phi^0(x,y) & \text{as } x \to -\infty, \\ T_p\,\phi^0(-x,y) & \text{as } x \to \infty. \end{cases}$$

It will be seen later that any of these scattering problems can be solved by utilizing two of them together.

The continuity of normal velocity across $x = 0$ ensures that the upper half-plane problems for the potentials $\hat{\phi}_j(x,y)$, $j = 1, 2$, can be reduced to quarter-plane problems by writing (see the book by Lamb [5, p. 517])

$$\hat{\phi}_j(x,y) = \begin{cases} \phi^0(-x,y) + \phi^0(x,y) + \phi_j^p(x,y), & (-1)^j x < 0, \\ -\phi_j^p(-x,y), & (-1)^j x > 0, \end{cases} \qquad j = 1, 2.$$

Then, in the account of the boundary value problems for $\hat{\phi}_j(x,y)$, $j = 1, 2$, the porous wave potential functions $\phi_j^p(x,y)$ defined on the domain $(-1)^{j+1}x > 0$ for $j = 1, 2$ satisfy equations (2.1)–(2.4) along with the boundary conditions

$$\phi^p_{j\,x}(0,y) + 2i\Gamma K[\phi_j^p(0,y) + \phi^0(0,y)] = 0, \quad y \in B,$$
$$\phi_j^p(0,y) + \phi^0(0,y) = 0, \quad y \in G$$

and

$$\phi_j^p(x,y) \to (R_p - 1)\phi^0(x,y) \quad \text{as } (-1)^j x \to \infty.$$

Also, the transmitted complex wave amplitude is now given by $T_p = 1 - R_p$.

Denoting the reflection and transmission amplitudes as R and T, respectively, in the scattering problem that involves a complete vertical solid barrier with two finite gaps in it, the associated quarter-plane problems for solid wave potentials $\phi_j^s(x,y)$ in the domain $(-1)^{j+1}x > 0$ for $j = 1, 2$ satisfy equations (2.1)–(2.4) along with the following boundary conditions with $T = 1 - R$:

$$\phi^s_{j\,x}(0,y) = 0, \quad y \in B,$$
$$\phi_j^s(0,y) + \phi^0(0,y) = 0, \quad y \in G$$

and

$$\phi_j^s(x,y) \to (R - 1)\phi^0(x,y) \quad \text{as } (-1)^j x \to \infty.$$

In the next section, analytical solutions to the problems for the porous wave potentials $\phi_j^p(x,y)$, $j = 1, 2$, are determined by the decomposition method. The decomposed problems are of finding solid wave potentials $\phi_j^s(x,y)$, $j = 1, 2$, and similar such auxiliary wave potentials.

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3. Method of solution

The boundary value problem for $\phi_j^p(x,y)$ that is described in the previous section is decomposed into two relatively easy, solvable problems by making use of an integral connection (see the paper by Manam and Sivanesan [10]) as given by

$$\begin{aligned}\phi_j^p(x,y) + iK\Gamma\int_0^x \Big[\{\phi^0(t,y)+\phi^0(-t,y)\} + \sum_{m=1}^{2}\phi_m^p((-1)^{j+m}t,y)\Big]\,dt \\ = \phi_j^s(x,y)+\psi_j(x,y), \quad (-1)^{j+1}x>0, \quad j=1,2. \end{aligned} \tag{3.1}$$

In equation (3.1), $\phi_j^s(x,y)$ is the solid wave potential described earlier and the auxiliary wave potential $\psi_j(x,y)$ satisfies (2.1)–(2.3). It may be verified that the integral term satisfies (2.1)–(2.3), because the partial derivatives satisfy $\phi^p_{1\,x}(0,y)=\phi^p_{2\,x}(0,y)$, for $y>0$ (see the paper by Manam and Sivanesan [9, Appendix]). The other boundary conditions that are satisfied by ψ_j are

$$\psi_j(0,y)=0, \quad y\in G, \tag{3.2}$$
$$\psi_{j\,x}(0,y)=0, \quad y\in B \tag{3.3}$$

and

$$\psi_j(x,y)\sim R_1^j\phi^0(x,y)+R_2^j\phi^0(-x,y) \quad \text{as } (-1)^{j+1}x\to\infty, \quad j=1,2. \tag{3.4}$$

Here R_k^j, $k=1,2$, are unknown constants. The condition (3.3) is due to the fact that $\phi_1^p(0,y)=\phi_2^p(0,y)$, $\quad y\in B$ [9, Appendix].

Also, by adding the relations in (3.1) after rewriting them in the same domain $x>0$, the porous wave potentials $\phi_j^p(x,y)$, $j=1,2$, are explicitly obtained as

$$\begin{aligned}\phi_j^p(x,y) &= [\phi_j^s(x,y)+\psi_j(x,y)] - \Gamma[\phi^0(x,y)-\phi^0(-x,y)] \\ &\quad - iK\Gamma\sum_{l=1}^{2}\int_0^x [\phi_l^s((-1)^{j+l}t,y)+\psi_l((-1)^{j+l}t,y)]\,dt, \quad (-1)^{j+1}x>0, \quad j=1,2.\end{aligned}$$

At this stage, the involved scattering wave amplitudes R_k^j, $j,k=1,2$, and R of the decomposed problems are related to R_p by (see the paper by Manam and Sivanesan [10])

$$(1+\Gamma)R_p = R+R_1^1, \quad \Gamma R_p = -R_2^1, \tag{3.5}$$
$$(1+\Gamma)R_p = R+R_2^2, \quad \Gamma R_p = -R_1^2. \tag{3.6}$$

Hence, the scattering wave amplitudes involved in the original problem as well as the auxiliary decomposed problems are determined from the equations (3.5)–(3.6), once the reflection amplitude R of an incident wave by the solid barrier is known.

Therefore, the original problem for the porous wave potentials is completely determined if one finds the decomposed wave potentials ϕ_j^s and ψ_j. In what follows, a solution method is provided first to solve a typical boundary value problem for $\chi(x,y)$, $x>0$, that satisfies (2.1)–(2.4), (3.2)–(3.3) and the radiating condition

$$\chi(x,y) \to \eta_1\,\phi^0(x,y) + \eta_2\,\phi^0(-x,y) \quad \text{as } x\to\infty, \tag{3.7}$$

where η_1,η_2 are constants. From this, one can determine the decomposed potentials $\psi_j(x,y)$, $j=1,2$. A similar procedure will be utilized to determine the solid wave potentials $\phi_j^s(x,y)$, $j=1,2$.

The most general form of the potential $\chi(x,y)$ that satisfies the equations (2.1)–(2.3) and (3.7) is

$$\chi(x,y) = \eta_1\,\phi^0(x,y) + \eta_2\,\phi^0(-x,y) + \int_0^\infty A(\xi)[\xi\cos(\xi y) - K\sin(\xi y)]e^{-\xi x}\,d\xi, \quad x>0.$$

Applying the boundary conditions (3.2) and (3.3), we obtain a pair of integral equations

$$\int_0^\infty \xi A(\xi)[\xi\cos(\xi y) - K\sin(\xi y)]\,d\xi = iK(\eta_1-\eta_2)e^{-Ky}, \quad y\in B,$$
$$\int_0^\infty A(\xi)[\xi\cos(\xi y) - K\sin(\xi y)]\,d\xi = -(\eta_1+\eta_2)e^{-Ky}, \quad y\in G.$$

They are rewritten in a different form as

$$\left(\frac{d}{dy}-K\right)\int_0^\infty \xi A(\xi)\sin(\xi y)\,d\xi = iK(\eta_1-\eta_2)e^{-Ky}, \quad y\in B,$$
$$\left(\frac{d}{dy}-K\right)\int_0^\infty A(\xi)\sin(\xi y)\,d\xi = -(\eta_1+\eta_2)e^{-Ky}, \quad y\in G.$$

By solving the above ordinary differential equations,

$$\int_0^\infty \xi A(\xi)\sin(\xi y)\,d\xi = \begin{cases} i(\eta_1-\eta_2)\sinh(Ky), & y\in(0,a_1),\\ P_1e^{Ky} - \dfrac{i}{2}(\eta_1-\eta_2)e^{-Ky}, & y\in(b_1,a_2),\\ -\dfrac{i}{2}(\eta_1-\eta_2)e^{-Ky}, & y\in(b_2,\infty),\end{cases} \tag{3.8}$$

$$\int_0^\infty A(\xi)\sin(\xi y)\,d\xi = \begin{cases} Q_1e^{Ky} + \dfrac{1}{2K}(\eta_1+\eta_2)e^{-Ky}, & y\in(a_1,b_1),\\ Q_2e^{Ky} + \dfrac{1}{2K}(\eta_1+\eta_2)e^{-Ky}, & y\in(a_2,b_2),\end{cases} \tag{3.9}$$

where P_1, Q_1 and Q_2 are arbitrary constants. Now, by defining

$$g(y) = \int_0^\infty A(\xi)\sin(\xi y)\,d\xi \quad \text{for } y\in G,$$

and by the inverse Fourier sine transform, we obtain from (3.8)

$$A(\xi) = \frac{2}{\pi\xi}\int_0^\infty P(y)\sin(\xi y)\,dy,$$

where

$$P(y)=\begin{cases} i(\eta_1-\eta_2)\sinh(Ky), & y\in(0,a_1),\\ P_1e^{Ky}-\dfrac{i}{2}(\eta_1-\eta_2)e^{-Ky}, & y\in(b_1,a_2),\\ \dfrac{-i}{2}(\eta_1-\eta_2)e^{-Ky}, & y\in(b_2,\infty),\\ g(y), & y\in G.\end{cases}$$

Then, by substituting $A(\xi)$ into equation (3.9), the unknown function $g(u)$ satisfies the weakly singular integral equation

$$\frac{1}{\pi}\int_G g(u)\log\left|\frac{y+u}{y-u}\right|du=f(y),\quad y\in G, \tag{3.10}$$

where

$$f(y)=\begin{cases}\hat{f}(y)+Q_1e^{Ky}, & y\in(a_1,b_1),\\ \hat{f}(y)+Q_2e^{Ky}, & y\in(a_2,b_2)\end{cases}$$

with

$$\begin{aligned}\hat{f}(y)=&-\frac{i}{\pi}(\eta_1-\eta_2)\int_0^{a_1}\sinh(Ku)\log\left|\frac{y+u}{y-u}\right|du\\ &+\frac{i}{2\pi}(\eta_1-\eta_2)\int_{(b_1,a_2)\cup(b_2,\infty)}e^{-Ku}\log\left|\frac{y+u}{y-u}\right|du\\ &-\frac{P_1}{\pi}\int_{b_1}^{a_2}e^{Ku}\log\left|\frac{y+u}{y-u}\right|du+\frac{1}{2K}(\eta_1+\eta_2)e^{-Ky},\quad y\in G.\end{aligned}$$

The general solution of the integral equation (3.10) can be obtained as (see the paper by Manam [7])

$$g(u)=\frac{A_0+A_1u^2}{S_1(u)}+\frac{2}{\pi S_1(u)}\int_G\frac{S_1(t)tf'(t)}{(u^2-t^2)}\,dt,\quad u\in G,$$

where

$$S_1(u)=\begin{cases}-\{(u^2-a_1^2)(b_1^2-u^2)(a_2^2-u^2)(b_2^2-u^2)\}^{1/2}, & u\in(a_1,b_1),\\ \{(u^2-a_1^2)(u^2-b_1^2)(u^2-a_2^2)(b_2^2-u^2)\}^{1/2}, & u\in(a_2,b_2)\end{cases}$$

and the arbitrary constants A_0 and A_1 can be obtained from the relations

$$\int_G ug(u)\,du=A_0\int_G\frac{u}{S_1(u)}\,du+A_0\int_G\frac{u^3}{S_1(u)}\,du$$

and

$$\int_G u^3g(u)\,du=A_0\int_G\frac{u^3}{S_1(u)}\,du+A_0\int_G\frac{u^5}{S_1(u)}\,du.$$

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In order to compute the integrals $\int_G ug(u)\,du$ and $\int_G u^3 g(u)\,du$, we use the integral equation (3.10). In the process, certain integrals are evaluated by the contour integration procedure. By multiplying the functions $(S_1(y))^{-1}$ and $y^2(S_1(y))^{-1}$ with the integral equation (3.10) and integrating over G,

$$\pi \int_G \frac{f(u)}{S_1(u)}\,du = I_1(a_1)\int_{a_1}^{b_1} g(u)\,du + I_1(b_2)\int_{a_2}^{b_2} g(u)\,du, \tag{3.11}$$

$$\pi \int_G u^2\frac{f(u)}{S_1(u)}\,du = I_2(a_1)\int_{a_1}^{b_1} g(u)\,du + I_2(b_2)\int_{a_2}^{b_2} g(u)\,du, \tag{3.12}$$

where

$$I_1(x) = \int_G \log\left|\frac{x+u}{x-u}\right|\frac{du}{S_1(u)} \quad \text{and} \quad I_2(x) = \int_G u^2\log\left|\frac{x+u}{x-u}\right|\frac{du}{S_1(u)}.$$

Then the integrals $\int_{a_1}^{b_1} g(u)\,du$ and $\int_{a_2}^{b_2} g(u)\,du$ are obtained from (3.11) and (3.12) as

$$\int_{a_1}^{b_1} g(u)\,du = \frac{\pi I_2(b_2)}{\Delta}\int_G \frac{f(u)}{S_1(u)}\,du - \frac{\pi I_1(b_2)}{\Delta}\int_G u^2\frac{f(u)}{S_1(u)}\,du$$

and

$$\int_{a_2}^{b_2} g(u)\,du = -\frac{\pi I_2(a_1)}{\Delta}\int_G \frac{f(u)}{S_1(u)}\,du + \frac{\pi I_1(a_1)}{\Delta}\int_G u^2\frac{f(u)}{S_1(u)}\,du,$$

respectively, where $\Delta = I_1(a_1)I_2(b_2) - I_1(b_2)I_2(a_1)$.

Similarly, by multiplying (3.10) with the function

$$T(y) = \left|\frac{(y^2-a_1^2)(y^2-a_2^2)}{(y^2-b_1^2)(y^2-b_2^2)}\right|^{1/2},$$

$$\int_G ug(u)\,du = \int_G f(u)T(u)\,du - \frac{\Delta}{\pi}\left[D_1(a_1)\int_{a_1}^{b_1} g(u)\,du + D_1(b_2)\int_{a_2}^{b_2} g(u)\,du\right],$$

where $D_1(x) = (I_3(x) - \pi x)/\Delta$ with

$$\begin{aligned} I_3(x) &= \int_G T(u)\log\left|\frac{x+u}{x-u}\right|du \\ &= \begin{cases} \pi(x-a_1) + I_3(a_1), & x \in (a_1, b_1), \\ \pi(x-b_2) + I_3(b_2), & y \in (a_2, b_2). \end{cases} \end{aligned}$$

Again, by multiplying (3.10) with the function $y^2T(y)$ and by integrating over G,

$$\begin{aligned} \int_G u^3 g(u)\,du = {}& -3D_2(a_1)\int_{a_1}^{b_1} g(u)\,du - 3D_2(b_2)\int_{a_2}^{b_2} g(u)\,du \\ & - 3C\int_G ug(u)\,du + 3\int_G u^2 f(u)\,T(u)\,du, \end{aligned}$$

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where $C = (b_1^2 + b_2^2 - a_1^2 - a_2^2)/2$ and $D_2(x) = \{I_4(x) - (\pi/3)x^3 - \pi C x\}/\pi$ with

$$I_4(x) = \int_G u^2 T(u) \log\left|\frac{x+u}{x-u}\right| du$$
$$= \begin{cases} \pi[\frac{1}{3}(x^3 - a_1^3) + C(x - a_1)] + I_3(a_1), & x \in (a_1, b_1), \\ \pi[\frac{1}{3}(x^3 - b_2^3) + C(x - b_2)] + I_3(b_2), & y \in (a_2, b_2). \end{cases}$$

Finally, the integrals

$$\int_G u g(u)\,du = \int_G f(u)T(u)\,du + L_1 \int_G \frac{f(u)}{S_1(u)}\,du + L_2 \int_G \frac{u^2 f(u)}{S_1(u)}\,du,$$
$$\int_G u^3 g(u)\,du = 3\int_G (u^2 - C) f(u)T(u)\,du + 3C_1 \int_G \frac{f(u)}{S_1(u)}\,du + 3C_2 \int_G \frac{u^2 f(u)}{S_1(u)}\,du,$$

where

$$L_1 = I_2(a_1)D_1(b_2) - I_2(b_2)D_1(a_1), \quad L_2 = I_1(b_2)D_1(a_1) - I_1(a_1)D_1(b_2),$$
$$C_1 = I_2(b_2)D_3(a_1) - I_2(a_1)D_3(b_2), \quad C_2 = I_1(a_1)D_3(b_2) - I_1(b_2)D_3(a_1)$$

with $D_3(x) = [CI_3(x) - I_4(x) + (\pi/3)x^3]/\Delta$. Since χ_x has an integrable singularity at the edges of the barrier, it is easy to see that the function $g(u)$ is bounded at a_j and b_j, $j = 1, 2$. Therefore, the bounded solution $g(u)$ of the integral equation (3.10) is

$$g(u) = \begin{cases} \dfrac{2T_1(u)}{\pi} \displaystyle\int_{a_1}^{b_1} \frac{t f'(t)}{T_1(t)(u^2 - t^2)}\,dt - \dfrac{2T_1(u)}{\pi} \displaystyle\int_{a_2}^{b_2} \frac{t f'(t)}{T_1(t)(u^2 - t^2)}\,dt, & u \in (a_1, b_1), \\ -\dfrac{2}{\pi T_1(u)} \displaystyle\int_{a_1}^{b_1} \frac{t T_1(t) f'(t)}{(u^2 - t^2)}\,dt + \dfrac{2}{\pi T_1(u)} \displaystyle\int_{a_2}^{b_2} \frac{t T_1(t) f'(t)}{(u^2 - t^2)}\,dt, & u \in (a_2, b_2) \end{cases}$$

provided that

$$A_0 - \frac{2}{\pi}\int_{a_1}^{b_1} \frac{t f'(t) q_1(t)}{T_1(t)}\,dt + \frac{2}{\pi}\int_{a_2}^{b_2} \frac{t f'(t) q_1(t)}{T_1(t)}\,dt = 0, \tag{3.13}$$

$$A_1 - \frac{2}{\pi}\int_{a_1}^{b_1} \frac{t f'(t)}{T_1(t)}\,dt + \frac{2}{\pi}\int_{a_2}^{b_2} \frac{t f'(t)}{T_1(t)}\,dt = 0, \tag{3.14}$$

$$A_0 - \frac{2}{\pi}\int_{a_1}^{b_1} t f'(t) T_1(t) q_2(t)\,dt + \frac{2}{\pi}\int_{a_2}^{b_2} t f'(t) T_1(t) q_2(t)\,dt = 0, \tag{3.15}$$

$$A_1 - \frac{2}{\pi}\int_{a_1}^{b_1} t f'(t) T_1(t)\,dt + \frac{2}{\pi}\int_{a_2}^{b_2} t f'(t) T_1(t)\,dt = 0, \tag{3.16}$$

where

$$T_1(t) = \left|\frac{(t^2 - a_1^2)(t^2 - b_1^2)}{(t^2 - a_2^2)(t^2 - b_2^2)}\right|^{1/2}, \quad q_1(t) = t^2 - a_1^2 - b_1^2, \quad q_2(t) = t^2 - a_2^2 - b_2^2.$$

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Therefore, the conditions (3.13)–(3.16) can be expressed as a system of linear equations for the unknowns η_1, η_2, P_1, Q_1 and Q_2 as

$$r_{11}\eta_1 + r_{12}\eta_2 + r_{13}P_1 + r_{14}Q_1 + r_{15}Q_2 = 0, \tag{3.17}$$
$$r_{21}\eta_1 + r_{22}\eta_2 + r_{23}P_1 + r_{24}Q_1 + r_{15}Q_2 = 0, \tag{3.18}$$
$$r_{31}\eta_1 + r_{32}\eta_2 + r_{33}P_1 + r_{34}Q_1 + r_{15}Q_2 = 0, \tag{3.19}$$
$$r_{41}\eta_1 + r_{42}\eta_2 + r_{43}P_1 + r_{44}Q_1 + r_{15}Q_2 = 0, \tag{3.20}$$

where the coefficients r_{ij} for $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4, 5$ are given in the Appendix.

Since $\phi^p_{j_x}$ and $\phi^s_{j_x}$ both have an integrable singularity at the edges of the barrier, it may be observed from (3.1) that ψ_{j_x} also has the same behaviour at the edges. Thus, in view of relation (3.4), the solid wave solution potentials $\psi_1(x, y)$ and $\psi_2(x, y)$ are found to be $\chi(x, y), x > 0$, with $\eta_1 = R^1_1, \eta_2 = R^1_2$, and $\chi(-x, y), x < 0$, with $\eta_1 = R^2_2, \eta_2 = R^2_1$, respectively.

By following the solution procedure that is demonstrated above, the potentials $\phi^s_j(x, y)$, $j = 1, 2$, that are associated with wave scattering by the solid vertical barrier with two gaps in it may be obtained as

$$\phi^s_j(x, y) = (R - 1)\,\phi^0(x, y) + \int_0^\infty B(\xi)[\xi\cos(\xi y) - K\sin(\xi y)]\,e^{(-1)^j\xi x}\,d\xi$$

for $(-1)^{j+1}x > 0$, $\quad j = 1, 2$. Here the unknown reflection amplitude R is determined from the system of linear equations

$$r_{11}R + r_{13}\hat{P}_1 + r_{14}\hat{Q}_1 + r_{15}\hat{Q}_2 = s_1,$$
$$r_{21}R + r_{23}\hat{P}_1 + r_{24}\hat{Q}_1 + r_{15}\hat{Q}_2 = s_2,$$
$$r_{31}R + r_{33}\hat{P}_1 + r_{34}\hat{Q}_1 + r_{15}\hat{Q}_2 = s_3,$$
$$r_{41}R + r_{43}\hat{P}_1 + r_{44}\hat{Q}_1 + r_{15}\hat{Q}_2 = s_4$$

with s_j, $j = 1, 2, 3, 4$, as listed in the Appendix. The function $B(\xi)$ is determined appropriately from the method. Thus, the linear equations (3.5)–(3.6) and (3.17)–(3.20) are solved to find explicit expressions for all the unknowns involved.

4. Numerical results

For the computation of the numerical results, a gap parameter μ is introduced such that $a_2 = b_1 + h(1 - \mu/2)$ and $b_2 = b_1 + h(1 + \mu/2)$ with $h + b_1 = (a_2 + b_2)/2$. Then h is the mean depth of the second gap from the first gap, and μ is the ratio of the second gap width to h with $0 \le \mu \le 2$.

In Figure 2, the reflection coefficient is plotted against the nondimensional wavenumber Kh in the case of the vertical solid barrier with two gaps in it. The reference (a_1, b_1) of the first gap on the barrier is fixed at various depths, and μ is taken from the set $\{0, 0.4, 0.8, 1.2, 1.8\}$, where $\mu = 0$ refers to the absence of a second

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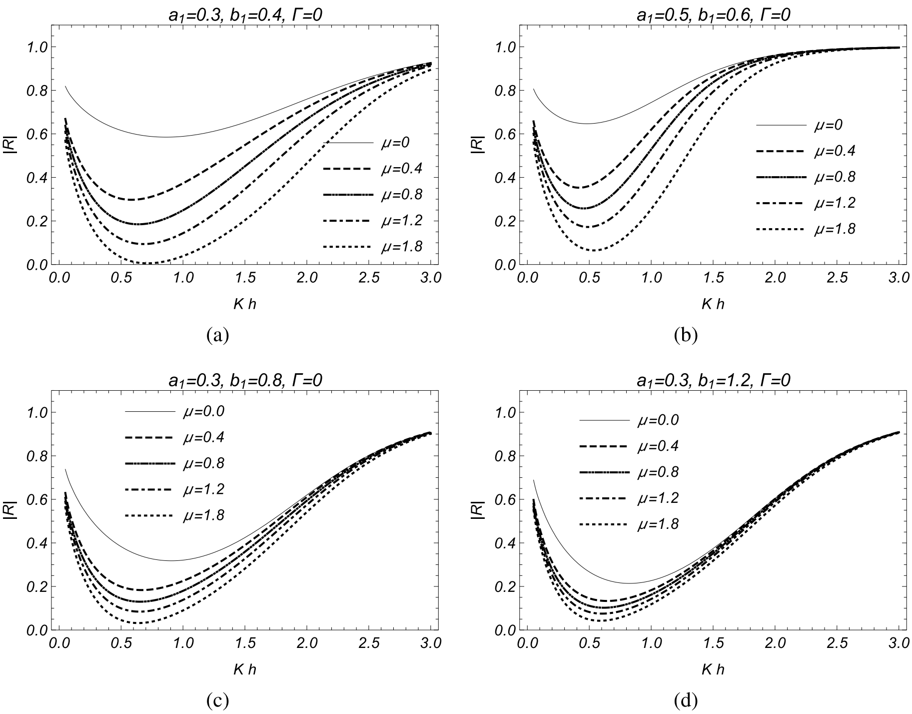

FIGURE 2. Reflection coefficient $|R|$ against Kh.

gap in the vertical barrier. As expected, an introduction of the second gap, that is, $\mu > 0$, induces transmission of waves through the gap, which causes a decrease in the reflection coefficient as shown in Figure 2(a)–(d). It is observed from Figure 2(a) that zero reflection occurs near $Kh = 0.6$ when the first gap is fixed at $(0.3, 0.4)$ and $\mu = 1.8$. It shows that the second gap plays a significant role in the transmission of waves when the first gap is relatively small and it is near the free surface. Moreover, a critical wavenumber exists at which complete transmission is possible when μ is sufficiently large and the first gap near the surface is sufficiently small. It may be seen from Figure 2(a) and (b) that the reflection of long waves increases when the depth of submergence of the first gap (a_1, b_1) increases from $(0.3, 0.4)$ to $(0.5, 0.6)$, while the reflection of very short waves is unaffected by the position of the gaps in the barrier. This may be attributed to the fact that wave energy mostly concentrates near the surface. This fact can also be observed in Figure 2(a), (c) and (d), since an increase in the width of the first gap causes a uniform reduction in the reflection across all wavenumbers. Moreover, the presence of the second gap is insignificant, as expected, when the first gap in the barrier is sufficiently large.

In Figure 3, reflection and total energy curves are plotted against Kh for the case of the vertical porous barrier with two gaps in it when the first gap $(a_1, b_1) = (0.3, 0.4)$. In Figure 3(a) and (b), reflection coefficients are plotted for various values of μ when

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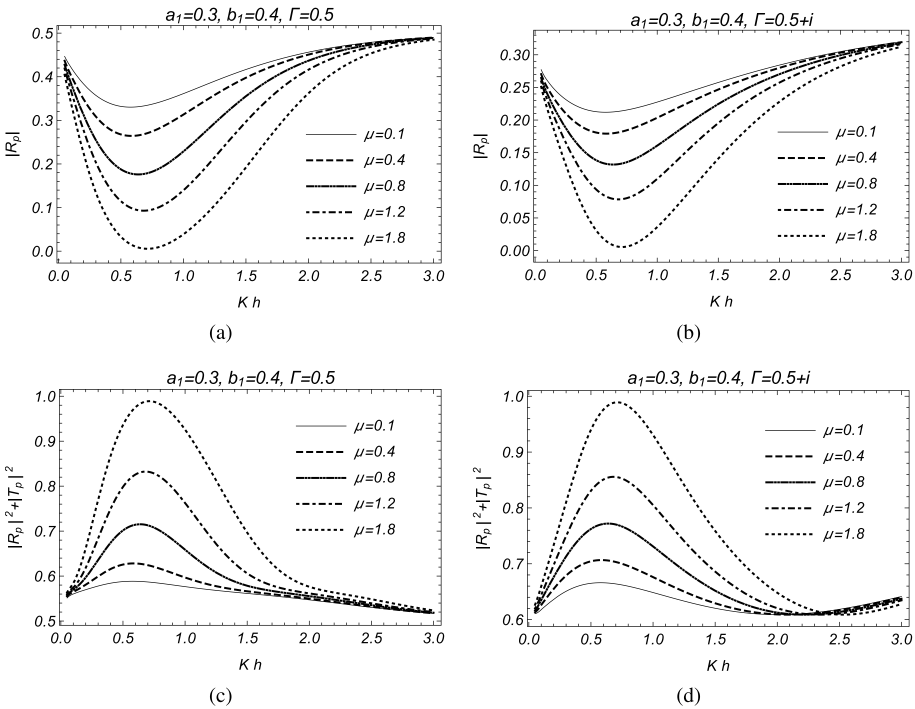

FIGURE 3. Reflection coefficient $|R_p|$ and total energy $|R_p|^2 + |T_p|^2$ against Kh.

the porous effect parameter $\Gamma = 0.5$ and $\Gamma = 0.5 + i$. Note that the overall reflection reduces due to the resistance and the inertial effects of the porous barrier. Energy curves are shown in Figure 3(c) and (d). A significant dissipation of wave energy is seen for moderate values of the gap parameter μ. More than 30% or 40% of wave energy has been dissipated for all $Kh > 1.6$ when Γ is 0.5 or $0.5 + i$. In other words, the inertial effect causes significant loss of energy in short waves. Interestingly, the energy dissipation is relatively smaller for the intermediate range of $0.5 < Kh < 1.0$ when the second gap is larger.

In Figures 4 and 5, the inertial and the resistance effects of the porous barrier on the reflection and the total energy curves are shown. Here the first gap $(a_1, b_1) = (0.3, 0.4)$ and $\mu = 0.4$ or 1.2 are fixed. For the moderate value of $\Gamma = 0.5$, we observe that 25% of the incident wave energy is getting reflected by the porous barrier at most of the frequencies, and double the amount of the energy is getting dissipated by the porous barrier. As the resistance effect in the barrier increases, the ability of the barrier to reflect the incident wave energy goes down while its ability to dissipate the wave energy goes up. The magnitude of reduction in the reflected energy is the same as the magnitude of dissipation in the total energy. However, the magnitude of the energy dissipation is significantly higher as compared to that of the wave reflection by the inertial effects of the porous barrier.

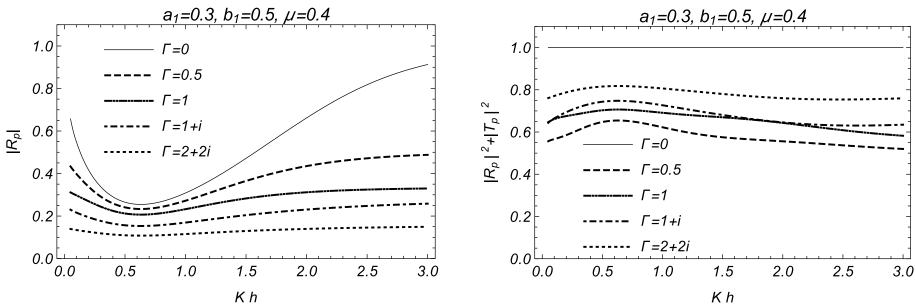

FIGURE 4. Reflection coefficient $|R_p|$ and total energy $|R_p|^2 + |T_p|^2$ against Kh when $\mu = 0.4$.

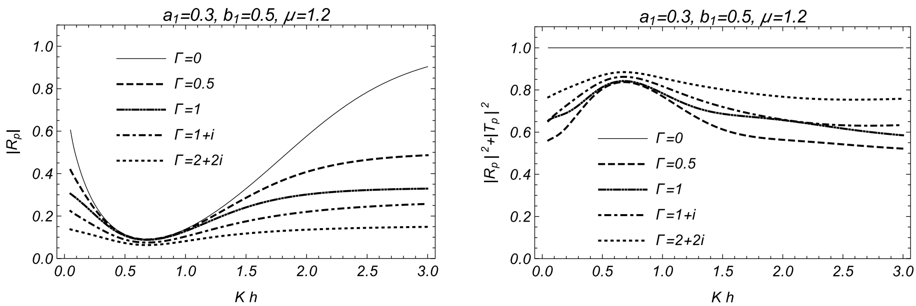

FIGURE 5. Reflection coefficient $|R_p|$ and total energy $|R_p|^2 + |T_p|^2$ against Kh when $\mu = 1.2$.

5. Conclusions

Scattering of water waves by a complete vertical porous barrier with two gaps of finite length is solved analytically. The required porous wave potential is explicitly obtained by decomposing the original problem into two resolvable problems. They are associated with scattering or bi-directional radiation of waves by a solid barrier of the same configuration. The decomposed problems are solved explicitly by the aid of a tested weakly singular integral equation method. The scattering quantities of the original problem are explicitly found in terms of the similar quantities involved in the decomposed problems. We also present numerical results for the reflection and the total energy. We find that the position of the second gap of the barrier does play a significant role in the wave reflection when the first gap is relatively small and is placed near the free surface for waves of moderate frequency. The present method can be extended to scattering problems involving a complete vertical porous barrier with many gaps or many barriers of finite length placed in a vertical line.

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Appendix

$$u(x) = -\int_{a_1}^{b_1} \frac{tq_1(t)}{T_1(t)(t^2-x^2)}\,dt + \int_{a_2}^{b_2} \frac{tq_1(t)}{T_1(t)(t^2-x^2)}\,dt,$$

$$\begin{aligned}
r_{11} &= -\frac{4i}{\pi^2}\int_0^{a_1} tu(t)\sinh(Kt)\,dt + \frac{2i}{\pi^2}\int_{(b_1,a_2)\cup(b_2,\infty)} tu(t)e^{-Kt}\,dt \\
&\quad + \frac{1}{\Delta_1}\int_G \frac{(\alpha_2 t^5 - \alpha_1 t^3)}{S_1(t)}\,dt + \frac{1}{\pi}\int_{a_1}^{b_1}\frac{te^{-Kt}q_1(t)}{T_1(t)}\,dt - \frac{1}{\pi}\int_{a_2}^{b_2}\frac{te^{-Kt}q_1(t)}{T_1(t)}\,dt,\\
r_{12} &= -r_{11} + \frac{2}{\pi}\int_{a_1}^{b_1}\frac{te^{-Kt}q_1(t)}{T_1(t)}\,dt - \frac{2}{\pi}\int_{a_2}^{b_2}\frac{te^{-Kt}q_1(t)}{T_1(t)}\,dt \\
&\quad + \frac{1}{\Delta_1}\int_G \frac{(\alpha_2 t^5 - \alpha_1 t^3)}{S_1(t)}\,dt + \frac{1}{\Delta_1}\int_G \frac{(\hat{\alpha}_2 t^5 - \hat{\alpha}_1 t^3)}{S_1(t)}\,dt,\\
r_{13} &= -\frac{4}{\pi^2}\int_{b_1}^{a_2} tu(t)e^{Kt}\,dt + \frac{1}{\Delta_1}\int_G \frac{(\beta_2 t^5 - \beta_1 t^3)}{S_1(t)}\,dt,\\
r_{14} &= -\frac{2K}{\pi}\int_{a_1}^{b_1}\frac{te^{Kt}q_1(t)}{T_1(t)}\,dt + \frac{1}{\Delta_1}\int_G \frac{(\gamma_2 t^5 - \gamma_1 t^3)}{S_1(t)}\,dt,\\
r_{15} &= \frac{2K}{\pi}\int_{a_2}^{b_2}\frac{te^{Kt}q_1(t)}{T_1(t)}\,dt + \frac{1}{\Delta_1}\int_G \frac{(\zeta_2 t^5 - \zeta_1 t^3)}{S_1(t)}\,dt,
\end{aligned}$$

$$v(x) = -\int_{a_1}^{b_1} \frac{t}{T_1(t)}\frac{dt}{(t^2-x^2)} + \int_{a_2}^{b_2} \frac{t}{T_1(t)}\frac{dt}{(t^2-x^2)},$$

$$\begin{aligned}
r_{21} &= -\frac{4i}{\pi^2}\int_0^{a_1} tv(t)\sinh(Kt)\,dt + \frac{2i}{\pi^2}\int_{(b_1,a_2)\cup(b_2,\infty)} tv(t)e^{-Kt}\,dt \\
&\quad + \frac{1}{\Delta_1}\int_G \frac{(\alpha_1 t - \alpha_2 t^3)}{S_1(t)}\,dt + \frac{1}{\pi}\int_{a_1}^{b_1}\frac{te^{-Kt}}{T_1(t)}\,dt - \frac{1}{\pi}\int_{a_2}^{b_2}\frac{te^{-Kt}}{T_1(t)}\,dt,\\
r_{22} &= -r_{11} + \frac{2}{\pi}\int_{a_1}^{b_1}\frac{te^{-Kt}}{T_1(t)}\,dt - \frac{2}{\pi}\int_{a_2}^{b_2}\frac{te^{-Kt}}{T_1(t)}\,dt \\
&\quad + \frac{1}{\Delta_1}\int_G \frac{(\alpha_1 t - \alpha_2 t^3)}{S_1(t)}\,dt + \frac{1}{\Delta_1}\int_G \frac{(\hat{\alpha}_1 t - \hat{\alpha}_2 t^3)}{S_1(t)}\,dt,\\
r_{23} &= -\frac{4}{\pi^2}\int_{b_1}^{a_2} tv(t)e^{Kt}\,dt + \frac{1}{\Delta_1}\int_G \frac{(\beta_1 t - \beta_2 t^3)}{S_1(t)}\,dt,\\
r_{24} &= -\frac{2K}{\pi}\int_{a_1}^{b_1}\frac{te^{Kt}}{T_1(t)}\,dt + \frac{1}{\Delta_1}\int_G \frac{(\gamma_1 t - \gamma_2 t^3)}{S_1(t)}\,dt,\\
r_{25} &= \frac{2K}{\pi}\int_{a_2}^{b_2}\frac{te^{Kt}}{T_1(t)}\,dt + \frac{1}{\Delta_1}\int_G \frac{(\zeta_1 t - \zeta_2 t^3)}{S_1(t)}\,dt,
\end{aligned}$$

$$\hat{u}(x) = -\int_{a_1}^{b_1} \frac{tT_1(t)q_2(t)}{(t^2-x^2)}\,dt + \int_{a_2}^{b_2} \frac{tT_1(t)q_2(t)}{(t^2-x^2)}\,dt,$$

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$$r_{31} = -\frac{4i}{\pi^2}\int_0^{a_1} t\hat{u}(t)\sinh(Kt)\,dt + \frac{2i}{\pi^2}\int_{(b_1,a_2)\cup(b_2,\infty)} t\hat{u}(t)e^{-Kt}\,dt$$
$$+ \frac{1}{\Delta_1}\int_G \frac{(\alpha_2 t^5 - \alpha_1 t^3)}{S_1(t)}\,dt + \frac{1}{\pi}\int_{a_1}^{b_1} tT_1(t)e^{-Kt}q_2(t)\,dt$$
$$- \frac{1}{\pi}\int_{a_2}^{b_2} tT_1(t)e^{-Kt}q_2(t)\,dt,$$
$$r_{32} = -r_{31} + \frac{2}{\pi}\int_{a_1}^{b_1} tT_1(t)e^{-Kt}q_2(t)\,dt - \frac{2}{\pi}\int_{a_2}^{b_2} tT_1(t)e^{-Kt}q_2(t)\,dt$$
$$+ \frac{1}{\Delta_1}\int_G \frac{(\alpha_2 t^5 - \alpha_1 t^3)}{S_1(t)}\,dt + \frac{1}{\Delta_1}\int_G \frac{(\hat{\alpha}_2 t^5 - \hat{\alpha}_1 t^3)}{S_1(t)}\,dt,$$
$$r_{33} = -\frac{4}{\pi^2}\int_{b_1}^{a_2} t\hat{u}(t)e^{Kt}dt + \frac{1}{\Delta_1}\int_G \frac{(\beta_2 t^5 - \beta_1 t^3)}{S_1(t)}\,dt,$$
$$r_{34} = -\frac{2K}{\pi}\int_{a_1}^{b_1} tT_1(t)e^{Kt}q_2(t)\,dt + \frac{1}{\Delta_1}\int_G \frac{(\gamma_2 t^5 - \gamma_1 t^3)}{S_1(t)}\,dt,$$
$$r_{35} = \frac{2K}{\pi}\int_{a_2}^{b_2} tT_1(t)e^{Kt}q_2(t)\,dt + \frac{1}{\Delta_1}\int_G \frac{(\zeta_2 t^5 - \zeta_1 t^3)}{S_1(t)}\,dt,$$
$$\hat{v}(x) = -\int_{a_1}^{b_1} \frac{tT_1(t)}{(t^2 - x^2)}\,dt + \int_{a_2}^{b_2} \frac{tT_1(t)}{(t^2 - x^2)}\,dt,$$
$$r_{41} = -\frac{4i}{\pi^2}\int_0^{a_1} t\hat{v}(t)\sinh(Kt)\,dt + \frac{2i}{\pi^2}\int_{(b_1,a_2)\cup(b_2,\infty)} t\hat{v}(t)e^{-Kt}\,dt$$
$$+ \frac{1}{\Delta_1}\int_G \frac{(\alpha_1 t - \alpha_2 t^3)}{S_1(t)}\,dt + \frac{1}{\pi}\int_{a_1}^{b_1} tT_1(t)e^{-Kt}\,dt - \frac{1}{\pi}\int_{a_2}^{b_2} tT_1(t)e^{-Kt}\,dt,$$
$$r_{42} = -r_{41} + \frac{2}{\pi}\int_{a_1}^{b_1} tT_1(t)e^{-Kt}\,dt - \frac{2}{\pi}\int_{a_2}^{b_2} tT_1(t)e^{-Kt}\,dt$$
$$+ \frac{1}{\Delta_1}\int_G \frac{(\alpha_1 t - \alpha_2 t^3)}{S_1(t)}\,dt + \frac{1}{\Delta_1}\int_G \frac{(\hat{\alpha}_1 t - \hat{\alpha}_2 t^3)}{S_1(t)}\,dt,$$
$$r_{43} = -\frac{4}{\pi^2}\int_{b_1}^{a_2} t\hat{v}(t)e^{Kt}\,dt + \frac{1}{\Delta_1}\int_G \frac{(\beta_1 t - \beta_2 t^3)}{S_1(t)}\,dt,$$
$$r_{44} = -\frac{2K}{\pi}\int_{a_1}^{b_1} tT_1(t)e^{Kt}\,dt + \frac{1}{\Delta_1}\int_G \frac{(\gamma_1 t - \gamma_2 t^3)}{S_1(t)}\,dt,$$
$$r_{45} = \frac{2K}{\pi}\int_{a_2}^{b_2} tT_1(t)e^{Kt}\,dt + \frac{1}{\Delta_1}\int_G \frac{(\zeta_1 t - \zeta_2 t^3)}{S_1(t)}\,dt,$$
$$\kappa_1(x) = 3(x^2 - C)T(x) + \frac{3(C_1 + x^2C_2)}{S_1(x)}, \quad \kappa_2(x) = T(x) + \frac{(L_1 + x^2L_2)}{S_1(x)},$$
$$\Delta_1 = \int_G \frac{u}{S_1(u)}\,du\int_G \frac{u^5}{S_1(u)}\,du - \Big(\int_G \frac{u^3}{S_1(u)}\,du\Big)^2,$$

$$
\begin{aligned}
\alpha_j &= -\frac{i}{\pi}\int_G \kappa_j(x)\int_0^{a_1} \sinh(Kt)\log\left|\frac{x+t}{x-t}\right| dt\,dx \\
&\quad + \frac{i}{2\pi}\int_G \kappa_j(x)\int_{(b_1,a_2)\cup(b_2,\infty)} e^{-Kt}\log\left|\frac{x+t}{x-t}\right| dt\,dx + \frac{1}{2K}\int_G \kappa_j(t)e^{-Kt}dt, \\
\hat{\alpha}_j &= -\alpha_j + \frac{1}{K}\int_G \kappa_j(t)e^{-Kt}\,dt, \\
\beta_j &= -\frac{1}{\pi}\int_G \kappa_j(x)\int_{b_1}^{a_2} e^{Kt}\log\left|\frac{x+t}{x-t}\right| dt\,dx, \\
\gamma_j &= \int_{a_1}^{b_1} \kappa_j(t)e^{Kt}\,dt, \quad \zeta_j = \int_{a_2}^{b_2} \kappa_j(t)e^{Kt}\,dt, \quad j = 1, 2, \\
s_1 &= r_{11} - \frac{1}{\pi}\int_{a_1}^{b_1} \frac{te^{-Kt}q_1(t)}{T_1(t)}\,dt + \frac{1}{\pi}\int_{a_2}^{b_2} \frac{te^{-Kt}q_1(t)}{T_1(t)}\,dt, \\
s_2 &= r_{21} - \frac{1}{\pi}\int_{a_1}^{b_1} \frac{te^{-Kt}}{T_1(t)}\,dt + \frac{1}{\pi}\int_{a_2}^{b_2} \frac{te^{-Kt}}{T_1(t)}\,dt, \\
s_3 &= r_{31} - \frac{1}{\pi}\int_{a_1}^{b_1} tT_1(t)e^{-Kt}q_2(t)\,dt + \frac{1}{\pi}\int_{a_2}^{b_2} tT_1(t)e^{-Kt}q_2(t)\,dt, \\
s_4 &= r_{41} - \frac{1}{\pi}\int_{a_1}^{b_1} tT_1(t)e^{-Kt}\,dt + \frac{1}{\pi}\int_{a_2}^{b_2} tT_1(t)e^{-Kt}\,dt.
\end{aligned}
$$

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