THE GRAVITATIONAL ZONES OF INFLUENCE OF THE PLANETS ACTING ON SMALL CELESTIAL BODIES

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ABSTRACT. Tisserand's definition of the "sphere of action" of a planet is based on the equality of tidal vs. gravitational acceleration ratios of the sun and planet. Öpik and others based their relation on equating the differential solar and planetary forces on a particle. Neither expression was formulated to describe the zone of influence surrounding a planet when considering the small, but significant, long-term perturbative effects of the planets on a particle's orbital elements. For the purpose of determining these effects on interplanetary dust we derive a zone of influence based on equating the gravitational forces of the sun and planet.

The general expression for the radial distance to the boundary of the planetary zone of influence on small particles can be written as

$$s = k \cdot f(a_p) \cdot g(m_p, \text{ other planetary parameters}) \qquad (1)$$

where k is a constant which depends on the nature of the problem under study, f and g are functions which may also have the same dependence as k, a_p is the planet's semimajor axis, and m_p is the planet's mass. The other parameters may include the eccentricity of the planet, as well as other orbital parameters and such factors as oblateness and axial tilt. Functions f and g are generally not separable but we have found that the effect of semimajor axis can be separated from the other parameters. The equation of Tisserand (1889) and Öpik (1951) can be written as

$$s = (1/M_\odot)^{2/5} a_p m_p^{2/5}, \text{ and} \qquad (2)$$

$$s = \tfrac{1}{2}(1/2M_\odot)^{1/3} a_p m_p^{1/3}, \text{ respectively.} \qquad (3)$$

If we equate the gravitational and radiation forces of the sun and planet, a different relationship is obtained. Consider a particle of mass m at a distance s from a planet of mass m_p and semimajor axis a_p, and at a distance r from the sun. The magnitude of the force on the particle due to the planet is

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$$F_p = Gm_p m/s^2 \quad , \tag{4}$$

while the magnitude of the force on the particle due to the sun is

$$F_\odot = GM_\odot m(1-\beta)/r^2 \quad , \tag{5}$$

where β is the ratio of radiation force to gravitational force. Since $a_p - s \leq r \leq a_p + s$, we can write r as $r = a_p + \delta s$ where δ is a number between -1 and +1 which depends on the planet-sun angle. Substituting the expression for r into equation (5) and substituting $M_\odot' = M_\odot(1-\beta)$ we can set the ratio of the solar force to the planetary gravitational force equal to some constant k^2, where

$$k^2 Gm_p m/s^2 = GM_\odot' m/(a_p+\delta s)^2 \text{ , which reduces to} \tag{6}$$

$$s = ka_p(m_p/M_\odot')^{1/2}[1-k\delta(m_p/M_\odot')^{1/2}]^{-1} \tag{7}$$

Equation (7) can be approximated by a simple power law when the mass of the planet and k are small, giving

$$s = (k/M_\odot'^{1/2})a_p m_p^{1/2} \quad . \tag{8}$$

Compare this with the results of Tisserand and Öpik, eqs. (2) and (3).

A computer simulation technique was used to find the most applicable expression for our study. Each encounter was run using a 10,000 step Cowell's method computation over the period of one synchronous orbit. For a particle in circular orbit at 10 A.U., the errors in eccentricity and semimajor axis incurred over one orbit were less than 1×10^{-5} and 5×10^{-9} A.U., respectively. The particle was initially in a circular orbit and the planet was given a fixed orbital eccentricity. The encounter occurred at perihelion or aphelion, depending on whether the particle's orbit was inside or outside the planet's orbit. The particle's orbital plane coincided with that of the planet, so that only subsolar and antisolar encounters were studied. The distance of closest approach was compared with either a_p or m_p assuming a constant Δe or a constant percentage change in a. Figure 1 shows the effect of a_p on the zone size

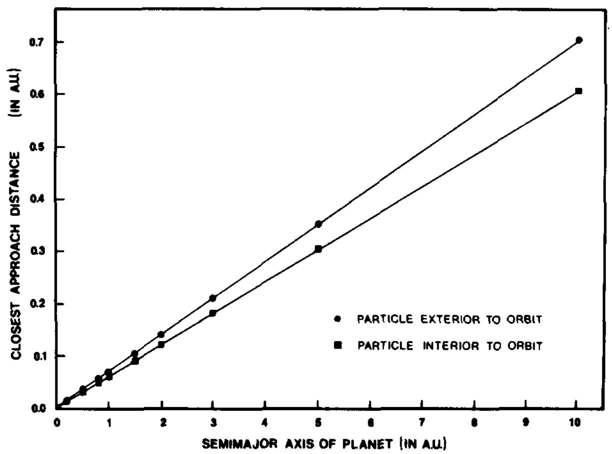

Figure 1. Relationship between a_p and s for a constant change in eccentricity (Δe=.0014). The planet's mass is 1 $M_\oplus$ and its eccentricity is 0.05.

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for a constant Δe, and verifies the linear relation, common to all three derivations, between the zone boundary and the semimajor axis. This result also shows that the uncoupling of the functional forms of a_p and m_p in equation (1) is valid. The constant change Δe= 0.0014 chosen in deriving figure 1 (and later 2 and 4) is the minimum value that guarantees that the zones of influence of the inner five planets do not overlap.

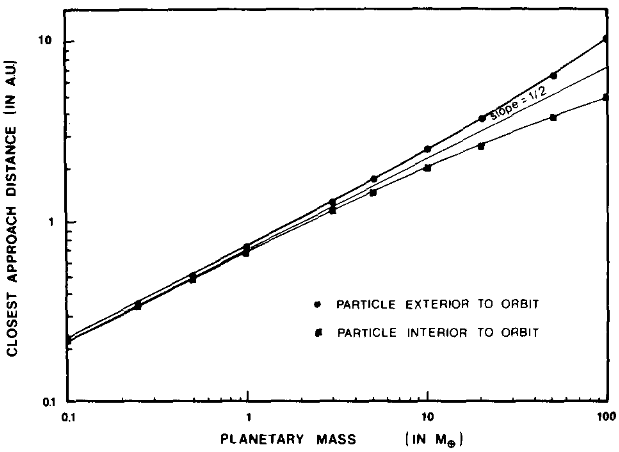

Figure 2. Relationship between m_p and s for a constant change in eccentricity (Δe=.0014). The planet's semimajor axis is 10 A.U. and its eccentricity is zero.

Figure 2 shows the effect of the planet's mass on the zone boundary. The general trend follows a power law most closely approximated by equation (8), and diverges from the power law in the same manner as equation (7). Figure 3 is a graphic representation of equation (7). While this and the previous figure show a striking similarity, the spread due to $[1-k\delta(m_p/M_\oplus)^{\frac{1}{2}}]^{-1}$ is greater than the spread observed in the simulations. This is a consequence of the fact that, during an encounter, δ assumes other values than 1.

Figure 4 shows the effect of planetary eccentricity on the zone boundary. As implied in equation (1), this effect cannot be separated from the functional dependence on planetary mass. This is evidenced by the fact that these curves match the results for high mass planets but diverge for those of low mass. This result deserves further investigation to determine the relation between eccentricity and the size of the zone of influence.

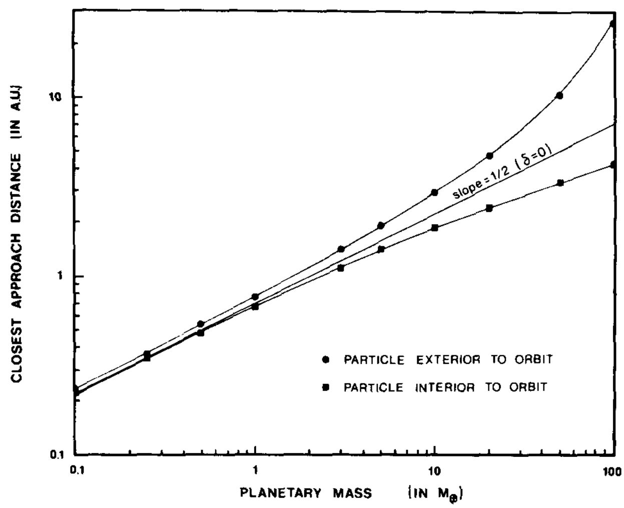

Figure 3. Theoretical curves relating m_p and s through equation (7). a_p is 10 A.U. and k is chosen to match the simulation value for $m_p = M_\oplus$. β is zero.

Figure 5 shows

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the relationship between planetary mass and the distance of closest approach when the desired effect is a constant percentage change in the semimajor axis of the particle, $\Delta a/a = 5x10^{-5}$, which is the value found when $\Delta e = .0014$. Unlike the result for a constant Δe, (see fig. 3) the boundary follows the form $s \propto (m_p/M_{\odot})^{3/8}$, a result which matches no theoretical formula yet derived. Note, also, the divergence from the power law for planets of high mass, similar to the form in equation (7).

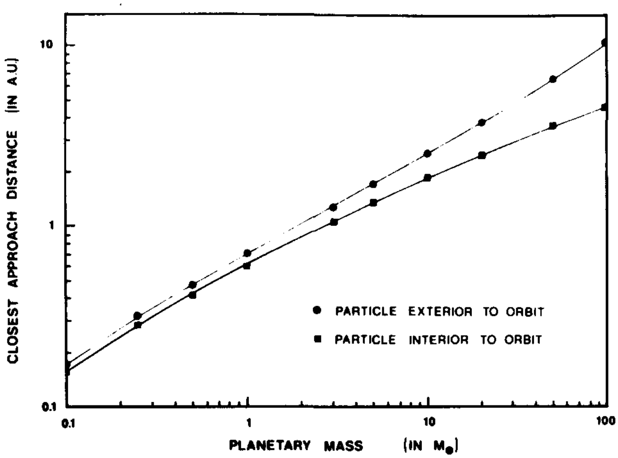

Figure 4. Relationship between m_p and s for a constant $\Delta e = .0014$. This is the same as Figure 2 except $e_p = 0.05$.

The form of the relation determining the zone of influence is highly dependent on the nature of the perturbing effect to be observed. We have shown that the expressions most applicable to interactions with orbiting particles involve powers of the planetary mass not formerly considered and that the size of the zone boundary depends upon the eccentricity. We are now studying the effect of eccentricity on the zone size for a constant $\Delta a/a$, and intend to extend this work to changes in inclination and ascending node.

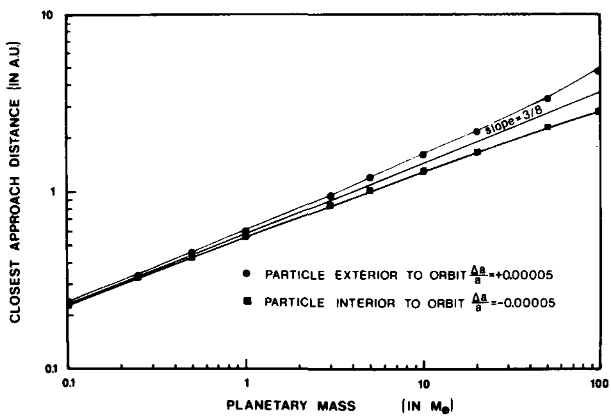

Figure 5. Relationship between m_p and s for a constant $\Delta a/a$ (= .00005). a_p is 10 A.U. and e_p is zero.

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REFERENCES:

1. Tisserand, F., *Traite de Mecanique Celeste Gauthier-Villars, Paris*, 1889.
2. Öpik, E. J., *Proc. Royal Irish Acad.* 54, 165-199, 1951.