

THE  
JOURNAL  
OF  
SYMBOLIC LOGIC

EDITED BY

ALONZO CHURCH    HARTLEY ROGERS, JR    ARTHUR N. PRIOR  
ALICE A. LAZEROWITZ    JULIA ROBINSON    AZRIEL LÉVY

*Managing Editor:* ALFONS BORGERS

*Consulting Editors:*

ALAN ROSS ANDERSON	HANS HERMES	ABRAHAM ROBINSON
C. A. BAYLIS	E. J. LEMMON	GENE F. ROSE
JONATHAN F. BENNETT	RICHARD MONTAGUE	BARKLEY ROSSER
PAUL BERNAYS	ANDRZEJ MOSTOWSKI	H. ARNOLD SCHMIDT
MARTIN DAVIS	GERT H. MÜLLER	KURT SCHÜTTE
FREDERIC B. FITCH	F. C. OGLESBY	DANA SCOTT
G. HASENJAEGER	STEVEN OREY	J. F. THOMSON
LEON HENKIN	RÓZSA PÉTER	A. R. TURQUETTE

VOLUME 31

1966

PUBLISHED QUARTERLY BY THE ASSOCIATION FOR SYMBOLIC LOGIC, INC WITH  
SUPPORT FROM THE U.S. NATIONAL SCIENCE FOUNDATION, FROM UNESCO  
THROUGH ICSU AND FROM INSTITUTIONAL AND CORPORATE MEMBERS

The four numbers of Volume 31 were issued at the following dates:

Number 1, pages 1–158, April 2, 1966.

Number 2, pages 159–302, August 3, 1966.

Number 3, pages 303–524, November 2, 1966.

Number 4, pages 525–737, January 1967.

All numbers of this volume are copyrighted by the Association for Symbolic Logic, Inc. Reproduction of copyrighted numbers of the Journal by photostat, photo-print, microfilm, or like process is forbidden, except by written permission of the Managing Editor.

## TABLE OF CONTENTS

On formulas in which no individual variable occurs more than twice. By STANISŁAW JAŚKOWSKI . . . . .	1
Postulates for implicational calculi. By C. A. MEREDITH. . . . .	7
On the indexing of classes of recursively enumerable sets. By A. H. LACHLAN . . . . .	10
Extensional interpretations of modal logics. By M. H. LÖB . . . . .	23
Algebraic semantics for modal logics, I. By E. J. LEMMON . . . . .	46
On a question of G. E. Sacks. By DONALD M. MARTIN . . . . .	66
Linear orderings under one-one reducibility. By PAUL R. YOUNG. . . . .	70
A reduction theorem for normal algorithms. By J. W. SAMSON. . . . .	86
Reviews . . . . .	98
Meeting of the Association for Symbolic Logic. By RUTH BARCAN MARCUS, WILLIAM BOONE and RICHARD MONTAGUE . . . . .	147
Abstracts of papers. . . . .	147
Notice on fellowship and research opportunities in mathematics . . . . .	157
Notebooks of Emil Post. . . . .	157
The Aristotelian Society: Essay prize-result . . . . .	158
A minimal pair of recursively enumerable degrees. By C. E. M. YATES . . . . .	159
Decidability and undecidability of extensions of second (first) order theory of (generalized) successor. By CALVIN C. ELGOT and MICHAEL O. RABIN . . . . .	169
Decision problems for multiple successor arithmetics. By J. W. THATCHER. . . . .	182
Algebraic semantics for modal logics, II. By E. J. LEMMON . . . . .	191
The undecidability of the Turing machine immortality problem. By PHILIP K. HOOPER . . . . .	219
Une généralisation de l'ultraproduit. By ROLAND FRAÏSSÉ. . . . .	235
Reviews . . . . .	245
Elections . . . . .	289
Meeting of the Association for Symbolic Logic. By J. N. CROSSLEY . . . . .	290
Abstracts of papers. . . . .	290
Notices of meetings of the Association for Symbolic Logic. . . . .	302
Trees and nest structures. By RAYMOND M. SMULLYAN . . . . .	303
Finite nest structures and propositional logic. By RAYMOND M. SMULLYAN . . . . .	322
Transfinite induction and bar induction of types zero and one, and the role of continuity in intuitionistic analysis. By W. A. HOWARD and G. KREISEL . . . . .	325
Pseudo-complements and ordinal logics based on consistency statements. By ROBERT A. DI PAOLA. . . . .	359
On definability of ordinals in logic with infinitely long expressions. By AKIKO KINO. . . . .	365
Hierarchies of computable groups and the word problem. By FRANK B. CANNONITO . . . . .	376
A supplement to Herbrand. By BURTON DREBEN and JOHN DENTON . . . . .	393
Pure three-valued Łukasiewiczian implication. By STORRS MCCALL and R. K. MEYER . . . . .	399
There are infinitely many Diodorean modal functions. By D. C. MAKINSON . . . . .	406
Short definitions of the ordinals. By KENNETH R. BROWN and HAO WANG . . . . .	409
Connexive implication. By STORRS MCCALL . . . . .	415
The impossibility of finding relative complements for recursively enumerable degrees. By A. H. LACHLAN . . . . .	434

A generalisation of productive set. By R. MITCHELL . . . . .	455
On formalizing the distinction between logical and factual truth. By WILLIAM H. HANSON . . . . .	460
Testing singly quantified tautologies. By GERALD STANDLEY. . . . .	478
Reviews . . . . .	481
Notice of the Third International Congress for Logic, Methodology and Philosophy of Science. . . . .	523
Notice of the new quarterly <i>Nous</i> . . . . .	524
Constructive order types, II. By JOHN N. CROSSLEY . . . . .	525
A theorem on minimal degrees. By J. R. SHOENFIELD. . . . .	539
Functions of propositions. By M. J. CRESSWELL . . . . .	545
Logical truth and logical implication. By WILLIAM S. HATCHER . . . . .	561
Non-null implication. By DAVID NELSON . . . . .	562
A note on universal sets. By A. H. LACHLAN . . . . .	573
Pure denumerable Łukasiewiczian implication. By R. K. MEYER. . . . .	575
Finite models for inequations. By M. D. GLADSTONE . . . . .	581
The theory of truth tabular connectives, both truth functional and modal. By GERALD J. MASSEY. . . . .	593
MIPC as the formalisation of an intuitionist concept of modality. By R. A. BULL	609
Syntactical and semantical properties of generalized quantifiers. By MITSURU YASUHARA . . . . .	617
An axiomatic system for the first order language with an equi-cardinality quantifier. By MITSURU YASUHARA. . . . .	633
On some alleged decision procedures for S4. By WILLIAM H. HANSON. . . . .	641
Reviews . . . . .	644
Notice on fellowships and research opportunities in mathematics . . . . .	681
Annual meeting of the Association for Symbolic Logic. By SIDNEY MORGENBESSER and ELLIOTT MENDELSON . . . . .	682
Abstracts . . . . .	682
Meeting of the Association for Symbolic Logic. By MARTIN DAVIS . . . . .	697
Abstracts . . . . .	697
List of officers and members of the Association for Symbolic Logic . . . . .	707

## ERRATA

### VOLUME 10

Page 135, line 13 from below. For «*Delboeuvs Bedeutung für die Logik*», read «*Delboeuvs Bedeutung für die Logik*».

Page 149, line 19 and line 4 from below. For 134(7), read 134(6).

### VOLUME 25

Page 293, line 13. For «*Librael*», read «*Liberal*».

### VOLUME 27

Page 91, line 12 from below. For  $R \supset E \times E$ , read  $R \subset E \times E$ .

Page 226, line 20 from below. For «*méthodogie*», read «*méthodologie*».

Page 368, line 4. Insert  $\sim$  over the last **a** in the line.

### VOLUME 28

Page 253, line 23. For  $F^*$ , read  $F_*$ .

Page 253, line 25. For  $(F_i)^*$ , read  $(F_i)_*$ .

Page 253, line 28. For  $F_*$ , read  $F'_*$ .

Page 337, column 1, line 10 from below. For «decision», read «decision».

Page 341, column 1, line 23. Insert «(Hungarian)» before XXVII.

Page 342, column 1, line 9 from below. For «fo», read «of».

Page 343, column 1, line 13. For «Raisiowa», read «Rasiowa».

Page 345, column 2, line 14. For 291, read 294.

### VOLUME 29

Page 48, line 10 from below. For «pretentions», read «pretensions».

Page 61, line 7 from below. For «simile», read «similarly», and for  $\Pi_k^{(C)}$ , read  $\Pi_1^{(C)}$ .

Page 62, line 15. For «simile», read «similarly».

Page 104, line 1. Insert  $\lambda$  before  $x\Psi(\Phi(v), x)$ .

Page 132, line 11 from below. Insert «[believes]» after «knows».

Page 133, line 30. Insert «'» after « $\sim(Ex)p$ ».

Page 139, line 3 from below. For «Corrigendum», read «*Corrigendum*».

Page 141, line 17 from below. The question mark should be italic.

Page 149, line 4 from below. For «shouls», read «should».

Page 149, line 3 from below. For «Marcud», read «Marcus».

Page 195, line 9 from below. «(1)» should be flush left.

Page 196, line 15. «(2)» should be flush left.

Page 196, line 13 from below. Insert a left parenthesis after «(y)».

Page 200, line 6. Insert a superscript  $l$  after  $\Sigma$ .

Page 213, line 11. For «*indépendent*», read «*indépendant*».

Page 213. Line 17 from below should be moved flush left.  
Page 216, lines 17 and 19. For 197(3), read 197(4).  
Page 216. Insert space between lines 3 and 2 from below.  
Page 217, line 3. Insert a period after «*mathematics*».  
Page 217, line 8. For «no. 4», read «no. 4A».  
Page 217, line 10. For «no. 5», read «no. 5A».  
Page ii, line 5. For «Januari», read «January».

#### VOLUME 30

Page 92, line 5 from below. For «Ladrière», read «Ladrière».  
Page 92, line 4 from below. For 546 **I**, read 546 **I**.  
Page 97, line 14. For all occurrences of  $v$ , read  $v$ , and for both occurrences of  $F$ , read  $F$ .  
Page 100, line 28. For «pp. 31–34», read «pp. 31–43».  
Page 101, line 13. Insert a left parenthesis before «See».  
Page 256, line 25. Delete the left parenthesis before the first occurrence of  $f(x, y)$  and one of the right parentheses before the right square bracket.  
Page 259, line 5 from below. For «no. 12», read «no. 12A».  
Page 360, line 27 from below. For «lu», read «1u».  
Page 361, line 1. For «doutbfu», read «doubtful».  
Page 361, line 3. For «not», read «no».  
Page 365, line 3. For «sic», read «*sic*».  
Page 380, line 13. Delete the period after  $K$ .  
Page 387, line 28. For «GRZEGORZYK», read «GRZEGORCZYK».  
Page 388, line 3. In the formula near the end of the line, a letter  $l$  should replace the figure 1.  
Page 392, line 27. For XVI 72, read XVII 152.  
Page 393, line 4. For XXX 000, read XXX 252.  
Page 393, line 10. For «*Reprezentation*», read «*Representation*».  
Page 394, line 26. For  $(TL_{pf})$ , read  $T(L_{pf})$ .

#### VOLUME 31

Page 111, line 22 from below. For «peripherically», read «peripherally».  
Page 112, line 10. For «*texts*», read «*texts*».  
Page 121, line 10. For  $((G, \mathfrak{R}, \mathbf{R})\Phi)$ , read  $((G, K, \mathbf{R})\Phi)$ .  
Page 131, line 25. Delete the dot over the aleph.  
Page 143, line 9. The parenthesis enclosing «*recenzja*» should be square brackets.  
Page 388, line 21. For «show to obtain», read «show how to obtain».

A new addition to B&N's  
COLLEGE OUTLINE  
SERIES  
\$1.50

## Modern Logic

Norman L. Thomas

KEYED  
TO YOUR  
STANDARD TEXT

*A new introductory text designed to offer your students deeper and more lasting insights into contemporary symbolic logic—concise, up-to-date approach!*

Modern Logic begins with an essentially intuitive presentation of sets and logical deduction. Then the more rigorous systems—Propositional and Class Calculus—are introduced as axiomatizations of their intuitive counterparts. A special chapter deals with the application of Class Calculus (Boolean Algebra) to the algebra of electrical circuits. The final chapter discusses topics in the philosophy of logic. **Special features are:** a system of designating the areas of Venn Diagrams by numbers, and a distinctive method for indicating the order of steps in constructing Truth Tables. Exercises and answers included. 236 pages.

AT YOUR BOOKSELLER

**Barnes & Noble, Inc.**

105 Fifth Avenue, New York, N. Y. 10003

## THE JOURNAL OF SYMBOLIC LOGIC

New Subscription Rates to  
Non-Members and Institutions\*:  
\$ 17 per yr.

Subscription Rates for Back  
Issues to Non-Members and  
Institutions\*:

Vols. 1-17

\$17 per volume; \$5 single issue

Vols. 18-25

\$20 per volume; \$6 single issue

Vols. 27-30

\$20 per volume; \$6 single issue  
(when available)

Please send orders to:

**Association for Symbolic Logic**  
**P.O. Box 6248**  
**Providence, R. I. 02904**

\* Institutions include libraries, laboratories,  
and research departments

## FROM POLAND STUDIA LOGICA

First published 1953.  
One or two volumes per year.

Papers on the whole area of logic, its history and didactics including critical reviews of the most important Polish and foreign publications in the field.

Articles in Polish, Russian, English, French or German with summaries in Polish, Russian or English.

Contents listed in Polish, Russian and English.  
Approximately \$ 5.00 per volume.

Available from

**Stechert-Hafner, Inc.**

The World's Leading International Booksellers

31 East 10th Street, New York, N. Y. 10003

Offices in England, France, Germany, Colombia

IS YOUR TYPEWRITER  
AS SCIENTIFIC AS YOUR  
SPECIALTY?

It can be, with TYPIT®

The Easy, Economical  
Attachment That TYPES  
ANY SYMBOL IN JUST 4  
SECONDS on your present  
machine.

OVER 1500 STOCK SYMBOLS  
ANY SYMBOL MADE TO ORDER

Write for free catalog!



**typit**® Mechanical Enterprises, Inc. Dept. 2412-A  
3127 Colvin Street, Alexandria, Va. 22314

SOLD IN EUROPE BY: Wilhelm Dreusicke & Co. KG  
1 Berlin 30  
Lectzowstrasse 3  
(British Sector)