On certain new connections between Legendre and Bessel Functions

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Let n be a positive integer. Then we know that[1], if $m > -1$,

$$\int_0^1 P_n(1-2y^2)\,y^{2m+1}\,dy = \tfrac{1}{2}(-1)^n \frac{\{\Gamma(m+1)\}^2}{\Gamma(m-n+1)\,\Gamma(m+n+2)}. \tag{1}$$

Consider the integral

$$I = \int_0^1 P_n(1-2y^2)\,J_0(2yz)\,y\,dy,$$

which is equal to[2]

$$\sum_{m=0}^{\infty} \frac{(-1)^m z^{2m}}{\{\Gamma(m+1)\}^2} \int_0^1 P_n(1-2y^2)\,y^{2m+1}\,dy.$$

On integrating term by term, we get

$$\begin{aligned} I &= \tfrac{1}{2}(-1)^n \sum_{m=0}^{\infty} \frac{(-1)^m z^{2m}}{\Gamma(m-n+1)\,\Gamma(m+n+2)} \\ &= \tfrac{1}{2} \sum_{m=0}^{\infty} \frac{(-1)^m z^{2m+2n}}{\Gamma(m+1)\,\Gamma(m+2n+2)} \\ &= (2z)^{-1} J_{2n+1}(2z). \end{aligned} \tag{2}$$

In a similar manner, we can prove the following results:

$$\int_0^1 P_n(1-2y^4)\,J_0(2yz)\,I_0(2yz)\,y^3\,dy = (8z)^{-1}\frac{d}{dz}\{J_{2n+1}(2z)\,I_{2n+1}(2z)\}, \tag{3}$$

$$\int_0^1 P_n(1-2y^4)\,\frac{d}{dy}\,[y^2\{\mathrm{ber}_1^2(2yz)+\mathrm{bei}_1^2(2yz)\}]\,dy = (-1)^n\{\mathrm{ber}_{2n+1}^2(2z)+\mathrm{bei}_{2n+1}^2(2z)\}, \tag{4}$$

and

$$\int_0^1 P_n(1-2y^4)\,\frac{d}{dy}\{y^2 J_1(2yz)\,I_1(2yz)\}\,dy = J_{2n+1}(2z)\,I_{2n+1}(2z). \tag{5}$$

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[1] Equation (1) follows at once by putting $x = 1 - 2y^2$, using Rodrigues' formula for $P_n(x)$, and integrating n times by parts. *Cf.* Cooke, *Proc. London Math. Soc.*, 23 (1924), xix, equ. (3).

[2] The process of arrangement and term by term integration can be easily justified.

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