Then $m=\frac{1}{2}(c_1+m_1)$, $m_1=\frac{1}{2}(c_2+m_2)$, $m_2=\frac{1}{2}(c_3+m_3)$ and so on, where c_r is either 1 or 0 according as the square of antilog m_{r-1} is greater or less than 10.

Thus $$m=\frac{c_1}{2}+\frac{c_2}{2^2}+\frac{c_3}{2^3}+\ldots+\frac{c_n}{2^n}+\frac{m_n}{2^n}.$$

R. F. MUIRHEAD.

Trigonometrical Ratios of (A ± B).

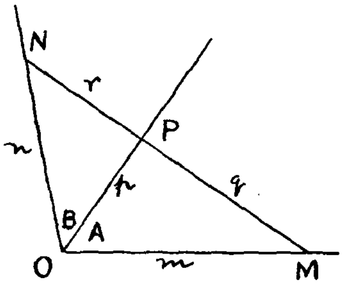

Fig. 1

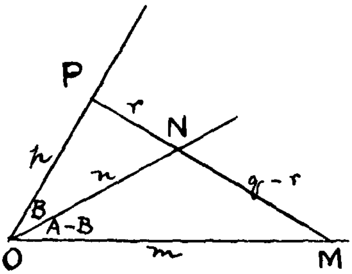

Fig. 2.

The formulae for sin $(A \pm B)$ and cos $(A \pm B)$ may be derived by the following method, the main attraction of which is the simplicity of the figures employed.

Consider the case in which A and B are both acute angles.

Let $\angle MOP=A$ and $\angle PON=B$, then in Fig. 1 $\angle MON=A+B$ and in Fig. 2 $\angle MON=A-B$. Through any point P on OP, the arm common to both angles, draw a perpendicular to OP meeting the other arms in M and N respectively.

Let $OM=m$, $ON=n$, $OP=p$, $MP=q$, $PN=r$.

From the triangle OMN we obtain

$$\sin(A \pm B)=\sin MON=\frac{2\,\Delta OMN}{mn}=\frac{p(q\pm r)}{mn}$$

$$=\frac{q}{m}\cdot\frac{p}{n}\pm\frac{p}{m}\cdot\frac{r}{n}$$

$$=\sin A\cos B\pm\cos A\sin B.$$

$$\cos(A \pm B) = \cos MNO = \frac{m^2 + n^2 - (q \pm r)^2}{2mn}$$

$$= \frac{(p^2 + q^2) + (p^2 + r^2) - (q^2 \pm 2qr + r^2)}{2mn}$$

$$= \frac{p^2 \mp qr}{mn}$$

$$= \frac{p}{m} \cdot \frac{p}{n} \mp \frac{q}{m} \cdot \frac{r}{n}$$

$$= \cos A \cos B \mp \sin A \sin B.$$

When A and B are unrestricted in size the perpendicular to the common arm may meet one or both of the other arms produced, but only slight modifications are required in the proofs.

ALEX. D. RUSSELL.