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CERTAIN GENERALIZATIONS OF PRESTARLIKE FUNCTIONS

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Abstract

The classes of prestarlike functions R_α, $\alpha \geqslant -1$, were studied recently by St. Ruscheweyh. The author generalizes and extends these classes. In particular the author obtains the radius of $R_{\alpha+1}$ for the class R_α, $\alpha \geqslant -1$.

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1. Introduction

Let A denote the set of analytic functions $f(z)$ in the unit disc E: $\{|z|<1\}$ normalized by $f(0)=0, f'(0)=1$. Let

$$D^\alpha f(z) = f(z) * \frac{z}{(1-z)^{\alpha+1}}, \quad \alpha \geqslant -1, \tag{1.1}$$

where $*$ denotes the Hadamard product (convolution) of two analytic functions in E. A function $f \in A$ is called *prestarlike* of order α, $\alpha \geqslant -1$, if and only if

$$\operatorname{Re} \frac{D^{\alpha+1} f(z)}{D^\alpha f(z)} > \frac{1}{2}, \quad z \in E. \tag{1.2}$$

We let R_α stand for the collection of prestarlike functions of order α.

Note that R_0 and R_1 are known classes of univalent functions that are starlike of order $\frac{1}{2}$ and convex respectively.

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Prestarlike functions, in a different parametrization, have already been studied by Ruscheweyh (1977) and also by Suffridge (1976). The subclasses of R_α where α is a non-negative integer were considered earlier by Ruscheweyh (1975).

Ruscheweyh (1977) obtained the basic relation

$$R_\alpha \subset R_\beta, \quad \alpha \geqslant \beta \geqslant -1. \tag{1.3}$$

Consequently all prestarlike functions of order α are univalent at least when $\alpha \geqslant 0$.

In this note we introduce the classes H_α where $f \in H_\alpha$ in $f \in A$ and if

$$\operatorname{Re} \frac{D^{\alpha+1} f(z)}{D^{\alpha+1} g(z)} > \frac{1}{2}, \quad z \in E, \quad \alpha \geqslant -1, \tag{1.4}$$

for some $g \in R_{\alpha+1}$. Note that when $\alpha = 0$, (1.4) shows that $\operatorname{Re}[f'(z)/g'(z)] > \frac{1}{2}$ for some convex function $g(z)$. Thus $f(z)$ is a univalent close-to-convex function (Kaplan (1952)).

In Section 2 we shall show that $H_{\alpha+1} \subset H_\alpha$, $\alpha \geqslant -1$. This particular result implies univalency of these classes at least when $\alpha = 0, 1, 2, \ldots$. In Section 3 we study special elements of R_α and H_α which have certain integral representation. In Sections 4 and 5 we consider the converse problems of the results of Sections 2 and 3. In particular the radius of $R_{\alpha+1}$ in R_α, $\alpha \geqslant -1$, is determined. Section 6 is devoted to further extensions and generalizations of the classes R_α and H_α along the concept of alpha-convex functions as introduced by Mocanu (1969).

2. The classes H_α

We shall prove the following

THEOREM 1. $H_{\alpha+1} \subset H_\alpha$, $\alpha \geqslant -1$.

PROOF. Let $f \in H_{\alpha+1}$ and $g \in R_{\alpha+2}$ be its associate function, see (1.4). Define $w(z)$ by

$$\frac{D^{\alpha+1} f(z)}{D^{\alpha+1} g(z)} = \frac{1}{1+w(z)}. \tag{2.1}$$

Here $w(z)$ is a regular function in E with $w(0) = 0$ and $w(z) \neq -1$ for $z \in E$. Since by (1.1) $g \in R_{\alpha+2}$ then $g \in R_{\alpha+1}$ it suffices to show that $|w(z)| <|$, $z \in E$.

Taking the logarithmic derivative of both sides of (2.1) and utilizing the identity

$$\frac{z}{(1-z)^{\alpha+2}} = \frac{z}{(1-z)^{\alpha+1}} * \left[\frac{\alpha}{\alpha+1}\frac{z}{1-z} + \frac{1}{\alpha+1}\frac{z}{(1-z)^2}\right], \quad \alpha > -1, \tag{2.2}$$

one gets

$$\frac{D^{\alpha+2}f(z)}{D^{\alpha+2}g(z)}=\frac{1}{1+w(z)}-\frac{zw'(z)}{(\alpha+2)(1+w(z))^2}\frac{D^{\alpha+1}g(z)}{D^{\alpha+2}g(z)}. \tag{2.3}$$

Equations (2.3) should yield $|w(z)|<1$ for all $z\in E$, for otherwise by a lemma of Jack (1971) there exists $z_0\in E$ such that $z_0 w'(z_0)=mw(z_0)$, $|w(z_0)|=1$ and $m\geqslant 1$. Applying this result to (2.3) we get

$$\frac{D^{\alpha+2}f(z_0)}{D^{\alpha+2}g(z_0)}=\frac{1}{1+w(z_0)}-\frac{mw'(z_0)}{(\alpha+2)(1+w(z_0))^2}\frac{D^{\alpha+1}g(z_0)}{D^{\alpha+2}g(z_0)}.$$

Since

$$\operatorname{Re}\frac{1}{1+w(z_0)}=\frac{1}{2},\quad \operatorname{Re}\frac{D^{\alpha+2}g(z_0)}{D^{\alpha+1}g(z_0)}>\frac{1}{2}$$

and $w(z_0)/(1+w(z_0))^2$ is real and positive, we conclude that

$$\operatorname{Re}\frac{D^{\alpha+2}f(z_0)}{D^{\alpha+2}g(z_0)}<\frac{1}{2}.$$

This is a contradiction to the assumption that $f\in R_{\alpha+1}$. Hence $f\in H_\alpha$ when $\alpha>-1$.

The case $\alpha=-1$ is simple. Since

$$f\in H_0\Rightarrow \operatorname{Re}\frac{f'(z)}{g'(z)}>\frac{1}{2}$$

for some

$$g\in R_1\,(\text{convex})\Rightarrow \operatorname{Re}\frac{f(z)}{g(z)}>\frac{1}{2}$$

by a generalization of a lemma due to Libera (1965). Thus $f\in H_{-1}$. This complete the proof of Theorem 1.

REMARK 1. Theorem 1 provides a partial answer to a much deeper problem of whether $H_\alpha\subset H_\beta$ for all $\alpha\geqslant\beta\geqslant -1$.

REMARK 2. Another interesting problem is that of determining $\alpha_0=\inf\alpha$ where every $f\in R_\alpha$ is univalent in E. It is clear that $-1\leqslant\alpha_0\leqslant 0$.

3. Special elements of R_α and H_α

Let

$$h_\gamma(z)=\sum_{j=1}^{\infty}\frac{\gamma+1}{\gamma+j}z^j,\quad \operatorname{Re}\gamma\geqslant\tfrac{1}{2}\alpha,\quad \operatorname{Re}\gamma>-1. \tag{3.1}$$

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The following is a straightforward extension of Ruscheweyh (1975), Theorem 5.

THEOREM 2. *Let* $\operatorname{Re}\gamma \geqslant \frac{1}{2}(\alpha-1)$, $\gamma \neq -1$. *Then* $F \in R_\alpha$ *where*

$$F(z) = f(z) * h_\gamma(z) = \frac{1+\gamma}{z^\gamma}\int_0^z t^{\gamma-1} f(t)\,dt$$

and $f \in R_\alpha$. *In particular* $h_\gamma(z)$ *(as given by* (3.1)*) are elements of* R_α.

We state without proof the following extension of Theorem 2, since its proof uses basically the same method that we employed in the proof of Theorem 1.

THEOREM 3. *Let* $\operatorname{Re}\gamma \geqslant \frac{1}{2}\alpha$ *and let* $f \in H_\alpha$. *Then* $F \in H_\alpha$ *where*

$$F(z) = f(z) * h_\gamma(z) = \frac{1+\gamma}{z^\gamma}\int_0^z t^{\gamma-1} f(t)\,dt.$$

REMARK 3. Theorems 2 and 3 extend and generalize some pioneering results of Libera (1965).

4. The radius of $R_{\alpha+1}$

In this section we raise the natural question of finding the largest disc E_r: $\{|z|<r\}$, $0<r\leqslant 1$, so that if $f \in R_\alpha$ then

$$\operatorname{Re}\frac{D^{\beta+1} f(z)}{D^\beta f(z)} > \frac{1}{2}, \quad \beta > \alpha, \quad z \in E_r.$$

Theorem 4 provides a partial answer to this rather interesting problem.

THEOREM 4. *Let* $f \in R_\alpha$, $\alpha \geqslant -1$. *Then*

$$\operatorname{Re}\frac{D^{\alpha+2} f(z)}{D^{\alpha+1} f(z)} > \frac{1}{2}$$

holds for $|z| < r_\alpha$ *where*

(4.1) $$r_\alpha = (\alpha + 2(\alpha+3)^{\frac{1}{2}})^{\frac{1}{2}}(\alpha+4+2(\alpha+3)^{\frac{1}{2}})^{-\frac{1}{2}}.$$

This result is sharp.

PROOF. For $f \in R_\alpha$, let $p(z)$ be the regular function defined in E by

(4.2) $$\frac{D^{\alpha+1} f(z)}{D^\alpha f(z)} = \frac{1}{2}(p(z)+1), \quad z \in E, \quad \alpha \geqslant -1.$$

Here $p(0) = 1$ and $\operatorname{Re} p(z) > 0$ in E.

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Logarithmic differentiation of (4.2) and the identity (2.2) should yield

$$\frac{D^{\alpha+2}f(z)}{D^{\alpha+1}f(z)}-\frac{1}{2}=\frac{1}{2(\alpha+2)}\left[1+(1+\alpha)p(z)+2\frac{zp'(z)}{p(z)+1}\right]. \tag{4.3}$$

In order to find r_α we need to determine the absolute minimum of the right-hand side of (4.3) as $p(z)$ varies in the disc $|p(z)-a|<\rho$ where

$$a=\frac{1+r^2}{1-r^2},\quad \rho=\frac{2r}{1-r^2},\quad |z|=r.$$

However, the right-hand side of (4.3), denoted by $\psi(w_1, w_2)$, $w_1 = p(z)$, $w_2 = zp'(z)$, is analytic in the w_2-plane and in the half-plane $\operatorname{Re} w_1 > 0$. Consequently by Robertson (1963) the $\min \operatorname{Re}\psi(p, zp')$ should occur for functions in the form

$$p(z)=\lambda_1\frac{1+e^{i\theta}z}{1-e^{i\theta}z}+\lambda_2\frac{1+e^{-i\theta}z}{1-e^{-i\theta}z},\quad \lambda_1+\lambda_2=1,\quad \lambda_1\geqslant 0,\quad \lambda_2\geqslant 0. \tag{4.4}$$

We may now use the technique of Zmorovič (1969) to conclude that

$$\min \operatorname{Re}\psi(p,zp')\equiv\psi_\rho(p)=\frac{1}{2}\left[\operatorname{Re}p(z)-\frac{1}{(\alpha+2)}\frac{\rho^2-\rho_0^2}{|p(z)+1|}\right], \tag{4.5}$$

where $\rho_0=|p(z)-a|<p$. Let $p(z)=a+\xi+i\eta$, $p(z)+1=Re^{i\varphi}$, $R^2=(a+\xi+1)^2+\eta^2$. Then we can readily show that $\min\psi_\rho(p)$ in the disc $|p-a|<\rho$ is achieved on the diameter of the circle $|p-a|=\rho$, that is when $\eta=0$. Thus our problem is reduced to minimizing $l(R)$ on the segment $a-\rho+1\leqslant R\leqslant a+\rho+1$, where

$$l(R)\equiv\psi_\rho(\xi)=\frac{1}{2(\alpha+2)}[(\alpha+3)R-(\alpha+4+2a)+2(1+a)R^{-1}]. \tag{4.6}$$

Clearly this minimum must occur for $R=\bar{R}$, where

$$\bar{R}=(2+2a)^{\frac{1}{2}}(\alpha+3)^{-\frac{1}{2}}. \tag{4.7}$$

While $\bar{R}<a+\rho+1$ is always true, $\bar{R}>a-\rho+1$ is valid only when

$$r>\frac{a+2}{\alpha+4}. \tag{4.8}$$

Consequently (4.6) and (4.7) show that $l(\bar{R})=0$ if

$$4a^2-4(\alpha+2)a+\alpha^2-8=0$$

which upon replacing a by $(1+r^2)/(1-r^2)$ we get r_α, as given by (4.1), as the smallest positive root. r_α in this case is the desired radius provided it satisfies (4.8). However, this is obviously true.

On the other hand, if $\bar{R}\leqslant a-\rho+1$ then the absolute minimum of $l(R)$ on the closed diameter occurs at $a-\rho+1$. In this case $l(a-\rho+1)=0$ shows that $r_\alpha=1$.

Because of what we have mentioned above $r_\alpha = 1$ would be the desired radius if $1 = r_\alpha \leqslant (\alpha+2)/(\alpha+4)$ which is impossible. Hence r_α as given by (4.1) is the radius of $R_{\alpha+1}$ for the class R_α, $\alpha \geqslant -1$.

To determine the extremal function $f_\theta(z)$ we note that (4.4) can be written in the form

$$p(z) = \lambda_1(a+\rho e^{i\psi_1}) + \lambda_2(a+\rho e^{i\psi_2}),$$

where $\psi_1 + \psi_2 \equiv 0 \pmod{2\pi}$. Hence

$$p(z) = a + \rho\cos\psi_1 + i\rho(\lambda_1 - \lambda_2)\sin\psi_1.$$

Since the minimum of $l(R)$ was realized at a point on the diameter which is not an endpoint, it then follows from the above that $\sin\psi_1 \neq 0$ while $\lambda_1 - \lambda_2 = 0$ or $\lambda_1 = \lambda_2$. Consequently the corresponding form of $p(z)$ for $f_0(z)$ is

$$p(z) = \frac{1}{2}\frac{1+ze^{i\theta}}{1-ze^{i\theta}} + \frac{1}{2}\frac{1+ze^{-i\theta}}{1-ze^{-i\theta}}.$$

Thus the extremal functions are rotations of $f_0(z)$ which is determined by

$$\frac{D^{\alpha+1}f_0(z)}{D^\alpha f_0(z)} = \frac{p(z)+1}{2} = \frac{1-z\cos\theta}{1-2z\cos\theta+z^2},$$

where $\cos\theta$ is the solution of

$$\bar{R} = \operatorname{Re}(p(z)+1) = 1 + (1-r_\alpha^2)(1-2r_\alpha\cos\theta + r_\alpha^2)^{-1},$$

r_α, $\bar{R}$ are given by (4.1) and (4.7), respectively. This completes the proof of this theorem.

REMARK 4. Interesting special cases correspond to $\alpha = 0, -1$. For $\alpha = 0$, $r_0 = (2.3^{\frac{1}{2}} - 3)^{\frac{1}{2}}$ is the radius of convexity for the class of starlike functions of order $\frac{1}{2}$. This is a well-known result due to MacGregor (1963).

5. Some converse theorems

In this section we consider the converse of Theorems 2 and 3.

THEOREM 5. *Let $F \in R_\alpha$, $\alpha \geqslant 0$ and $\Gamma = \operatorname{Re}\gamma > \frac{1}{2}(\alpha-1)$. Let $f(z)$ be the unique solution of $F(z) = h_\gamma(z) * f(z)$ where $h_\gamma(z)$ is given by* (3.1). *Then*

$$\operatorname{Re}\frac{D^{\alpha+1}f(z)}{D^\alpha f(z)} > \frac{1}{2}$$

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is valid in $|z|<r_{\alpha,\gamma}$, *when* $r_{\alpha,\gamma}$ *is the smallest positive root of*

$$(\Gamma-\alpha)r^2+(\alpha+3)r-\Gamma-1=0. \tag{5.1}$$

This result is sharp.

PROOF. Let $q(z)$ be the regular function in E defined by

$$\frac{D^{\alpha+1}F(z)}{D^{\alpha}F(z)}=q(z). \tag{5.2}$$

Here $q(0)=1$ and $\operatorname{Re} q(z)>\frac{1}{2}$ for $z\in E$. A simple generalization of a result in the proof of Ruscheweyh (1975), Theorem 5, shows that

$$\frac{D^{\alpha+1}f(z)}{D^{\alpha}f(z)}-\frac{1}{2}=q(z)-\frac{1}{2}+z\frac{q'(z)}{\gamma-\alpha(\alpha+1)q(z)}. \tag{5.3}$$

However, it is well known that for $|z|=r<1$

$$|zq'(z)|\leqslant\frac{2r}{1-r^2}(\operatorname{Re}q(z)-\tfrac{1}{2}). \tag{5.4}$$

Thus (5.3) and (5.4) give for $|z|=r$

$$\operatorname{Re}\left(\frac{D^{\alpha+1}f(z)}{D^{\alpha}f(z)}-\frac{1}{2}\right)\geqslant\operatorname{Re}\left(q(z)-\frac{1}{2}\right)\left(1-\frac{2r}{(1-r)(1+\Gamma+(\Gamma-\alpha)r)}\right). \tag{5.5}$$

Now the right-hand side of (5.5) is positive provided that $r<r_{\alpha,\gamma}$ where $r_{\alpha,\gamma}$ is the smallest positive root of (5.1).

For sharpness we consider $F(z)=z/(1-z)$. In this case

$$f(z)=\frac{z(1+\gamma-\gamma z)}{(1+\gamma)(1-z)^2}.$$

It is a simple matter to verify that

$$(z^{\gamma}D^{\alpha+1}F(z))'=(1+\gamma)z^{\gamma-1}D^{\alpha+1}f(z)$$

and

$$(z^{\gamma}D^{\alpha}F(z))'=(1+\gamma)z^{\gamma-1}D^{\alpha}f(z).$$

Using these for our special functions $f(z)$ and $F(z)$, we obtain

$$\begin{aligned}\frac{D^{\alpha+1}f(z)}{D^{\alpha}f(z)}&=\left(z^{\gamma}\frac{z}{(1-z)^{\alpha+2}}\right)'\Big/\left(z^{\gamma}\frac{z}{(1-z)^{\alpha+1}}\right)'\\&=\frac{1}{1-z}+\frac{z}{(1-z)(1+\gamma+(\alpha-\gamma)z)}\end{aligned}$$

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and consequently

$$\frac{D^{\alpha+1}f(z)}{D^{\alpha}f(z)}=\frac{1}{2} \quad \text{for } z=-r_{\alpha,\gamma}.$$

This completes the proof of this theorem.

Next theorem is a converse of Theorem 3 in the sense of Theorem 5. This theorem follows from (2.3) and Theorem 5 and we omit the proof.

THEOREM 6. *Let* $F\in H_{\alpha}$ *and* $G\in R_{\alpha+1}$ *be its associate function. Let* f *and* g *be the unique solution of* $F(z)=h_{\gamma}(z)*f(z)$ *and* $G(z)=h_{\gamma}(z)*g(z)$ *with* $\Gamma=\mathrm{Re}\,\gamma\geqslant\frac{1}{2}\alpha$, *respectively. Then*

$$\mathrm{Re}\frac{D^{\alpha+1}f(z)}{D^{\alpha+1}g(z)} \quad \frac{1}{2}$$

for $|z|<R_{\alpha,\gamma}$, *where* $R_{\alpha,\gamma}$ *is the smallest positive root of*

$$(\Gamma-\alpha-1)r^{2}+(\alpha+4)r-\Gamma-1=0.$$

This result is also sharp.

REMARK 5. Theorems 5 and 6 generalize and extend similar results of Livingston (1966).

6. Extensions of the classes R_{α} and H_{α}

In this section we extend the notion of prestarlikeness and of its generalization along the concept of alpha-convex functions as introduced by Mocanu (1969).

We say $f\in R_{\alpha}(\beta)$, $\alpha\geqslant-1$ if $f\in A$ and

$$(1-\beta)\,\mathrm{Re}\frac{D^{\alpha+1}f(z)}{D^{\alpha}f(z)}+\beta\,\mathrm{Re}\frac{D^{\alpha+2}f(z)}{D^{\alpha+1}f(z)}>\frac{1}{2}, \quad z\in E,$$

and for some $\beta\geqslant 0$.

We also say that $f\in H_{\alpha}(\beta)$, $\alpha\geqslant-1$, if $f\in A$ and if exists $g\in R_{\alpha+2}$ such that

$$(1-\beta)\,\mathrm{Re}\frac{D^{\alpha+1}f(z)}{D^{\alpha+1}g(z)}+\beta\,\mathrm{Re}\frac{D^{\alpha+2}f(z)}{D^{\alpha+2}g(z)}>\frac{1}{2}, \quad z\in E,$$

and for some $\beta\geqslant 0$.

A proof similar to that used in Theorem 1 should yield these results:

THEOREM 7. (a) $R_\alpha(\beta) \subset R_\alpha$, $\alpha \geqslant -1$, $\beta \geqslant 0$, (b) $H_\alpha(\beta) \subset H_\alpha$, $\alpha \geqslant -1$, $\beta \geqslant 0$.

When $\alpha = 0, 1, 2, \ldots$ part (a) was shown by Al-Amiri (1978). Part (b) reduces to Theorem 1 when $\beta = 1$.

Also using the technique of Theorem 4 one can easily prove:

THEOREM 8. *Let* $f \in R_\alpha$, $\alpha \geqslant -1$. *Then*

$$(1-\beta)\,\mathrm{Re}\,\frac{D^{\alpha+1}f(z)}{D^\alpha f(z)} + \beta\,\mathrm{Re}\,\frac{D^{\alpha+2}f(z)}{D^{\alpha+1}f(z)} > \frac{1}{2}$$

holds for $|z| < r_\alpha(\beta)$ *where* $r_\alpha(\beta)$ *is given by*

$$r_\alpha(\beta) = (\alpha+2-2\beta+2\Delta)^{\frac{1}{2}}(\alpha+2+2\beta+2\Delta)^{-\frac{1}{2}}, \quad \Delta = (\beta(\alpha+2+\beta))^{\frac{1}{2}}.$$

The result is sharp.

The case $\alpha = 0, 1, 2, \ldots$ was treated by Al-Amiri (1978). We also note here that Theorem 8 reduces to Theorem 4 when $\beta = 1$.

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