# Appendix C
# Useful relations in the treatment of collective modes

In this appendix we give some simple relations used in the treatment of collective surface vibrations in the harmonic approximation.

## C.1 Limit on the multipolarity of collective surface vibrations

Collective surface vibrations can be self-sustained modes provided the ripples they produce on the surface contain many particles, so that the surface can be viewed as a continuous elastic medium. In other words (see Fig. C.1(a))

$$\frac{2\pi R}{2\lambda} \gg d, \tag{C.1}$$

where $R = 1.2A^{1/3}$ fm is the nuclear radius, $\lambda$ is the multipolarity of the surface mode and

$$d = \left(\frac{\frac{4\pi}{3}R^3}{A}\right)^{1/3} \approx 2\,\text{fm} \tag{C.2}$$

is the mean distance between nucleons. From equations (C.1) and (C.2) one obtains (see Fig. C.1(b))

$$\lambda \ll 2A^{1/3} \approx 10 \tag{C.3}$$

for a nucleus with mass number $A \sim 120$. This result agrees well with the experimental fact that collective states in medium-heavy mass nuclei have multipolarities $\lambda \leq 5$.

## C.2 The relation between $\hat{F}$ and $\hat{\alpha}$

The operator $\hat{F}$ defined in equation (8.29) is restricted, in the random phase approximation, to either create or destroy particle–hole excitations, i.e.

$$\hat{F} = \sum_{\nu_k \nu_i} \left\{ \langle \nu_k | F | \tilde{\nu}_i \rangle \Gamma^{\dagger}_{\nu_k \nu_i} + \langle \tilde{\nu}_i | F | \nu_k \rangle \Gamma_{\nu_k \nu_i} \right\}. \tag{C.4}$$



https://doi.org/10.1017/9781009401920.015 Published online by Cambridge University Press

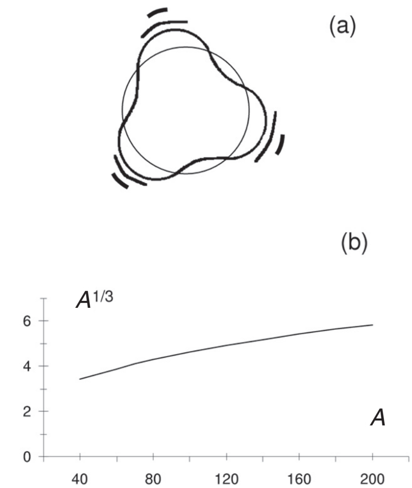


Figure C.1. (a) Schematic representation of an octupole surface wave. (b) The quantity $(A)^{1/3}$ as a function of $A$ for medium-heavy nuclei.

Making use of equation (8.43) and the corresponding equation for $\Gamma_{\nu_k\,\nu_i}$ one can write equation (C.4) in terms of the RPA boson operators $\Gamma^\dagger_\alpha$ and $\Gamma_\alpha$, according to

$$\begin{aligned}\hat{F} &= \sum_{\substack{\nu_k\,\nu_i\\ \alpha'}} \left\{ \frac{\Lambda_{\alpha'}\,|\langle\tilde{\nu}_i|F|\nu_k\rangle|^2}{(\varepsilon_{\nu_k}-\varepsilon_{\nu_i})-\hbar\omega_{\alpha'}}\Gamma^\dagger_{\alpha'} - \left(-\frac{\Lambda_{\alpha'}\,|\langle\tilde{\nu}_i|F|\nu_k\rangle|^2}{(\varepsilon_{\nu_k}-\varepsilon_{\nu_i})+\hbar\omega_{\alpha'}}\right)\Gamma_{\alpha'} \right.\\ &\qquad \left. + \frac{\Lambda_{\alpha'}\,|\langle\tilde{\nu}_i|F|\nu_k\rangle|^2}{(\varepsilon_{\nu_k}-\varepsilon_{\nu_i})-\hbar\omega_{\alpha'}}\Gamma_{\alpha'} - \left(-\frac{\Lambda_{\alpha'}\,|\langle\tilde{\nu}_i|F|\nu_k\rangle|^2}{(\varepsilon_{\nu_k}-\varepsilon_{\nu_i})+\hbar\omega_{\alpha'}}\Gamma^\dagger_{\alpha'}\right)\right\}\\ &= \sum_{\alpha'}\Lambda_{\alpha'}\sum_{\nu_k\,\nu_i}\frac{|\langle\tilde{\nu}_i|F|\nu_k\rangle|^2\;2(\varepsilon_{\nu_k}-\varepsilon_{\nu_i})}{(\varepsilon_{\nu_k}-\varepsilon_{\nu_i})^2-(\hbar\omega_{\alpha'})^2}(\Gamma^\dagger_{\alpha'}+\Gamma_{\alpha'})\\ &= \sum_{\alpha'}\frac{\Lambda_{\alpha'}}{\kappa}(\Gamma^\dagger_{\alpha'}+\Gamma_{\alpha'}) = \sum_{\alpha'}\sqrt{\frac{\hbar\omega_{\alpha'}}{2C_{\alpha'}}}(\Gamma^\dagger_{\alpha'}+\Gamma_{\alpha'}) = \hat{\alpha},\end{aligned} \tag{C.5}$$

where use has been made of equation (8.39).

In other words, $\hat{F}$ and $\hat{\alpha}$ are the single-particle and the collective representations of the same operator.

https://doi.org/10.1017/9781009401920.015 Published online by Cambridge University Press