PART IV

THE ORIGIN OF STRUCTURE IN THE EXPANDING UNIVERSE

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THE FORMATION OF GALAXIES IN FRIEDMANNIAN UNIVERSES

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Abstract. A short review of the theory of the formation of galaxies and clusters of galaxies is given within the framework of the non-linear theory of gravitational instability. Probable initial conditions are discussed as well as processes which bring about the origin of galaxies and clusters of galaxies. Possible observational tests are discussed.

1. Introduction

Any theory of the formation of galaxies has to account for three groups of facts.

(1) The existence of galaxies and clusters of galaxies with densities 10^2 to 10^6 times the mean cosmological density and probably the existence of superclusters with densities some 2 or 3 times greater than the mean density. An important specific property of galaxies is their rotation [1, 2, and 3].

(2) The Hubble law for galaxies in the distance range 20 to 3000 Mpc (i.e. $0.03 < z < 0.5$) [2]; the absence of angular variations in the brightness temperature of the relic radiation (certainly $\Delta T/T < 10^{-3}$) [4, 5]. This second group of facts proves that departures from the Friedmann model are small, especially those on the largest scale and those in the recent past.

(3) The agreement of the relic radiation spectrum with the equilibrium Planck formula ($\Delta T/T < 0.05$) [6, 7]; the probable agreement of the primordial chemical composition with the theoretical predictions of Friedmann's theory [8, 9]; the absence of any strong annihilation radiation in the χ-ray background spectrum [10].

The first group of facts excludes the Friedmann cosmological solution with exact isotropy and uniformity (with uniform entropy) as a possible exact description of the Universe. The third group of facts extends the evidence that the perturbations were small even further back into the past, to $z \approx 10^{10}$, corresponding to a time approximately 1 s after the big bang.

2. The General Picture

It is only natural therefore to build a theory of the formation of galaxies beginning from the Friedmann solution with small perturbations which grow to sufficient amplitude at the moment when galaxies form. The gravitational (Jeans) instability gives the necessary mechanism of growth. The overall scheme consists of the following consecutive parts:

(1) Redshift $z > 1300$: small adiabatic perturbation (simultaneous perturbations of temperature and matter density with $3(\Delta T/T) = (\Delta \varrho_m/\varrho_m)$) superimposed on the standard cosmological solution for the hot model.

(2) Redshift $1300 > z > 10$. Neutral gas density perturbations grow but remain less than unity.

M. S. Longair (ed.), Confrontation of Cosmological Theories with Observational Data, 213–225.

(3) Redshift 4–5 $< z <$ 10. The formation of dense gas clouds due to further growth of the perturbations, ending with shock wave formation. The mass of the clouds is of the order of 10^{13} to 10^{15} $M_\odot$ (clusters of galaxies).

Among accompanying physical processes, the following should be mentioned:

(i) The formation of clouds is accompanied by their fragmentation into galaxies.

(ii) The rotational motion and perhaps even turbulence acquired in the shock wave explain the rotation of galaxies.

(iii) The birth of the first quasars.

(iv) The radiation of the shock wave, of the first quasars and of young galaxies ionizes the cold intergalactic gas which escaped shock wave compression and direct shock ionization.

(v) Perturbations on the largest scale (10^{16} to 10^{17} $M_\odot$ and greater) are relatively smaller; they are frozen with amplitude less than unity (probable before cloud encounters) at some $z \leqslant 3$. The corresponding peculiar velocities decay in the time interval $3 \geqslant z > 0$.

This behaviour of perturbations is characteristic for open cosmological models. For simplicity of presentation we present calculations made for a definite set of input data. Their possible range of variation will be considered later.

We adopt a Hubble constant of 50 km s^{-1} Mpc^{-1}, density parameter $\Omega = 2q_0 = \varrho/\varrho_c = 0.1$ corresponding to $\bar{\varrho} = 5 \times 10^{-31}$ g cm^{-3} and relic radiation temperature 2.7 K. All data are given for the present epoch. This choice leads to the redshifts of equality of radiation and matter density and of hydrogen recombination coinciding at $z = 1300$, $t_{\text{rec}} = 3 \times 10^{13}$ s. The cosmological model is open with radius $a = 10^4$ Mpc and age $t = 1.9 \times 10^{10}$ yr.

3. The Initial Spectrum and Evolution of Density Perturbations

The initial metric perturbations are assumed to be of amplitude $h = 10^{-4}$ (dimensionless) with a flat spectrum, independent of space scale. Using the classical results of Lifshitz [11, 24] on the general relativistic theory of perturbations, perturbations of density are calculated. In the radiation-dominated era $z > 1300$ they are given by three different formulae for three ranges of scale.

The scale is characterized by masses $M = (4\pi/3)\bar{\varrho}_0\kappa^{-3}$ or $M = (1/6\pi^2)\bar{\varrho}l^3$; $l = 2\pi/\kappa$ is the wavelength, ϱ_0 the density of matter measured at the present day. Theory gives for the period before recombination, $z > 1300$

$$M < M_D; \qquad \frac{\delta\varrho}{\varrho} = 10^{-4}\, e^{-(M_D/M)^{2/3}},$$

$$M_D < M < M_J; \qquad \frac{\delta\varrho}{\varrho} = 10^{-4},$$

$$M_J < M; \qquad \frac{\delta\varrho}{\varrho} = 10^{-4}\left(\frac{M_J}{M}\right)^{2/3}.$$

Here M_J is the Jeans mass, which gives the boundary of the region of gravitational

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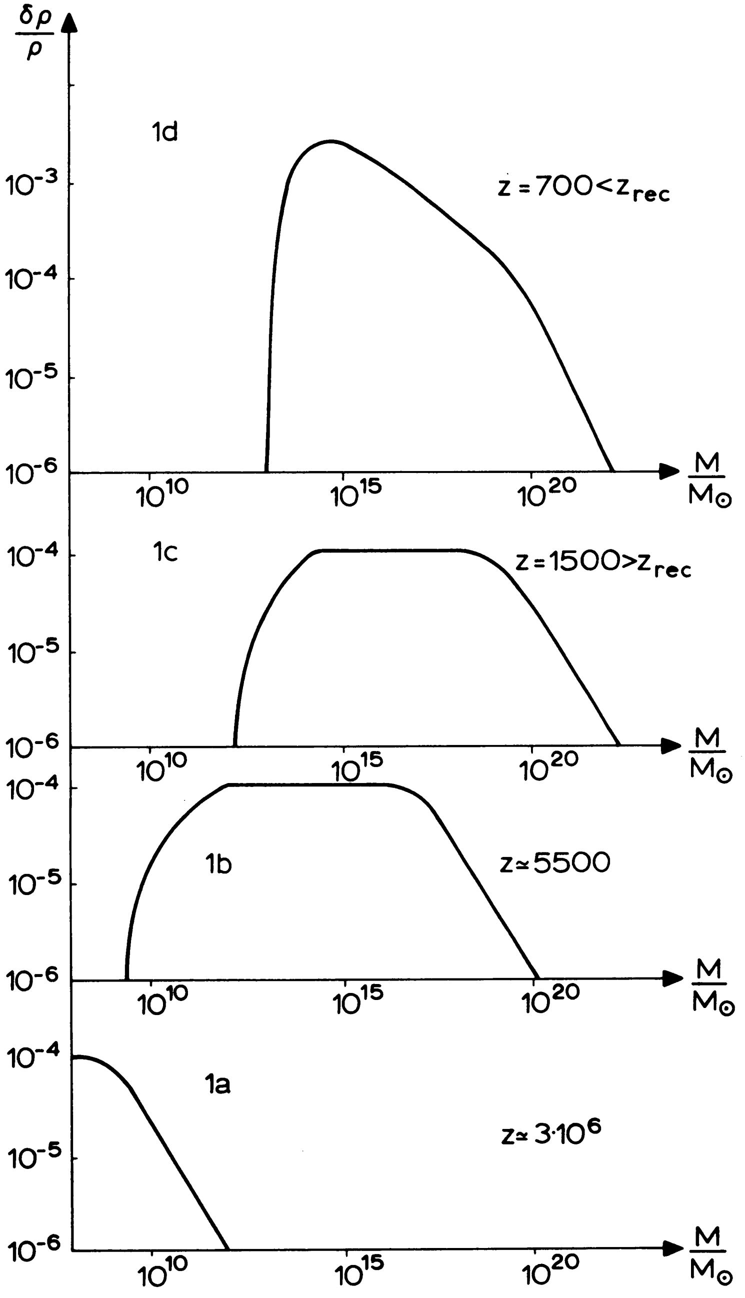

Fig. 1. The evolution of the spectrum of density perturbations. It is supposed that $\delta\varrho/\varrho \propto M^{-2/3}$ in the long wave region.

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instability [12]. The mass M_D (also called the Silk mass) is the characteristic mass for dissipation due to viscosity and thermal conductivity [13–15]. Both are time dependent, $M_J \propto t^{3/2}$; $M_D \propto t^{9/4}$. Large scale perturbations (last entry $M > M_J$) grow as $\delta\varrho/\varrho \propto t$. Dependence on z instead of t is used in Figure 2, $t \sim z^{-2}$.

The evolution of the density perturbation spectrum is illustrated in Figures 1 and 2. The perturbations are small enough, so that all predictions of the exact hot Friedmann cosmological model (nuclear reactions, radiation spectrum, radiation isotropy) are undisturbed.

The detailed calculations of the evolution of perturbations after recombination result in rather small perturbations at the present day having $\delta\varrho/\varrho < 0.1$ for $l > 150$ Mpc. The maximum of perturbation spectrum corresponds to a mass of the order (but somewhat greater than) M_D. On this scale $\delta\varrho/\varrho \geqslant 1$ is achieved long before the present day. A unique initial spectrum explains (or rather we should say describes) the birth of gravitationally bound systems such as galaxies, clusters of galaxies, etc., their clustering without gravitational binding on intermediate scales and the uniformity on the largest scale which is equal to the radius of the Universe or its horizon. Many details of the theory are already given in published papers [16–23].

4. Physics of the Nonlinear Stages

In this report we concentrate on physical processes connected with the late stage when formerly small perturbations have grown large and give gravitationally bound systems. This stage occurs much later than recombination.

Neutral gas does not interact with radiation and its own pressure is small. The motion of such gas, neglecting its pressure, is equivalent to the free motion of a set of independent particles under the influence of a self-consistent gravitational field. It is well known that the particle trajectories intersect on two dimensional surfaces, analogous to caustic surfaces in geometrical optics. For cosmological problems this type of intersection was pointed out in [24, 25, 26].

Thin layers of gas strongly compressed in one dimension are formed (see Figure 3). We call them pancakes. New gas layers falling onto these pancakes and lose their velocity in the shock wave. Kinetic energy is transformed into heat and the gas acquires a high temperature. The dense central part of the pancake cools swiftly due to radiation, but the outlying layers remain hot up to the present epoch. Typical profiles of temperature and density are given in Figures 3 and 4. Detailed calculations are described in [27–31] and the complicated cooling processes and the different behaviour of different layers are pointed out in the latest paper [30].

Due to the assumed amplitude and statistical nature of the initial perturbations, the first pancakes begin to be born at $z = 10$, when on the average $\delta\varrho/\varrho < 1$, but the maximum rate of pancake formation corresponds to $z = 4$–6.

The spread of the process in time (or in z) is a very general result in the statistical theory. The absolute values of z mentioned above depend on the adjustment of the otherwise arbitrary initial amplitude of the perturbations and also on Ω (or q_0) and H_0.

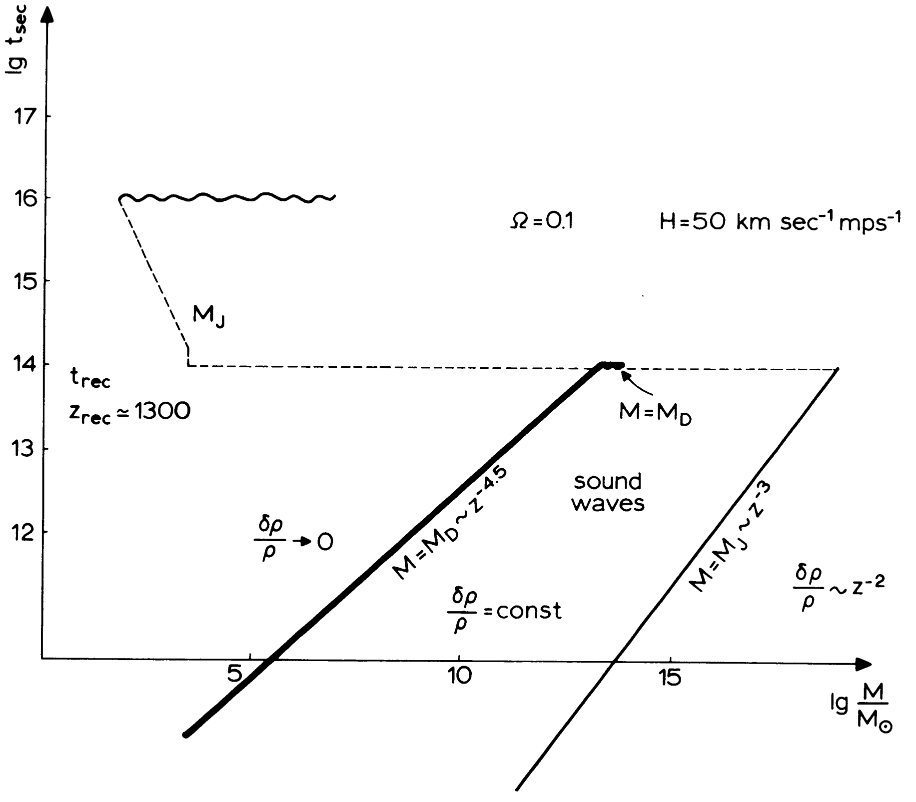

Fig. 2. Diagram of the evolution of gravitational instability and dissipation regions in the hot Friedmann Universe. The region of gravitational instability is situated to the right of the line $M = M_J$. The region of dissipation of perturbations is situated to left of line $M = M_D$.

The pancakes born first near $z = 10$ give later, at $z = 4$–5, the most massive systems having $M \sim 10^{15}\ M_\odot$. Pancakes born later give bodies of more modest mass, of the order of $M \sim 10^{13}$ to $10^{14}\ M_\odot$. A guess at the distribution of masses of different systems according to this theory is given in Figure 5.

5. The Origin of the Rotation of Galaxies

There is a widespread belief that the rotation of galaxies is impossible to explain in the adiabatic perturbation theory. The theorem is invoked which states that irrotational motion of an ideal fluid remains irrotational under the action of gravitational forces. It is sometimes said that the explanation of rotation is the privilege of turbulent theories. These statements are erroneous and in recent years it has been shown that there are several mechanisms for inducing rotation.

Peebles [34] calculated the tidal action of protogalaxies and galaxies which leads to their acquiring angular momentum. In [32] (see also [33]) the crucial role of shock waves where Helmholtz' theorem on irrotational motion is broken was pointed out.

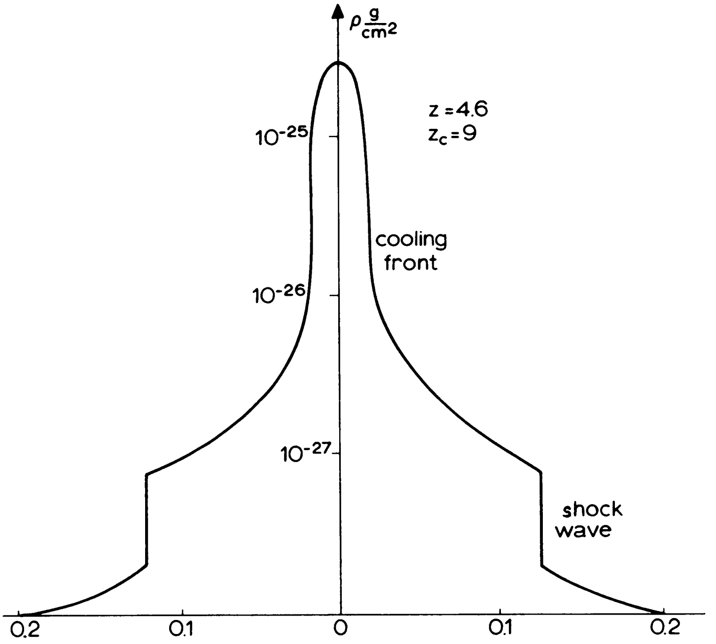

Fig. 3. The density profile of protoclusters. The unit of length is equal to the characteristic distance between clusters.

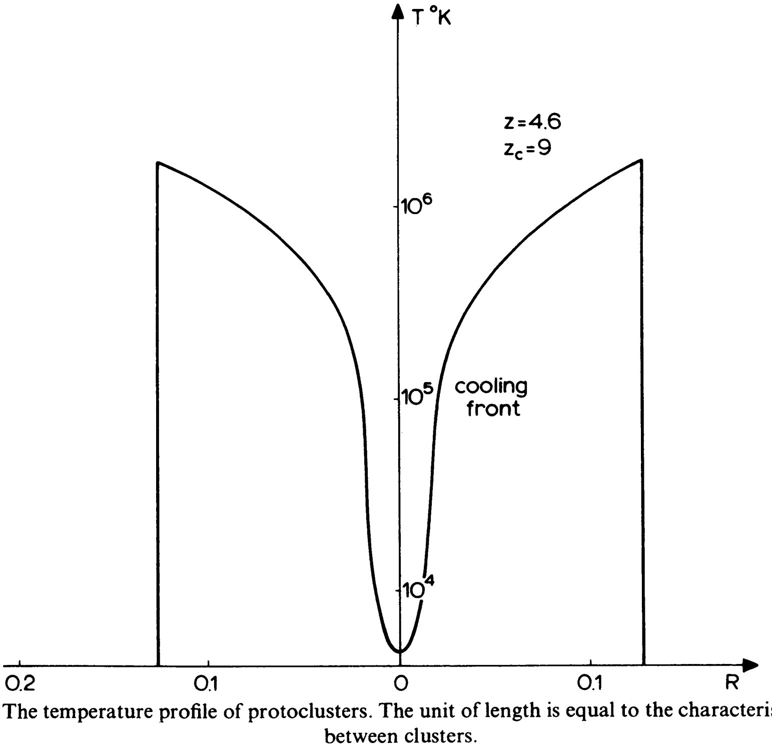

Fig. 4. The temperature profile of protoclusters. The unit of length is equal to the characteristic distance between clusters.

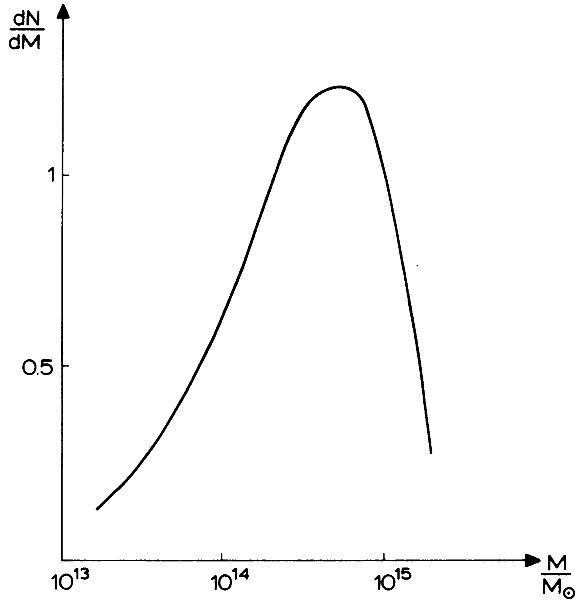

Fig. 5. Probable mass distribution of protoclusters.

In the outlying parts of the pancake the compressed gas executes rotational motion with rotational velocities not much different from the sound velocity.

It is possible that the motion is turbulent – but it is *secondary* turbulence in compressed gas at $z < 10$ and not primaeval turbulence at $z > 1300$, a remnant of a vortex-type singularity, according to Ozernoi (see his report).

The physical conditions (temperature, density) vary strongly across the compressed gas, the turbulence and rotation depending on the coordinates along the pancake surface. Therefore one can predict the formation of various very different types of system.

6. The Fragmentation of Clusters of Galaxies

The central cool region fragments under the action of gravitational and thermal instabilities. The central part of the pancake has less rotation and one expects [30] the formation of more massive galaxies, $M/M_\odot \sim 10^{12}$ with small angular momentum per unit mass, $l \sim (0.1 \text{ to } 0.01)\, l_G$, where $l_G = 6 \times 10^{29}\ \text{cm}^2\ \text{s}^{-1}$ is the angular momentum (per unit mass) of our Galaxy. Probably in the central part giant elliptical galaxies are formed. The very cool regions perhaps give quasars [28].

The outer parts of the disc are expected to give spiral galaxies with $l \approx l_G$ and smaller mass, 10^{10} to $10^{11}\ M_\odot$, due to strong rotational motion and turbulence. We expect more spiral galaxies in the outer parts of a cluster of galaxies [30]. The rotation of the pancake (cluster of galaxies) as a whole – if any – should be smaller since it can only result from tidal forces [34, 35]. It should be pointed out that if the Hubble expansion occurs in a supercluster of galaxies, the virial theorem is inapplicable for

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mass determinations and the elliptical form does not prove that the cluster rotates.

The formation of magnetic field in this theory occurs at the late stages, $z < 10$, due to rotation and the dynamo effect. This point of view is different from Harrison's, where magnetic field is generated in the radiation dominated era, $z > 1300$, due to initial turbulence. Another possibility is that the magnetic field in stars spreads on a galactic scale during supernovae explosions [36, 37, 38].

7. Heating and Ionization of the Intergalactic Gas

Of particular importance is the problem of the intergalactic – and inter-cluster – gas, its evolution and its interaction with galaxies. In our picture gas *moderately* heated by the shock wave cooled down to 10^4 K and from it galaxies are formed. Gas *strongly* heated by the shock wave ($T > 5 \times 10^5$ K) remains hot. Part of it is captured in clusters of galaxies but the rest is free. A sizeable fraction of the gas does not encounter the shock wave, and this gas is ionized and slightly heated by the radiation of pancakes and perhaps also by the radiation of quasars or young galaxies [28, 39, 40]. The spectrum of radiation emitted by a single pancake depends only weakly on the moment at which the pancakes 'burn' and is given in Figure 6.

The density of neutral hydrogen outside the pancake is given by the curve in Figure 7 (calculations see [39]). The solid dots define upper limits to the density of neutral hydrogen which would absorb less than one half of the radiation with $R_\nu >$

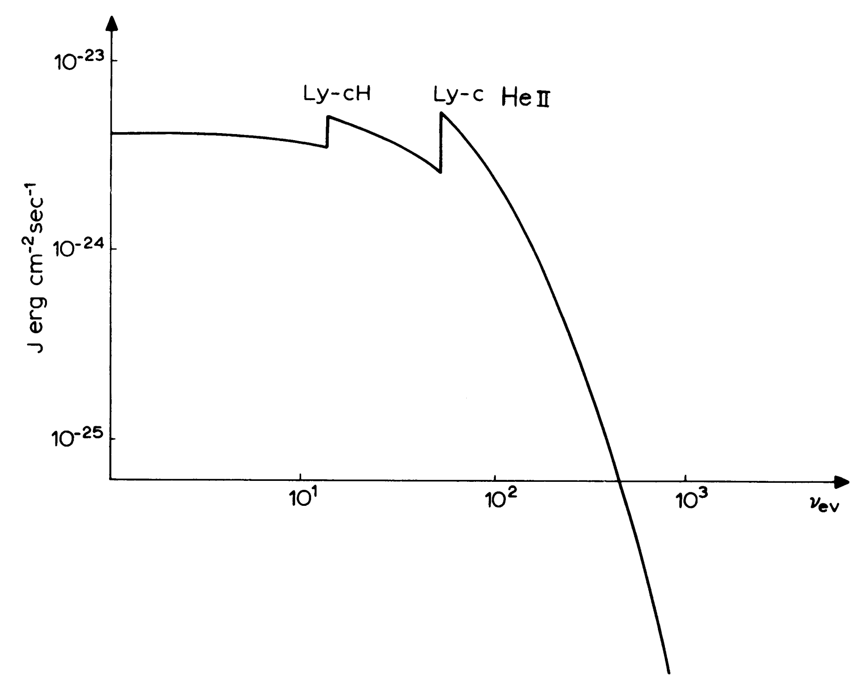

Fig. 6. An approximate spectrum of the emission per unit area of a protocluster.

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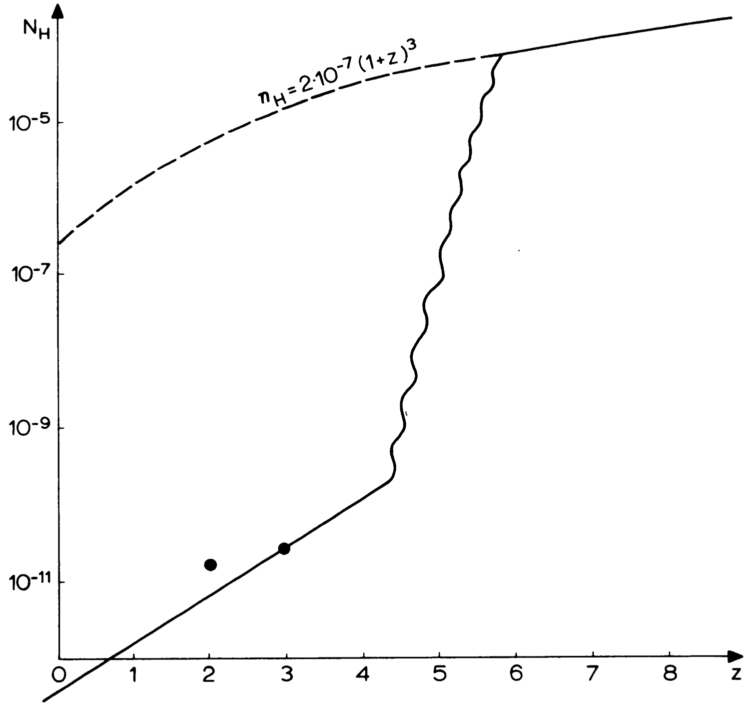

Fig. 7. The dependence of the density of H I as a function of redshift z in the Universe.

$> [(R_\nu)\,\mathrm{L}\alpha]/(1+z)$. This upper limit to the H I density is compatible with the lack of absorption in the spectra of distant quasars according to the Gunn and Peterson argument.*

The overall radiation spectrum of pancakes and the intergalactic gas is given in Figure 8. It is compared with observations collected in the review article [41].

8. Observational Tests of the Theory

The best, most sensitive detector of background radiation is provided by the neutral hydrogen haloes of galaxies [42]. The scheme proposed here does not contradict any observation of hard radiation or of absorption of radiation by neutral gas. Of particular interest is the possibility of direct observation of the 21 cm emission from the central dense part of the pancake before the fragmentation of this gas [28, 43]. In galaxies most of the matter is in the form of stars and the mass of gas is of the order of a few percent. But protoclusters of galaxies in the prestellar stage have unique properties: their masses are in the form of neutral gas with kinetic and spin temperatures up to about 10^4 K. Their angular dimensions are of the order 1′–10′. The high emissivity makes it possible that such radio-emitting regions might be detectable at wave-

* Afterthought: see also the discussion of the spectra of the quasars with largest redshift with Maarten Schmidt (pp. 255–256).

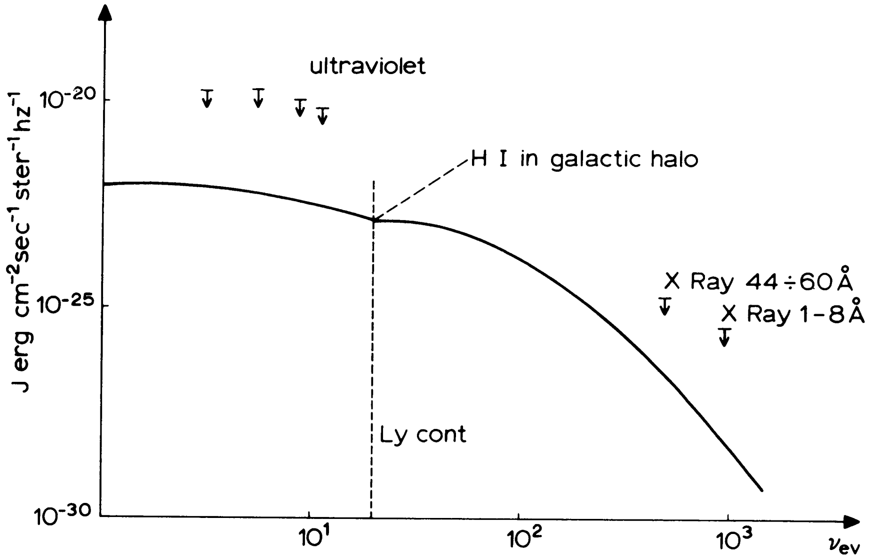

Fig. 8. The background radiation due to the emission of protoclusters and the intercluster gas.

lengths 80 to 200 cm (due to the effects of redshift $z \sim 3$ to 9). The outer zone of the pancake, where recombination is actually occurring could emit lines due to highly excited atomic levels, but the emission measure is small and only in closed world models ($\Omega > 1$) is there any hope of observing these lines.

A large part of the recombination energy is radiated in a single line, Lα. The possibility of observing redshifted Lα from pancakes is analyzed in [44].

Partridge and Peebles [45] argue that young galaxies have high percentages of massive bright stars so that during the first 10^8 yr [46, 47] the overall brightness of galaxies is much greater than the present day or average brightness. It seems to us that one should check whether the formation of stars is not spread out over a longer time interval with a corresponding decrease of the maximum brightness.

The quasars, their observation and theoretical interpretation, will also give clues to many cosmological problems. The $\log N - \log S$ curve for radio sources and the $N(z)$ curve for quasars are similar, there being a huge evolutionary effect at $z \sim 3$ and a decrease or at least no increase at $z > 3$ [48–50]. It is natural to assume that most galaxies were formed about the same time, $z \sim 3$ to 5. If quasars are in fact galactic nuclei in a particular stage of evolution, this point of view adds additional support [51, 52]. Perturbations on the biggest scale make the distribution of matter, including quasars and galaxies, non-uniform. A detailed statistical study of the quasar distribution as a function of z and angular coordinates can perhaps give some information on these perturbations.

A classical method of studying large scale perturbations is by temperature fluctuations in the relic radiation. The amplitude chosen (10^{-4} in the metric) does not contradict the upper limit given by present observations [4].

9. Concluding Remarks

We have presented here the results for a definite choice of parameters. To what extent are the results sensible and what is the possible range of variation of the parameters? The average matter density should be derived from other investigations. The choice of the value $\Omega = 0.1$ is rather arbitrary and perhaps will be altered. The perturbation theory for low densities $\Omega \ll 1$ is consistent with arguments about the small peculiar velocities of galaxies [2, 40, 53] (see Tammann, this volume, p. 47).

If a unique power law for the initial perturbations is chosen, it must be $\delta\varrho/\varrho \propto M^{-2/3}$ corresponding to mass independent metric perturbations; at the singularity ($t = t_{\text{Planck}} = 10^{-43}$ s) the number density perturbations are smaller than the statistical value $\Delta N = N^{1/2}$ for elementary particles [55–57]. It is rather interesting that the short wave part of the perturbation spectrum can perhaps explain the entropy of the initially cold (at $t = 10^{-43}$ s) baryonic fluid [57].

Why do we adopt the point of view of initial adiabatic perturbations, instead of composition (adiabatic turbulent, entropy) fluctuations?

Entropy fluctuations can give the same results only if the primaeval spectrum is specially adjusted. The mass $M_D \sim 10^{13}\ M_\odot$ is singled out in adiabatic theory by dissipation, but it is not characteristic for entropy fluctuations.

The primaeval turbulence theory is now very fashionable but this theory has many difficulties. The cosmological model near the singularity must be non-Friedmannian [20, 59] and the theory of cosmological nucleosynthesis changes. The perturbations of velocity at the epoch of recombination in turbulent theory are 100 times greater than in adiabatic theory; this is hardly compatible with small temperature fluctuations in the relic radiation [4]. Turbulent theory leads to rather early formation of bound bodies with too high density [58].

A priori the theory of small initial adiabatic perturbations advocated in this report seems very unlikely and unnatural.

Given an immense big bang explosion at the beginning, why should one insist on rather exact uniformity and isotropy? Chaotic velocities and a chaotic distribution of matter and antimatter seem more appealing.

It is under the pressure of observational data from the relic radiation and the Hubble diagram that we feel the need for an explosion following the Friedmann picture with high precision ($\sim 10^{-4}$).

It was thought that adiabatic perturbations which are characterised by one number $\delta\varrho/\varrho$ might not give the necessary diversity of conditions. In fact the nonlinear picture being a consequence of adiabatic perturbations, includes tidal interactions and shock wave formation which induce vortices in the gas. The theory corresponds well to the observed picture of the different forms of galaxies, clusters, quasars etc.

We show that the logical consequences of this picture are in reasonable accord with the observed properties of the Universe. In particular, the observed rotation of galaxies does not contradict adiabatic perturbation theory.

References

[1] de Vaucouleurs, G. and Peters, W. L.: 1968, *Nature* **220**, 868.
[2] Sandage, A.: 1972, *Astrophys. J.* **178**, 1.
[3] Abell, G. O.: 1965, *Ann. Rev. Astron. Astrophys.* **3**, 1.
[4] Parijskij, Yu. N.: 1973, *Astron. Zh.* **50**, 453.
[5] Boynton, P. E. and Partridge, R. B.: 1973, *Astrophys. J.* **181**, 243.
[6] Blair, A. G., Beery, J. G., Edeskuty, F., Hiebert, R. D., Shipley, J. P., and Williamson, K. D., Jr.: 1971, *Phys. Rev. Letters* **27**, 1154.
[7] Sunyaev, R. A.: 1973, in C. DeWitt-Morette (ed.), 'Gravitational Radiation and Gravitational Collapse', *IAU Symp.* **64**, 193.
[8] Reeves, M., Audouze, J., Fowler, W. A., and Schramm, D. N.: 1973, *Astrophys. J.* **179**, 909.
[9] Wagoner, R. V.: 1973, *Astrophys. J.* **179**, 343.
[10] Steigman, G.: 1969, *Nature* **224**, 477.
[11] Lifshitz, E. M.: 1946, *Zh. Eksper. Teor. Phys.* **16**, 587.
[12] Jeans, J. H.: 1929, *Astronomy and Cosmology*, Cambridge.
[13] Silk, J.: 1968, *Astrophys. J.* **151**, 459.
[14] Peebles, P. J. E. and Yu, I. I.: 1970, *Astrophys. J.* **162**, 815.
[15] Chibisov, G. V.: 1972, *Astron. Zh.* **49**, 74, 286.
[16] Bonnor, W. B.: 1957, *Monthly Notices Roy. Astron. Soc.* **117**, 104.
[17] Peebles, P. J. E.: 1965, *Astrophys. J.* **142**, 1317.
[18] Guyot, M. and Zel'dovich, Ya. B.: 1970, *Astron. Astrophys.* **9**, 227.
[19] Sachs, R. K. and Wolfe, A. M.: 1967, *Astrophys. J.* **147**, 73.
[20] Zel'dovich, Ya. B. and Novikov, I. D.: 1970, *Astrofizika* **6**, 379.
[21] Doroshkevich, A. G. and Zel'dovich, Ya. B.: 1963, *Astron. Zh.* **40**, 807.
[22] Sunyaev, R. A. and Zel'dovich, Ya. B.: 1970, *Astrophys. Space Sci.* **7**, 3.
[23] Field, G. B.: 1972, *Ann. Rev. Astron. Astrophys.* **10**, 227.
[24] Lifshitz, E. M. and Khalatnikov, I. M.: 1963, *Usp. Fis. Nauk.* **80**, 391; *Adv. in Phys.* **12**, 185.
[25] Grishchuk, L. P.: 1967, *Zh. Eksper. Theor. Phys.* **53**, 1699.
[26] Oort, J. H.: 1969, *Nature* **224**, 1158.
Oort, J. H.: 1970, *Astron. Astrophys.* **7**, 381.
[27] Zel'dovich, Ya. B.: 1970, *Astrofizika* **6**, 119; *Astron. Astrophys.* **5**, 84.
[28] Sunyaev, R. A. and Zel'dovich, Ya. B.: 1972, *Astron. Astrophys.* **20**, 189.
[29] Doroshkevich, A. G., Ryabenki, V. S., and Shandarin, S. F.: 1973, *Astrofizika* **9**, 181.
[30] Doroshkevich, A. G. and Shandarin, S. F.: 1973, Preprint, Inst. Appl. Math. N7, *Astron. Zh.* **51**, 41.
[31] Doroshkevich, A. G. and Shandarin, S. F.: 1973, Preprint, Inst. Appl. Math. N55, *Astrofizika* **9**, 550.
[32] Doroshkevich, A. G.: 1973, *Astrophys. Letters* **14**, 11.
[33] Chernin, A. D.: 1970, *Pisma Zh. Eksper. Theor. Phys.* **11**, 317.
[34] Peebles, P. J. E.: 1969, *Astrophys. J.* **155**, 393.
[35] Doroshkevich, A. G.: 1970, *Astrofizika* **6**, 581.
[36] Mishystin, I. N. and Ruzmaikin, A. A.: 1971, *Zh. Eksper. Theor. Phys.* **61**, 441.
[37] Bisnovatyi-Kogan, G. S., Ruzmaikin, A. A., and Sunyaev, R. A.: 1973, *Astron. Zh.* **50**, 210.
[38] Harrison, E. R.: 1970, *Monthly Notices Roy. Astron. Soc.* **147**, 279.
[39] Doroshkevich, A. G. and Shandarin, S. F.: 1973, Preprint, Inst. Appl. Math. N59, *Astrofizika*, in press.
[40] Sunyaev, R. A.: 1971, *Astron. Astrophys.* **12**, 190.
[41] Longair, M. S. and Sunyaev, R. A.: 1971, *Usp. Fiz. Nauk* **105**, 41.
[42] Sunyaev, R. A.: 1969, *Astron. Zh.* **46**, 929.
[43] Novokreshenova, S. I. and Rudnisky, G. M.: 1973, *Astron. Zh.* **50**, 877.
[44] Kurt, V. G. and Sunyaev, R. A.: 1970, in L. Houziaux and H. E. Butler (eds.), 'Ultra-Violet Stellar Spectra', *IAU Symp.* **36**, 341.
[45] Partridge, R. B. and Peebles, P. J. E.: 1967, *Astrophys. J.* **147**, 808; *Astrophys. J.* **148**, 377.
[46] Salpeter, E. E.: 1955, *Astrophys. J.* **121**, 161.
[47] Tinsley, B. M.: 1972, *Astron. Astrophys.* **20**, 383.
[48] Ryle, M. and Longair, M. S.: 1967, *Monthly Notices Roy. Astron. Soc.* **136**, 123.

[49] Longair, M. S.: 1969, *Usp. Fiz. Nauk.* **99**, 229.
[50] Doroshkevich, A. G., Longair, M. S., and Zel'dovich, Ya. B.: 1970, *Monthly Notices Roy. Astron. Soc.* **147**, 139.
[51] Shklovsky, I. S.: 1963, *Astron. Zh.* **40**, 972.
[52] Field, G. B.: 1964, *Astrophys. J.* **140**, 1434.
[53] Sandage, A.: 1973, preprint.
[54] Tomita, K.: 1973, *Prog. Theor. Phys.* **50**, 1285.
[55] Doroshkevich, A. G., Zel'dovich, Ya. B., and Novikov, I. D.: 1968, Report at 5, *Grav. Symp. Tbilisi.*
[56] Harrison, E. R.: 1970, *Phys. Rev.* **D1**, 2726.
[57] Zel'dovich, Ya. B.: 1972, *Monthly Notices Roy. Astron. Soc.* **160**, 1p.
[58] Peebles, P. J. E.: 1971, *Astrophys. Space Sci.* **11**, 443.
[59] Ozernoy, L. M. and Chernin, A. D.: 1968, *Astron. Zh.* **45**, 1137.

DISCUSSION

Urbanik: Can the theory presented above explain the behaviour of brightest cluster members, namely the sharp upper limit to their masses and their independence of cluster composition?

Zel'dovich: The theory has not yet been developed to the point at which it can account for such details.

Afterthought: Detailed calculations of the thermal history of the pregalactic gas always give the same temperature $T \sim 10^4$ K at which cooling due to radiative losses stops and most of the hydrogen recombines. Perhaps this explains (at least partially) the important property used by Sandage and mentioned by Urbanik.

Icke: If the deviation from the Hubble expansion in a protocluster is about the same for all protoclusters, their final temperatures will be comparable. Hence the masses of the most massive galaxies will also be approximately equal because of the dependence of the Jeans' mass on temperature.

Field: One important part of your theory is the generation of vorticity by shock waves, followed by the contribution of this vorticity to density clumps which eventually appears as the angular momentum of galaxies. Will you please explain these two processes in greater detail?

Zel'dovich: In an oblique shock wave it is the normal component of velocity which changes. Take an incoming gas with

$$\frac{dv_x}{dy} = \frac{dv_y}{dx}; \quad v_z = 0; \quad \frac{d}{dz} = 0; \quad (\text{rot } v)_z = \frac{dv_x}{dy} - \frac{dv_y}{dx} = 0$$

and no vorticity. Let the shock be in the plane $y = \text{const}$. After the shock

$$v'_x = \frac{v_x}{4}; \quad dx' = \frac{dx}{4}$$

but v_y and d/dy are preserved.

Therefore

$$\frac{dv'_x}{dy'} = \frac{1}{4}\frac{dv_x}{dy}; \quad \frac{dv'_y}{dx'} = 4\frac{dy}{dx}$$

and therefore

$$\frac{dv'_x}{dy'} \neq \frac{dv'_y}{dx'}.$$

Thus vorticity is created from irrotational motions.

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