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Wind Acceleration in Pulsar Magnetospheres

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Abstract. To explain the pulsar wind acceleration and the energy conversion from the electromagnetic energy to the plasma's kinetic energy between the pulsar and the nebula's shock, we study relativistic cold ideal MHD winds in non-radial magnetic field geometry and clarify the critical conditions of trans-fast MHD winds.

1. Introduction

The luminosity of the Crab nebular is compatible to the rotational energy loss rate estimated from the observed spin down rate of the central pulsar. It is clear that pulsar wind streams from the central pulsar to the surrounding nebula. There are direct observational evidences that wind or jet exists around several pulsars.

The pulsar-nebula interaction has been investigated by many authors. Recent progression of the pulsar magnetosphere theories gives some restrictions on pulsar wind models. That is, pair-creation models on the polar cap restrict initial wind velocity or initial σ-value, where σ is the ratio between the Poynting flux and the particle energy flux and the initial σ value is very large because of strong magnetic field. On the other hand, synchrotron nebula shock models also restrict the terminal velocity of the wind and require the weak magnetization of the wind. Then, we believe that the pulsar wind should be a trans-fast MHD wind, which is ejected from a plasma source located near the pulsar and goes away to the nebula after accelerating at somewhere in the magnetosphere.

2. Pulsar Magnetosphere

In a pulsar magnetosphere, the magnetically dominated wind ejected from the pair-creation region must be accelerated by $\boldsymbol{J}\times\boldsymbol{B}$ force on the way to the nebula shock, where the kinetic energy dominated wind is required (Kennel & Coronity 1984). The previous many theories have been discussed in a radial field configuration (e.g., Michel 1969, Okamoto 1978, Camenzind 1986b, Takahashi 1991).

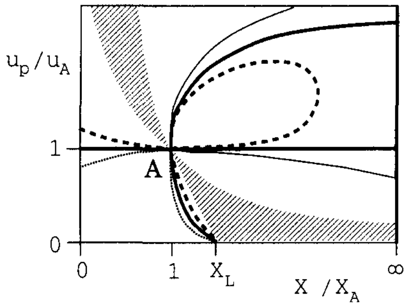

Figure 1. The $N = 0$ curves on the (x, u_p)-plane. Thick curves, dotted curves and thin curves denote the cases of $\Phi' = 0$, $\Phi' < 0$ and $\Phi' > 0$. The hatched regions are forbidden regions of wind solutions.

However, it was shown that in the radial magnetic field the expected pulsar wind acceleration and energy conversion do not obtained. The Mickel's minimum energy solution gives an upper-limit of terminal velocity with the magnetization parameter σ_∞.

To be clear the energy transport from the initial magnetically dominated wind to the final particle energy dominated wind, we study relativistic ideal MHD winds by assuming *non-radial* field geometries. The critical condition at the fast point is analyzed and whether the Poynting flux dominated wind is converted into the particle dominated wind is discussed in a stationary and axisymmetric pulsar magnetosphere.

3. Trans-fast MHD winds

For a stationary and axisymmetric ideal MHD flows, the total energy E, the total angular momentum L, the angular velocity Ω_F of magnetic field lines and the particle number flux per magnetic tube conserve along a magnetic field line, which gives a stream line of plasma (e.g., Camenzind 1986a). The poloidal wind equation is given by

$$(u_p)' = \frac{N(x, u_p; \Phi; \tilde{L})}{D(x, u_p; \Phi; \eta, \tilde{L})} , \tag{1}$$

where we have defined $(') \equiv d/dx$ and $\Phi \equiv x^2 B_p$. The functions N and D are expressed in terms of the radius $x (\equiv r\Omega_F/c)$ from the rotation axis, the poloidal 4-velocity $u_p (\equiv \sqrt{u_x^2 + u_z^2})$, the poloidal magnetic field $B_p (\equiv \sqrt{B_x^2 + B_z^2})$ and the conserved quantities η and $\tilde{L} (\equiv L/E)$. The intersections of the $N = 0$ curve and the $D = 0$ curve on (x, u_p)-plane denote the Alfvén point (x_A, u_A) and the fast point (x_F, u_F). Though this equation presents the pulsar wind solution, the wind parameters are restricted at the Alfvén point and the fast point, where $D = 0$ and the condition $N = 0$ is also required at the same points.

Figure 1 shows the $N = 0$ curves for cases $\Phi' < 0$, $\Phi' = 0$ and $\Phi' > 0$. In the case of $\Phi' < 0$, we see a loop of $N = 0$ curve on (x, u_p)-plane, and we can show that at least one intersection must exist between the Alfvén point and infinity as

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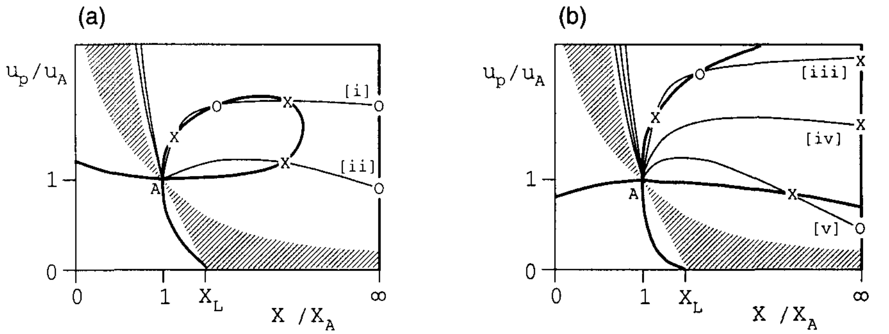

Figure 2. Variation of the $D=0$ curve (thin curve) with the $N=0$ curves (thick curves). Location of the fast points is depend on the η-values and the field configuration; (a) $\Phi' < 0$ and (b) $\Phi' > 0$. For larger η-value, the $D = 0$ curve labeled [i] or [iii] makes crossings with the $N = 0$ curve, where marks 'X' and 'O' denote X-type (physical) and O-type (unphysical) fast points, but this X-point presents a disk wind. In contrast to this, for small η-value, the $D = 0$ curve labeled [ii] makes a crossing with the $N = 0$ curve, this point can give a reasonable pulsar wind solution (see Figure 3). The curve labeled [v] makes a decelerated trans-fast wind solution. The wind solution related to the curve [iv] makes a 'Michel-type' wind solution (The fast point only locates at infinity).

seen in Figure 2a (see Takahashi 1993, Takahashi & Shibata 1996). This means that the trans-fast MHD wind is always available for the field geometry satisfying $\Phi' < 0$, while it is no necessary to create the crossing at a finite distance for $\Phi' \geq 0$ (see curve [iv] in Figure 2b). We can also show that the intersection for the case of $\Phi' \geq 0$ only gives a constant or decelerated wind solution and a disk wind solution injected from the plasma source located at far outside the star surface, which are not acceptable as a pulsar wind solution (see Takahashi 1991 for the wind with $\Phi' = 0$).

4. Conclusions

We researched restrictions on the wind parameters by analyzing the critical condition at the fast point. Then, we found condition of field lines to generate the X-type fast point at a finite distance; that is, for field configurations $\Phi' < 0$, the accelerated trans-fast pulsar wind can be realized (This results have already been pointed out by Begelman & Li 1994), and the highly accelerated wind is obtained for very small injection rate. Figure 3 demonstrates the pulsar wind solution. Because of the strong magnetic field, the Alfvén point locates very close to the light cylinder. We see that the huge acceleration occurs around the fast magnetsonic point.

We can also demonstrate the energy conversion in the pulsar magnetosphere (see Takahashi & Shibata 1996). In cases of $\delta \sim O(1)$, where δ is defined by

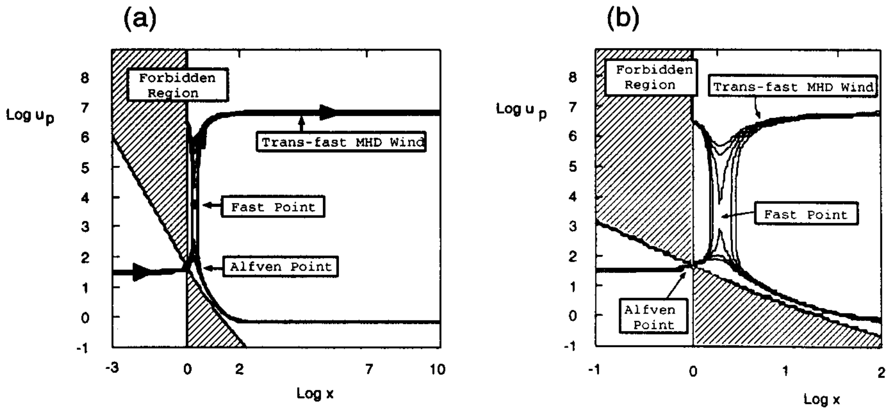

Figure 3. Trans-fast MHD wind ($x_A = 0.999999$). The magnetic field line is given by $z = Cx^{1.5}$, which satisfies $\Phi' < 0$. The fast point is allowed only near the Alfvén point; we set $x_F = 1.985299$.

$B_p \propto x^{-(2+\delta)}$ ($\delta > 0$), we can obtain a reasonable energy conversion in the pulsar wind, which occurs beyond the fast point. The converted kinetic energy is cased by the toroidal motion of plasma. (We should note that, beyond the fast point, the poloidal velocity has already reached the terminal velocity as seen in Figure 3).

Unfortunately, we do not know the magnetic field configuration from observation directly. However, for pulsar's parameters estimated from some observations, we will be able to explain both the wind acceleration and the conversion from the electromagnetically dominated wind to the particle kinetic energy dominated wind, which is require from both the pair-creation models and the synchrotron nebular models.

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