A LINEAR AND NONLINEAR STUDY OF MIRA

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Abstract. Both linear and nonlinear calculations of the 331 day, long period variable star Mira have been undertaken to see what radial pulsation mode is naturally selected. Models are similar to those considered in the linear nonadiabatic stellar pulsation study of Ostlie and Cox (1986). Models are considered with masses near one solar mass, luminosities between 4000 and 5000 solar luminosities, and effective temperatures of approximately 3000 K. These models have fundamental mode periods that closely match the pulsation period of Mira. The equation of state for the stellar material is given by the Stellingwerf (1975ab) procedure, and the opacity is obtained from a fit by Cahn that matches the low temperature molecular absorption data for the Population I Ross-Aller 1 mixture calculated from the Los Alamos Astrophysical Opacity Library. For the linear study, the Cox, Brownlee, and Eilers (1966) approximation is used for the linear theory variation of the convection luminosity. For the nonlinear work, the method described by Ostlie (1990) and Cox (1990) is followed. Results showing internal details of the radial fundamental and first overtone modes behavior in linear theory are presented. Preliminary radial fundamental mode nonlinear calculations are discussed. The very tentative conclusion is that neither the fundamental or first overtone mode is excluded from being the actual observed one.

1. Background

Mira variables have been studied observationally for hundreds of years, because many of them are bright enough to be seen with the naked eye or with small telescopes, and their periods are so long that constant attention is not needed. Theoretical studies, however have been far fewer, mostly because the extensive and deep convection is difficult to cope with in the context of stellar pulsation. In this paper we present new theoretical results, but a definitive conclusion for which radial mode of pulsation is actually occurring still eludes us.

The basic parameters of Mira variables are their pulsation periods (or many periods for the red semiregular variables), their radii, and their luminosities. Surface compositions seem to be at least hydrogen rich as for most stars, even though these stars are obviously remnants of more massive stars that have undergone significant mass loss. Thus the composition of the pulsating envelopes, while surely homogeneous, may not be that for normal solar type stars. Numerous composition anomalies are known. For example, these red giants are often in spectral classes R, N, and S. Mira masses are unknown and disputed, but most seem to come from stars with original mass of less than 2 solar masses. More massive progenitors become red supergiants

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and even Mira variables, but their red supergiant lifetimes must be short as they rapidly lose mass to create planetary nebulae and ultimately white dwarfs. Most Miras have currently near only one solar mass.

The evolution of the Mira stars is just at the Hayashi line on the Hertzsprung-Russell diagram, because their luminosity is very large for their mass. The vary low mass envelopes result from the high luminosity blowing out the matter to large radii. The low temperature of the matter gives it a high opacity, and consequently strong convection to carry the high luminosity.

Nonlinear calculations to study the behavior of the Mira pulsations have been made by Wood (1974), by Tuchman, Sack, and Barkat (1978, 1979), and by Perl and Tuchman (1990). We at Los Alamos have been trying to do these calculations for 10 years, but we have not been able to be satisfied that our results are correct. We believe that the neglect of turbulent pressure in all previous calculations is a significant deficiency, and the instantaneous adaptation of convection by the Israel authors may be a bad approximation. Studies of the upper atmospheres of Miras have also been carried out by numerous groups using a driving piston boundary condition near the photosphere, with the latest papers being by Bowen (1988) and Beach (1990). The results of these various studies give conflicting conclusions as to the pulsation mode of Mira. Our conclusion is that either the fundamental or the first overtone mode is allowed, but our demonstration is not yet secure.

2. Time-Dependent Convection Procedure

In both our linear and nonlinear studies we have adopted the approximate time-dependent theory of Cox, Brownlee, and Eilers (CBE, 1966). The concept is that the instantaneous conditions suggest a convection luminosity that cannot be realized because the turbulent eddies have an inertia that the hydrodynamic forces cannot overcome rapidly. Thus there is a lag behind the desired luminosity that can be represented by the formula for the convection luminosity increment:

$$dL_c = \frac{dt}{\tau}[\bar{L}_c(t) - L_c(t)],$$

where $\tau(t)$ is the mean eddy lifetime at time t at the level of interest in the convection zone, and $\bar{L}_c$ is the instantaneous convection luminosity desired. If τ is small, convection can adapt well, but for Mira variables it is often comparable to the pulsation period Π. The instantaneous luminosity is given by

$$\bar{L}_c = Ae^{i\omega t},$$

where ω is $2\pi/\Pi$, and A is the amplitude for completely adapting convection.

The solution for such a model with time-dependent convection is that the convection luminosity increment, with the τ taken constant in time, is

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$$\delta L_c(t) = B\, e^{i(\omega t - \theta)},$$

where

$$B = \frac{A}{\sqrt{(1 + (2\pi\tau/\Pi)^2}},$$

and

$$\theta = \tan^{-1}(2\pi\tau/\Pi).$$

The amplitude B actually realized is often much smaller than the amplitude A. This is because as the convection is struggling to reach its amplitude, the pulsation configuration changes sign rapidly, overcoming any convection luminosity changes before they can be attained.

Implementation of this time-dependent convection involves first calculating the partial derivatives of the convection luminosity with respect to the temperature and density on both sides of an interface between Lagrangian mass shells in the stellar model. These are calculated during model construction. They are then converted to derivatives with respect to the perturbations of our linear theory, that is δr and $T\delta S$. Then these terms are added to the existing matrix elements (coefficients of the perturbation variables) that represent the linearized momentum and energy equations for the stellar mass elements. Allowance is made for the fraction of the total luminosity due to convection, the actual reduced amplitude from the CBE procedure, and the cosine of the lag angle θ. Additional matrix elements are necessary for the imaginary parts of the convection luminosity variations also, and they use the sine of the lag angle instead of the cosine.

For many stellar models, the logarithmic derivatives of the convection luminosity are huge, maybe over 200! For radiation they rarely get about 15 for the temperature derivative and much smaller for the density derivative. Thus matrix elements are sometimes greatly increased, and the nonadiabatic eigenvector and eigensolution can be considerably different from the adiabatic one.

3. Mira Models

For this work we have constructed many models. The one discussed in detail here has one solar mass, a luminosity of 4000 solar luminosities, and an effective surface temperature of 3000 K. The opacity used is given by a fit by Cahn to the Ross-Aller 1 solar composition with $X = 0.70$ and $Z = 0.02$. The Stellingwerf (1975ab) analytic equation of state is used throughout. For both linear and nonlinear studies 60 mass shells have been used. Table I gives additional interior details.

The construction of our envelope models necessarily includes turbulent pressure using only the convective velocity at the exterior interface of each

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TABLE I
Mira Variable Star Model

Mass: $1.0M_{\odot}$ ($1.989 \times 10^{33} g$)
Luminosity (constant through model): 1.53×10^{37} erg/s ($4000L_{\odot}$)
Effective Temperature: 3000 K (1.628×10^{13} cm)
Composition (constant through model): $X = 0.70$ $Z = 0.02$
Mass Shells: 60
Last Shell Mass: $7 \times 10^{28} g$ ($\tau = 3 \times 10^{-2}$)
Central Ball Mass: $9.2 \times 10^{32} g$ ($q = 0.46$)
Central Ball Radius Fraction: 0.04
Mass Ratios: range from 1.3 to 1.0 to 1.1 to 0.965
Convection Zone Temperature Range: 2.6×10^{3} K to over 2.8×10^{5} K
Radiative Luminosity Minimum: 2.0×10^{-5} at 15,800 K
Mixing Length/Pressure Scale Height: 2.60
Turbulent Pressure
Time Dependent Convection
Fundamental Mode Period and Kinetic Energy Growth Rate: 330 days, 1.23
First Overtone Mode Period and Kinetic Energy Growth Rate: 152 days, 0.73

mass shell, because as the integration proceeds, the interior interface convection is not yet known. When this model and its linear theory eigenvector is used to start the hydrodynamic calculations, turbulent pressure from both sides of a mass shell is needed. This is produced linearly over the first hydro period, and it affects the entire mean structure as well as the nonlinear solution. The relaxation can be done rather easily, because the time scale of the layers where the turbulent pressure is significant is typically a small number of pulsation periods.

4. Linear Results

Plots of the work per pulsational cycle to drive or damp pulsations are given in the five figures. The first one shows the hydrogen ionization driving that has been considered for years as the sole cause of the instability. This driving occurs just exterior to the strong convection zone top near 9000 K, because at the usual hydrogen driving temperature of 11,000 K, convection is too strong to allow the κ effect periodic radiation blocking. Figure 1 presents the strong fundamental and weaker overtone net driving, with extremely small damping.

Figure 2 shows a very different situation where our time-dependent convection is allowed to adapt completely to the current configuration. This means that θ is zero at all times for all mass zones. Again the fundamental mode is more strongly driven. The overtone has two peaks in its driving be-

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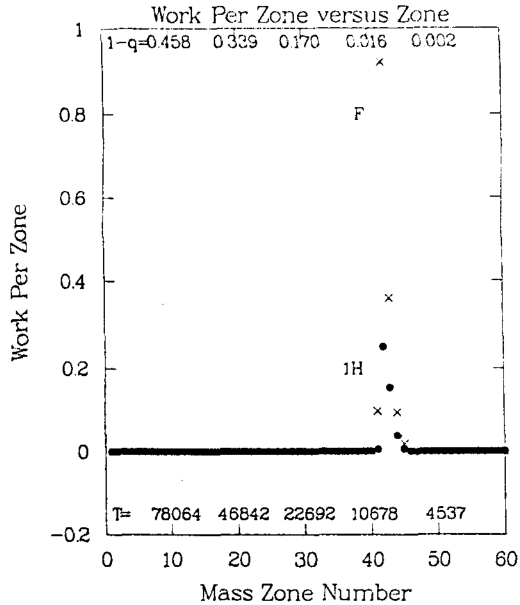

Fig. 1. The work per zone each pulsation cycle versus zone number and external mass fraction for a linear solution with frozen-in convection.

cause it has a node in its oscillation eigenvector at a temperature just over 11,000 K. The driving for both modes occurs between about one and ten percent of the mass of the star, but the damping extends deeply to almost half of the mass.

Figure 3 presents our best linear interpretation of what occurs for driving and damping in Mira stars. Here θ is allowed to be greater than zero, as appropriate for convection lagging. Note first that there are two fundamental mode peaks and three overtone peaks in the driving, whereas completely adapting convection gave only one for the fundamental and two for the overtone.

Figure 4 shows the time-dependent convection case when turbulent pressure is ignored. Again significant differences can be seen. The fundamental mode period is changed from 330 days to 343 days, but the growth rate is unchanged. For the overtone, the period is changed from 152 days to 153 days, and the growth rate is decreased from 0.73 to 0.44 per cycle relative to our best case in figure 3.

An interpretation of these time-dependent cases is displayed as Figure 5. Here $\Gamma_3 - 1$ is plotted versus the same zone numbers. This quantity reveals the relation between the temperature, density, and entropy variations as

$$\delta T/T = (\Gamma_3 - 1)\delta\rho/\rho + \delta S/c_v.$$

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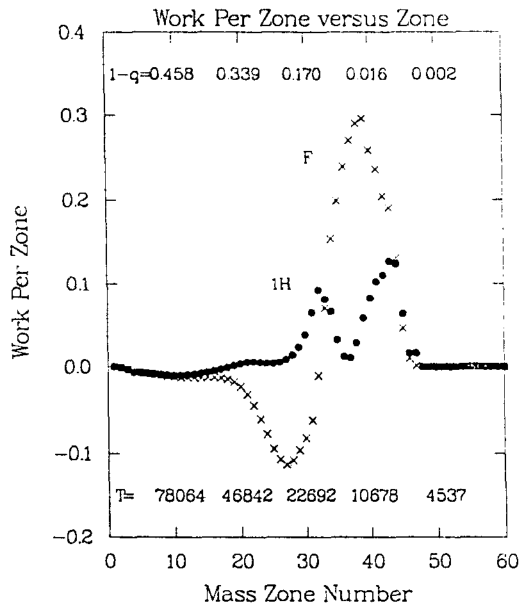

Fig. 2. The work per zone each pulsation cycle versus zone number and external mass fraction for a linear solution with completely adaptive convection.

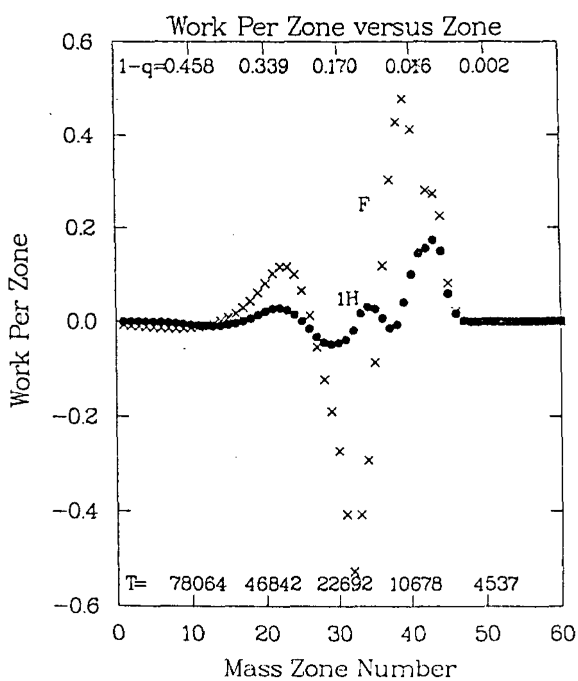

Fig. 3. The work per zone each pulsation cycle versus zone number and external mass fraction for a linear solution with the properly lagging adaptive convection.

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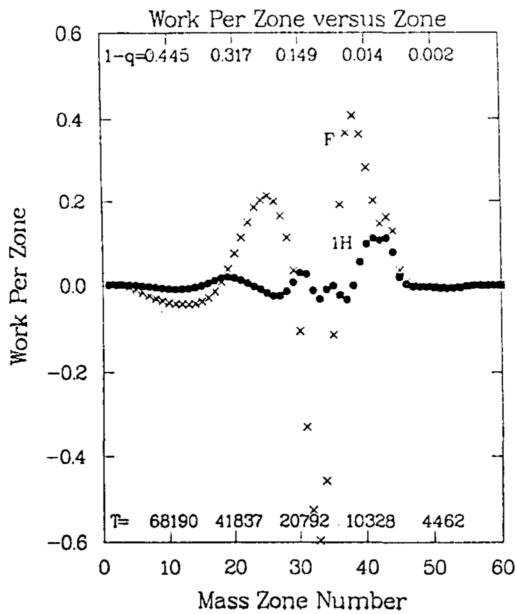

Fig. 4. The work per zone each pulsation cycle versus zone number and external mass fraction for a linear solution with the properly lagging adaptive convection, but no turbulent pressure.

Places where $(\Gamma_3 - 1)$ is small are where the temperature fluctuations are small and where convection cannot be modulated so much. These then are places where the pulsation damping is smaller. Convection seems to drive when the $\Gamma_3 - 1$ is rapidly changing giving a rapid change in space of the temperature eigenvector. Note that the driving comes largely from the spatial variations of δr and $T\delta S$ in the eigenvector. The other factors in the convection luminosity variations, such as the fraction of the luminosity being carried by convection, the CBE amplitude, and the lag angle are only very slowly varying throughout the model.

5. Nonlinear Results

Our procedures for nonlinear calculations were described by Ostlie (1990) and Cox (1990). The main features are: Turbulent pressure is included. Turbulent viscosity, while small, is included. There is weighted spatial averaging of convective velocities from neighboring interfaces at the previous time step, with an adjustment for their velocities relative to the interface of interest. This spatially averaged velocity for an interface is time lagged according to the CBE procedure, and this velocity is then used in the mixing length luminosity formula. Finally, for an interface with a subadiabatic gradient,

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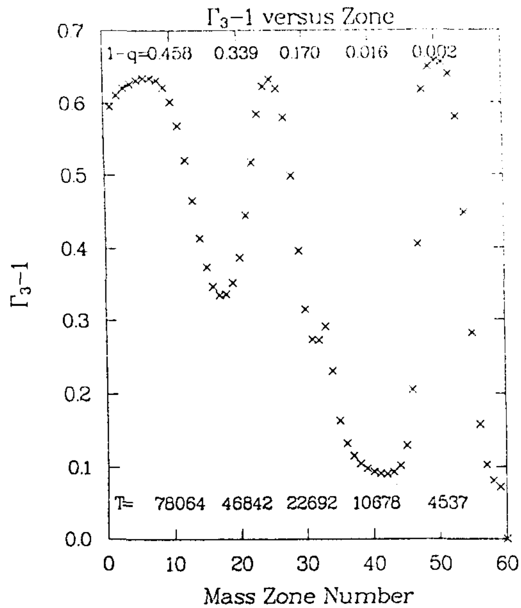

Fig. 5. The $\Gamma_3 - 1$ versus mass zone number and external mass fraction for the 3000 K, 5000 solar luminosity model.

the spatially average convective velocity is decreased in time by the mixing length theory dragging formula, and the luminosity at this subadiabatic interface is taken to be negative.

Current results for our nonlinear work include a fundamental mode case at the very long period of 998 days with an amplitude of about one kilometer per second. We have found that most models tend to decay in amplitude as they expand and cool from the hydrostatic models. Many more calculations are in progress.

6. Current Conclusions

Since turbulent pressure plays a significant role in limiting Mira pulsation amplitudes, hydrodynamic calculations without this real pressure, such as those by all earlier calculations, are probably not realistic.

Completely adapting convection produces a different phasing of the convection luminosity than that for the correct lagging, and thus results using completely adapting convection are not highly accurate.

Our use of masses at and above one solar mass produces unobserved low pulsation amplitudes, indicating that Mira masses are small.

For masses as low at 0.9 solar mass, both fundamental and overtone modes grow rapidly in amplitude and do not reach realistic limiting values.

Most important, fundamental mode pulsation for Mira variables is not excluded.

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