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# IRVING FISHER, RAGNAR FRISCH, AND THE ELUSIVE QUEST FOR MEASURABLE UTILITY

BY
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*Commitment to the behaviorist approach to utility theory, to the usefulness of mathematics in economic analysis, and to equalization of the marginal utility of income as a principle of just taxation brought Irving Fisher and Ragnar Frisch to attempt to measure the marginal utility of income and led them to collaborate in forming the Econometric Society and sponsoring the establishment of the Cowles Commission, institutions advancing economic theory in connection to mathematics and statistics, and led Frisch to pioneer an axiomatic approach to utility and microeconomic theory.*

## I. INTRODUCTION

Ragnar Frisch (1926, 1932, 1947) acclaimed Irving Fisher (1892) as the pioneer of what Frisch termed the "behavioristic" or ordinalist approach to consumer choice theory and utility,[1] preceding Vilfredo Pareto and differing from Francis Edgeworth's hope to measure sensations. While preference ordering suffices for analysis of individual choice of goods, the shift in neoclassical choice theory from cardinal to ordinal utility had strengthened the positive theory of demand while weakening the normative foundation for welfare economics. Seeking to restore the welfare basis for using economics to guide policy decisions, Frisch (1926, 1932) and Fisher (1927) sought to measure marginal utility

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[1] As Edwards (2016) and Herfeld (2019) explain, by "behavioristic" Frisch referred to empirically meaningful, interpersonally observable study of consumer behavior, not to the behaviorist psychology of his time, which stressed conditioned response rather than preference-based choice. See Thurstone (1927) on the efforts of psychologists to use just-noticeable differences as a unit of measurement at the time when Frisch and Fisher were attempting to measure the marginal utility of money.

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by identifying some commodity class whose marginal utility was independent of the consumption of other classes of commodities. Fisher (1927) linked this goal to Edgeworth's principle that just taxation should equalize the utility of taxes paid across people (see also Fisher 1937, one of many examples of his continuing concern with this goal).[2] A progressive income tax could be justified by Edgeworth's principle only by interpersonal comparison of utility levels. Frisch and Fisher were unsuccessful in identifying such a commodity class, or in devising any method of measuring utility that could be used to evaluate the justice of an income tax schedule or otherwise guide public policy. Further research on cardinal utility, such as by John von Neumann and Oskar Morgenstern (1944) and by Fisher's protégé William Vickrey (1945), looked instead to comparison of risky situations, using probability weights to measure the marginal utility of wealth, while researchers such as Abraham Wald (1940) and Allen Wallis and Milton Friedman (1942) looked to determination of indifference surfaces rather than utility functions.[3]

Nonetheless, the failed attempts by Frisch and Fisher to measure the marginal utility of money had important consequences, beyond the stimulus their writings gave to Vickrey (1945) and beyond the literature on utility measurement. Frisch (1926), followed up by Franz Alt ([1936] 1971, 1937), pioneered the axiomatic approach to utility theory (see Bjerkholt and Dupont-Kieffer 2007), and Frisch drew attention to Fisher's priority over Pareto in the history of choice theory.[4] The shared concern of Frisch and Fisher with measurement of marginal utility brought Frisch to Yale as a visiting professor in 1930–31, leading to their collaboration in founding the Econometric Society at the end of 1930 and then in late 1931 and early 1932 to persuade the Econometric Society to accept Alfred Cowles's offer to finance both a journal (*Econometrica*) and a research foundation affiliated with the society (the Cowles Commission for Research in Economics).[5] Yale's offer to Frisch to remain as a full professor of economics at the end of his visiting position was a major factor stimulating the

[2] Frisch, as editor of *Econometrica*, allotted fifty-five pages to Fisher (1937) and made it the year's lead article.

[3] On the history of attempts to measure utility, from the marginalists of the 1870s to behavioral economics, see Moscati (2019), and, from Bentham to neuroeconomics, see Narens and Skryms (2020); as well as Colander (2007), Chaigneau (1998, 2014), and Mueller (2020) on Edgeworth; and Chipman (1998), Dupont-Kieffer (2003, 2013), and Herfeld (2019) on Frisch's contribution.

[4] Fisher's doctoral dissertation was accepted in 1891 and then published in 1892, so it preceded Pareto's series of journal articles (Pareto 1892–93), Pareto's first major publication, but not by long. Pareto ([1892–93] 2007) viewed utility as being measurable in principle, adopting an ordinalist approach in Pareto ([1900] 2008) (see Moscati 2019, p. 81). Frisch, although familiar with Pareto's later, much better-known books, may not have known Pareto's early articles. The question of the integrability of a utility function, that is, the conditions under which the existence of a utility function is guaranteed if there are more than two commodities, was raised by Antonelli ([1886] 1971) and Fisher (1892) but taken up independently only by Pareto ([1906] 1971). On ordinal utility and indifference curves, see also Johnson and Sanger ([1894] 1968) and Johnson (1913).

[5] The founding of the Econometric Society and the Cowles Commission is of course discussed in every history of econometrics (e.g., Epstein 1987; Morgan 1990; Louçã 2007, with Louçã particularly attentive to Frisch's role) or of the Cowles Commission (Christ 1952; Hildreth 1986) but with little attention to what shared interest brought Fisher and Frisch together for the 1930–31 academic year, providing the occasion for them to organize the Econometric Society. Hildreth (1986, p. 1) reports only that Fisher and Charles F. Roos "joined with others to organize the Econometric Society" without naming Frisch. Christ (1952, p. 6) refers to "Frisch (who was again in the United States, this time as a visiting professor at Yale)" without indicating why Frisch happened to spend that academic year at Yale.

University of Oslo to appoint Frisch as a full professor, just five years after his doctorate (and with only two years of formal study in economics, as distinct from mathematical statistics), and to establish a research laboratory for him. Ivan Moscati (2019, p. 121) reports that, after his year visiting Yale, Frisch returned to Oslo as a professor of economics and statistics and as director of the newly established Institute of Economics but not that the professorship so early in Frisch's career and the creation of the institute resulted from Fisher persuading Yale to try to hire Frisch. The shared scientific interest of Fisher and Frisch, over and above their place in the history of utility measurement, led to Frisch pioneering an axiomatic approach to utility and microeconomic theory and was crucial both for Frisch's academic career and for the establishment of the Econometric Society, *Econometrica,* and the Cowles Commission, institutions that were transformative in advancing formal mathematical and statistical methods in economics.

## II. IRVING FISHER ON MEASURABLE UTILITY

Irving Fisher (1927, pp. 157–158) regretted that:

> The basic importance of this concept [marginal utility[6]] has been partially lost sight of because of the growth of statistical economics and the lack hitherto of any method of showing that such a purely psychical magnitude is at least capable of being measured, granted the necessary data.… If so-called 'marginal utility' of anything (or, as I prefer to say, if the want-for-one-more unit of anything) is a true mathematical quantity, should not the marginal want be measurable?

Fisher added, "In my first economic publication [1892, pp. 11–24, 86–89], I endeavored to show that this magnitude is measurable—in theory, at least. My object here is to go one step further and to show that even the problem of statistically measuring it should not be considered impossible." In a footnote (1927, p. 158n), he stated, "So far as I know this is the only attempt (other than Edgeworth's therein cited [in Fisher 1892]) of treating 'utility' or 'want' as a definite mathematical quantity."

Fisher there drew attention to a remarkable aspect of his and Edgeworth's works. W. Stanley Jevons, Carl Menger, and Léon Walras, the marginalist pioneers of the 1870s, had reasoned in terms of cardinal utility. In contrast, Edgeworth (1881) and Fisher (1892) pioneered an ordinal approach to preferences and choice, drawing indifference curves (before Vilfredo Pareto and William E. Johnson) and, for more than two commodities, indifference surfaces, and showing that choice theory did not require knowing the unobservable marginal utility of any good but only the ratio between the marginal utilities of any two goods, the marginal rate of substitution

[6] Fisher (1927, p. 157), writing in honor of John Bates Clark, praised Clark for "his discovery, independently of Jevons, Menger, and Walras or their anticipators, of the concept of 'Marginal Utility.' … He is the only American who has that honor." Although Fisher (1927) cited his dissertation (Fisher 1892) on the same page, he did not draw an explicit parallel to his own independent rediscovery of general equilibrium analysis, managing to obtain the books of Walras and Edgeworth only as he was about to submit his completed thesis: "The equations in Chapter I, §10, were found by me two years ago, when I had read no mathematical economist except Jevons" (Fisher 1892, p. 4).

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represented by the slope of the indifference curve, a curve given (in principle) by observable choices of an individual. All that needed to be observed was whether an individual preferred bundle A to bundle B, not how many utils of pleasure or desire or want the individual would derive from either bundle. And yet, having shown that cardinally measurable utility was not needed to analyze choice by an individual among bundles of goods, Edgeworth (1881, 1887) and Fisher (1892, pp. 11–24, 86–89) concerned themselves with how to measure marginal utility whereas Jevons, Menger, and Walras did not.[7] While ordinal utility, without interpersonal comparisons of utility, sufficed for understanding individual demand for goods, Edgeworth (1919) and Fisher (1927) later advanced interpersonal equalization of the utility of taxes paid as the criterion for just taxation. In the case of Fisher (1892), concern with measuring utility, even if cardinally measurable utility was not needed to analyze individual choice, was consistent with his engineering background as a student of the physicist Josiah Willard Gibbs, which contributed to Fisher's focus on concrete results. For his 1891 dissertation and 1892 book, Fisher did not merely rediscover the Walrasian system of simultaneous equations for general equilibrium but, acting as an engineer and under Gibbs's guidance, constructed a hydraulic mechanism to simulate the determination of equilibrium prices and quantities, an anticipation of the much later development of computable general equilibrium that required Fisher to assume specific measures of utility represented by the shape of the vessels holding the water (Scarf 1967; Dimand 2019, ch. 2).

Edgeworth (1881, 1887) appealed to experimental psychology, equating a util to the smallest perceptible increment in pleasure or sense of well-being (see Colander 2007). Fisher (1892, p. 5) protested, "This foisting of Psychology upon Economics seems to be inappropriate and vicious. Others besides Prof. Edgeworth have done it. Gossen and Jevons appeared to regard the 'calculus of Pleasure and Pain' as part of the profundity of their theory.… The result has been that 'mathematics' has been blamed [by J. K. Ingram] for 'restoring the metaphysical entities previously discarded.'" Edgeworth took no offense at the allegation of viciousness, reviewing Fisher's thesis enthusiastically in the *Economic Journal*, inviting Fisher to Oxford, visiting Fisher in New Haven, and, as editor of the *Economic Journal*, publishing Fisher's first four journal articles.[8] In contrast to Edgeworth's psychological approach, Fisher (1892) looked to the observable behavior of economic agents rather than the psychology behind that behavior. A great admirer of Fisher's dissertation, Ragnar Frisch, described what Fisher did in his thesis as "the behavioristic approach to the quantitative definition of economic utility … the choice point of view in the analysis of utility is perhaps best known in connection with Pareto's name. However, the theory of choice was, in fact, first introduced and developed with great consistency by Fisher" (Frisch's lectures at Yale in 1930,[9] in Bjerkholt and Qin 2010, pp. 83–84).

[7] See Daniel Ellsberg (1954) on the history of concepts of utility, and Chaigneau (1998, 2014) on Edgeworth.
[8] See Robert Loring Allen (1993, pp. 55, 63–64, 67, 104, 112, 133, 170) on the long friendship and intellectual exchange between Edgeworth and Fisher, and Chaigneau (2014) on their disagreement over the role of psychology in economics.
[9] Although Frisch said this while visiting Fisher's university, Yale, he had made a similar statement of Fisher's priority over Pareto in Frisch (1926).

## III. FISHER AND FRISCH AT YALE IN 1930–31

Fisher and Frisch first met when Frisch was in the United States on a Rockefeller Fellowship in 1927–28, at which time Frisch told Fisher about Frisch (1926), if Fisher had not previously heard of it. Their shared commitment to "the choice point of view in the analysis" led to Fisher (1927) and Frisch (1926, 1932; Bjerkholt 1995, pt. I), and, together with their shared support of the applicability of mathematics to economics (e.g., Fisher 1930), initiated a conversation between them that brought Frisch to Yale, where he lectured on "A Dynamic Approach to Economic Theory" from February to December 1930 (Bjerkholt and Qin 2010), a visiting professorship that resulted in Frisch and Fisher collaborating in the creation of the Econometric Society (Bjerkholt 1998, 2017). Yale's offer to Frisch of a professorship provoked the University of Oslo to promote him to full professor of economics and statistics from July 1931. As Olav Bjerkholt and Duo Qin (2010, p. 11) observe, the University of Oslo did so even though, since Frisch was a largely self-taught mathematician and statistician, "his [formal] educational background was unimpressive—only a two-year economics program at the University of Oslo," although he successfully defended a doctoral dissertation in statistics in Oslo in 1926.

Frisch (1926) and Fisher (1927) both approached measurement of marginal utility of money by seeking some class of commodities whose marginal utility was independent of the consumption of other classes of commodities. In the absence of such a commodity class, Fisher and Frisch, who were both index number theorists (Fisher 1922; Frisch 1930, 1936), each favored an approximation based on knowing that the constant-utility index of the cost of living is bounded by the Paasche and Laspeyres indexes. Bjerkholt and Qin (2010, p. 21) report, "After Frisch's arrival at Yale in 1930, [Fisher and Frisch] set out to write jointly a monograph on utility measurement. Their plan was somehow aborted; instead Frisch completed the manuscript of a monograph entitled *New Methods for Measuring Marginal Utility* while at Yale in 1931 and had it published in 1932." Frisch (1932) was dedicated to "Irving Fisher, the pioneer of utility measurement." Frisch (1947) recalled that he and Fisher quarreled over Fisher's refusal to adopt the concise mathematical style and axiomatic approach of Frisch (1926), even though, as Bjerkholt and Qin (2010, p. 21) observe, Frisch (1932) "does not pursue axiomatization or dynamization of the utility concept."

Although Frisch and Fisher never wrote their projected joint monograph, they collaborated in other ways during Frisch's year at Yale. They joined with Charles Roos of Cornell and Joseph Schumpeter on a circular letter in June 1930 soliciting interest in creating an international society for the advancement of economic theory in connection with mathematics and statistics. Then, in late November, they followed this with a letter of invitation to the organizing meeting of the Econometric Society at the end of December at the annual meeting of the American Economic Association and the American Statistical Association. The following year Alfred Cowles 3rd wrote to Fisher (the founding president of the Econometric Society) offering to finance what became *Econometrica* and the Cowles Commission for Research in Economics. Fisher's acquaintance with the Cowles family (having been a Yale undergraduate and member of the undergraduate senior society Skull and Bones at the same time as Cowles's father

and uncle),[10] the trust between Fisher and Frisch, and a visit by Frisch to Cowles in Colorado Springs combined to reassure the European members of the society's council, who were understandably skeptical of windfall offers of funding in the depths of the Depression (Bjerkholt 1998, 2017; Dimand 2019). Fisher and Frisch were the most active of the five members of the commission's advisory council (appointed by the Econometric Society), as well as speaking at Cowles summer research conferences in Colorado, and Frisch edited *Econometrica* from its beginning in 1933 until 1960.

Fisher had suggested forming a group or society to promote quantitative and mathematical economics, under the aegis of the American Association for the Advancement of Science, as early as 1912 (see Dimand 2019, pp. 204–205), but no concrete action was taken until Frisch's visiting position at Yale in 1930. Such a society would no doubt have eventually been formed, but Frisch's visiting appointment as Fisher's temporary colleague determined the timing, which resulted in the Econometric Society, upon Fisher's reassurance and Frisch's visit to Colorado to investigate, being in a position to respond when Alfred Cowles became disillusioned with stock market forecasters. In the wake of the 1929 Wall Street crash, Cowles, an investment counselor in Colorado Springs and heir to a publishing fortune (his grandfather was a cofounder of the *Chicago Tribune*), lost confidence both in the stock market letters to which he subscribed and in the one that he produced. Seeking to demonstrate statistically that stock market forecasters did no better than random predictions (see Cowles 1933, 1944; Dimand and Veloce 2010),[11] Cowles asked the advice of Harold Thayer Davis, an Indiana University mathematics professor who summered in Colorado Springs and who told him of the newly established Econometric Society. Given Cowles's devastating critique of stock forecasts, he needed, and demonstrated, great tact in approaching Fisher, sensitive and defensive about his notorious gaffe about stock prices reaching a permanently high plateau in October 1929.[12] The Econometric Society was a small-scale organization when Cowles guaranteed a minimum of $12,000 a year for the society, a journal, and a research laboratory: the society's total assets were $24.13 when Cowles began more than two decades as the society's treasurer in April 1932. The shared interest of Fisher and Frisch in measuring marginal utility, which brought Frisch to spend an academic year with Fisher and thus determined the timing of the creation of the Econometric Society, had institutional consequences for the advancement of formal mathematical and statistical methods in economics.

[10] Walter Friedman (2014, p. 201) states that Cowles, who graduated from Yale in 1913 and was tapped for the Yale senior society Skull and Bones, had studied with Fisher, but neither Cowles nor Fisher said so during the negotiations leading to Cowles's financing of the Econometric Society. Fisher's reassurance that Cowles's offer was not a crank letter (as suspected by the Econometric Society's vice-president François Divisia, its secretary-treasurer Charles Roos, and other council members) was based on having known Cowles's father and uncle as fellow undergraduates in the 1880s (see Christ 1952, p. 8).

[11] Brown, Goetzmann, and Kumar (1998) unfortunately conflate Cowles (1933 and 1944) into a single 1934 article. They thus miss Cowles's 1944 concession that one forecaster did manage to consistently outperform the market (unnamed, but, judging from the dates of the predictions, William Peter Hamilton in his *Wall Street Journal* editorials), which rendered redundant their laborious demonstration that Cowles's strictures on forecasters did not apply to Hamilton (see Dimand and Veloce 2010). Walter Friedman (2014) cites Cowles (1933) and Brown, Goetzmann, and Kumar (1998) but not Cowles (1944).

[12] See Fisher (1997, vol. 10, pp. 3–26), an address to the District of Columbia Bankers Association on October 23, but see also McGrattan and Prescott (2004) for an argument that Fisher was right.

## IV. FRISCH ON MEASUREMENT OF UTILITY

"Intermediate between mathematics, statistics and economics, we find a new discipline which, for lack of a better name, may be called *econometrics*," announced Ragnar Frisch (1926, p. 386).

> Econometrics has as its aim to subject abstract laws of theoretical political economy or 'pure' economics to experimental and numerical verification, and thus to turn pure economics, as far as it is possible, into a science in the strict sense of the word. The econometric study that I shall present is an attempt to realize the dream of [William Stanley] Jevons[13]: to measure the variation in the marginal utility of economic goods. I shall give special attention to the variation in the marginal utility of money.

He thereby discarded as an evasion of the problem the assumption by Alfred Marshall (and, at least implicitly, by Jules Dupuit half a century before Marshall) of a constant marginal utility of money for sufficiently small changes in consumption bundles. Although Frisch published his essay in a Norwegian series of mathematical publications, he did so in French and stated, "The study was made during a stay in Paris, 1923," when he was studying mathematics there. Frisch was already familiar with at least some of Fisher's work when writing his 1926 study, mentioning Fisher's preference for Charles Gide's term "desirability" rather than "utility" (Frisch 1926, p. 395, perhaps with Fisher 1918 in mind) and discussing the basis on which "Mr. Fisher and Mr. Pareto after him have studied the static equilibrium of exchange" (Frisch 1926, p. 391, clearly with reference to Fisher 1892 since Fisher was stated to precede Pareto). Bjerkholt and Ariane Dupont-Kiefer (2009) report that Frisch had already studied Fisher (1892) while in Paris.

The Paris connection was important to Frisch's involvement with Fisher, the formation of the Econometric Society, and measurement of marginal utility. Upon publication, Frisch sent a copy of Frisch (1926) to François Divisia (still famous for the Divisia index), initiating an exchange of fourteen letters leading to the creation at the end of 1930 of the Econometric Society, of which Frisch was the moving spirit (and from 1933 founding editor of *Econometrica*), Divisia the first vice-president and second president, and Fisher the first president (Bjerkholt 1998, 2017). In 1933 Frisch gave a series of lectures on econometrics in Paris, in the prestigious Poincaré series noted, for example, for Bruno De Finetti's lectures on subjective probability (De Finetti [1937] 1964; see also De Finetti 1935 on utility measurement). While *New Methods of Measuring Marginal Utility* (Frisch 1932), drafted at Yale, was forthcoming, Frisch's French summary of the book, presented to the first European meeting of the Econometric Society in Lausanne in September 1931, was published in *Revue d'Économie Politique* with an expository introduction by Jacques Moret that was as lengthy as Frisch's paper (Moret and Frisch 1932). Frisch had first encountered Fisher's work through Moret's translation of Fisher's dissertation (Fisher 1917). Moret, a follower of the Lausanne school of general equilibrium (Moret 1915), was, along with Divisia, one of the four French economists among twenty-six recipients of a letter from Fisher, Frisch, Charles Roos, and Joseph Schumpeter about organizing an Econometric Society (letter of

[13] Frisch cited the paragraph "Numerical Determination of the Laws of Utility" by Jevons ([1871] 1911, p. 146).

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invitation, 17 June 1930, and list of invitees on the Econometric Society website). Moret shared both Frisch's admiration for Fisher's dissertation as the origin of the choice-theoretic approach to utility and Fisher's and Frisch's interest in trying to measure marginal utility, and published a short review of Frisch (1932) in the next issue of *Revue d'Économie Politique* after his and Frisch's articles (Moret 1932).

Where Fisher (1927) began with a policy-relevant problem, that of justifying a progressive income tax by Edgeworth's principle of equal sacrifice (equalizing marginal utility of income across persons, Edgeworth 1919), the first numbered section (after the unnumbered introduction) of Frisch (1926) presented two sets of three axioms (axioms of choice, transitivity, and addition), first for an individual at a given position choosing between two displacements and then for an individual who knows that on two different occasions he will be in two different positions, choosing among displacements from those positions. Independently of the merits of his method to measure marginal utility, Frisch (1926) is thus important as a pioneering axiomatization of utility theory, an approach he did not take further in Frisch (1932) but did discuss in Yale lectures (Bjerkholt and Qin 2010, pp. 93–94) and which was carried further, with reference to Frisch (1932) and in a presentation in Oslo, by Franz Alt ([1936] 1971, 1937). The axiomatization of utility theory, continued by John von Neumann and Oskar Morgenstern in the 1940s for maximization of expected utility and culminating in the work of Gérard Debreu (1959),[14] paralleled the axiomatization of probability theory, notably by Andrei N. Kolmogorov in the 1930s. Frisch's axiomatic presentation was a major difference between his and Fisher's essays on measuring marginal utility, leading to what Frisch (1947) termed "quite an argument" between them.

Based on his axioms, Frisch (1926, pp. 401–402) ruled out the famous function of "moral expectation" or marginal utility proposed by Daniel Bernouilli ([1738] 1954),[15] which in Frisch's notation expressed the function $g(r)$ as a constant divided by $(r - a)$, where r stands for wealth and $a$ for some constant, as well as the utility function proposed by Charles Jordan (1924) equating $g(r)$ to a constant divided by the square of $(r - a)$. According to Frisch, the simplest function for the marginal utility of money consistent with his axioms would be a constant divided by $(\log r - \log a)$.

Where Fisher (1927) proposed only a method for measuring marginal utility, if it was possible to define a class of commodities such that its marginal utility was independent of the consumption of other classes of commodities, Frisch (1926) attempted to actually perform such a calculation, using for illustration monthly data from June 1920 to December 1922 for French real income, sales of sugar at the Union des Coopérateurs, and the price of sugar divided by the cost-of-living index. Frisch used more extensive

[14] Debreu had spent the spring of 1950 in Oslo working with Frisch. However, Debreu (1959, 1983) never cited Frisch or Fisher or Franz Alt in any context. Werner Hildenbrand, in his introduction to Debreu (1983, p. 3), acknowledges, "Indeed there had already been a few axiomatic treatments of economic theory prior to Debreu" but, apart from papers by Abraham Wald about the existence of general equilibrium in the proceedings of Karl Menger's Vienna mathematical colloquium in the 1930s (of which Alt was a participant), mentions only von Neumann and Morgenstern (1944) before Cowles monographs of Tjalling Koopmans and Kenneth Arrow in the early 1950s, omitting any mention of Frisch (1926) and Alt ([1936] 1971) as pioneering axiomatizations of choice and utility theory. Since Morgenstern hired Alt to tutor him in mathematics and, as journal editor, published Alt ([1936] 1971), he was familiar with Alt's contribution before the writing of von Neumann and Morgenstern (1944).

[15] Frisch cited a German translation of Bernoulli, published in Leipzig in 1896.

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data in his 1932 monograph. As Fisher remarked in his 1942 book on income taxation (reprinted in Fisher 1997, vol. 12, p. 264), Frisch's 1932 statistical results, following from his novel procedures of measurement, "were, however, startlingly different from what many people would have expected. According to his results (very tentative, of course), if a man's income is halved, his marginal want for one more dollar is not *more* than doubled, as commonly supposed—not even *doubled*—but *less* than doubled. Or conversely if his income is doubled his desire for one more dollar, although reduced, is reduced by less than half" so that Edgeworth's principle of equal sacrifice would imply "not progressive, but *regressive* rates" (Fisher's italics).

## V. FURTHER CONTRIBUTIONS TO THE FISHER–FRISCH INVESTIGATION OF MEASURABLE UTILITY

A few authors followed up Frisch's studies. Frederick Waugh (1935, p. 376) credited Fisher (1927) and Frisch (1926, 1932) with having "shown that under certain conditions it is possible to measure quantitatively the variation in the marginal utility of money from one period of time to another or from one group of persons to another," although he cautioned that the results of Fisher and Frisch applied to the marginal utility of money of selected groups of consumers, not necessarily representative of "the whole populations of France and the United States." Like Fisher, Waugh cited the principle of "equal sacrifice" in taxation as a motivation for his study. Waugh, at the US Bureau of Agricultural Economics when his article appeared, studied with Frisch in 1932–33, on a one-year fellowship from the Social Science Research Council, and applied Frisch's methods to US data on income and on the prices and consumption of sugar, coffee, meat, and butter: "With the kind assistance of Professor Frisch and his assistants at the Universitets Økonomiske Institutt in Oslo, these data were studied both by graphic and by mathematical-statistical methods" (Waugh 1935, p. 381). Waugh (1935, p. 385) reached "the conclusion that *in the period 1916 to 1921 and again in the period 1922 to 1932 the per capita consumption of foods was almost constant from year to year* … if the average diet in a given period was constant, we can assume that the marginal utility of that diet was very nearly constant and thus we have a convenient and reliable base with which to compare other utilities" (italics in original). Later writers did not, however, find such a convenient simplification offered by the data.

In contrast to Waugh's enthusiasm for Frisch's approach, Roy G. D. Allen (1933, p. 187) rejected

> the validity of an 'approximation' adopted at the very outset [of Frisch 1932] … taking the marginal utility of money as a function, not of the individual's income and the prices of all the various consumers' goods on the market, but of two variables only, i.e. the income and the price of living … not an approximation at all; it can only be regarded as an extra, and most serious, assumption. Further, the making of the assumption seems to take from the notion of money marginal utility most of its essential and distinctive meaning.

(See Dupont-Kieffer 2013 on the debate between Frisch and Allen.)

But, contrary to Allen's insistence on the full set of variables that theory states should be arguments of a utility function, Fisher and Frisch, apart from the exact representation

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possibility if the marginal utility of one commodity class was independent of the consumption of other commodity classes, sought to approximate empirical reality by making use of the true constant-utility index of the cost of living being bounded by the Paasche and Laspeyres indexes. Fisher's candidate for such an index would be the Fisher ideal index, the geometric mean of the Paasche and Laspeyres indexes (Fisher 1922), which satisfied more of Fisher's seven "statistical tests" or criteria for a desirable index than any other index formula (Frisch 1930, 1936, were to show that no index could satisfy more than six of the seven criteria; see Dimand 2019 and my introduction and afterword to Fisher 1997, vol. 7). Like Allen, Harold Hotelling (1932) and Henry Schultz (1933, 1938), while sympathetic to Frisch's attempt to find an indirect method of measuring utility, were skeptical of the auxiliary assumptions (such as a consumer's utility from a commodity depending only on the quantity of that commodity) that Frisch needed to get concrete results (Moscati 2019, p. 122).

Schultz held "that the feelings of the human heart are not directly measurable by any known procedure, and I doubt whether such a procedure will ever be invented. But it is these feelings which continually prompt us to buy and sell, borrow and lend, labor and rest, produce and consume; and *it is from the quantitative effects of these feelings that we can estimate their comparative intensity*" (1933, p. 110; Schultz's italics). Schultz noted that data limitations obliged Frisch to try to assume the existence of an average person, rather than analyzing individuals. "Although the lack of the necessary data compels Frisch in his numerical illustrations to make this assumption and even less plausible assumptions," argued Schultz (1933, p. 111), "there can be doubt but that, had the relevant data for an individual been available, Frisch's procedure would give his money flexibility, i.e., the relative change in his marginal utility of money corresponding to a relative change in his income when changes are infinitesimal." Schultz (1933, pp. 114–115) noted that Frisch's assumption that the marginal utility of money is a function only of income and the cost of living, while less general than the assumption of Fisher (1927) that the marginal utility of money depends on income and the prices of commodities, "makes it possible to use observations relating to *different price situations* in order to determine points on the *same utility curve*" (Schultz's italics).

Frisch's 1926 axiomatic approach to measurement of marginal utility (which contrasted to Fisher's more discursive, non-axiomatic presentation)[16] was taken up by the mathematician and physicist Franz Alt, who, in addition to publishing in German in the Austrian journal *Zeitschrift für Nationalökonomie* (Alt [1936] 1971), came to Oslo to present his results at the International Congress of Mathematicians in 1936 (summarized in English as Alt 1937), placing his research in a discourse with Frisch. Alt ([1936] 1971, p. 430) mentioned Frisch (1932) "and a few shorter articles by the same author" (not further identified) but as writings in which "methods for the practical measurement of utility of commodities were proposed—provided that certain conditions were satisfied—but whereas there the conditions were usually not specified precisely or completely, we have been concerned mainly with the formulation of these conditions, that is, with the logical side of the problem," which overlooks the axiomatic approach of Frisch (1926).

[16] See Bjerkholt and Dupont-Kieffer (2007). Frisch also adopted an axiomatic approach in his Yale lectures in 1930 and his Poincaré lectures in Paris in 1933, but these lectures were only published more than three quarters of a century later (Bjerkholt and Dupont-Kieffer 2009; Bjerkholt and Qin 2010), so it was only his publications, most notably Frisch (1926, 1932), that could stimulate further developments such as that by Alt.

Instead, Alt focused on Oskar Lange (1934), which had sketched a proof of the existence of a utility function without offering the proof itself or giving a complete list of the necessary conditions.[17] John Chipman, Leonid Hurwicz, Marcel Richter, and Hugo Sonnenschein (1971, pp. 327–329) noted that Alt's first three axioms, regarding transitivity and connexity of binary preference relations, corresponded to the axioms stated by Frisch, but Alt went further to established necessary and sufficient conditions for solvability, making use of the topological property of connectedness used two decades later by Gérard Debreu: "To a large extent many of the most essential features of contemporary models of measurable utility were anticipated in this pioneering study by Franz Alt" (p. 329).

In addition to Alt's study, another important contribution to economic theory was inspired by the attempts of Frisch and Fisher to measure utility. William Vickrey's "Measuring Marginal Utility by Reactions to Risk" (1945) transformed the discussion of measuring marginal utility by appealing to the treatment of expected utility by von Neumann and Morgenstern (1944), recognizing the importance of that analysis even before the axiomatization of rationality as expected utility maximization in an appendix to the 1947 second edition. Fisher (1927), cited in Vickrey's first and last footnotes, and Frisch (1932), cited in Vickrey's second footnote, found a utility function defined up to a positive linear transformation by assuming that commodity classes could be defined so that the marginal utility of one commodity class is independent of the consumption of another class. Vickrey showed that comparing risky alternatives allowed deriving such a utility function without the Fisher–Frisch assumption: the probability weights attached to two risky alternatives to make an individual indifferent between the gamble and a sure thing served as an index of utility. Because those probability weights are bounded between zero and one, that index of utility is measurable. Vickrey, a Nobel laureate who was president of the American Economic Association seventy-five years after Fisher, was Fisher's last and most distinguished student. Vickrey graduated in engineering and mathematics from Yale and went on to doctoral study in economics at Columbia in 1935, the year Fisher retired from Yale. Fisher, whose PhD was jointly in mathematics and political economy and who began his teaching career as an assistant professor of mathematics, was the only Yale economist of that era likely to attract a Yale mathematics major to become an economist. Fisher thanked Vickrey in a 1939 *American Economic Review* on double taxation of savings for "kindness and patience in reading and criticizing sundry preliminary drafts of this article," and in the preface of a 1942 book on income taxation for reading most of the manuscript and making important suggestions (Fisher 1997, vol. 12, pp. 118, 161), while Vickrey's dissertation and first book devoted a chapter to Fisher's income concept and expenditure tax proposal (Dimand and Koehn 2002).

While Vickrey (1945) is the outstanding example of the influence of the Fisher–Frisch connection within economics, William P. Fisher Jr. (2021) draws attention to another beyond economics. Following from Frisch's 1930 article on the possibility of an

[17] Alt, who had written a 1932 dissertation in mathematics supervised by Karl Menger, was hired to tutor Oskar Morgenstern in mathematics. At an afternoon tea at Morgenstern's house, he was shown a preprint of Lange's article by Paul Sweezy, one of the editors of the *Review of Economic Studies*, and soon began a letter (in English) to Lange about Lange's postulates that became an article (in German) in Morgenstern's journal (Alt and Akera 2006, pp. 8–9; Moscati 2019, pp. 107–109). Chipman et al. (1971) translated Frisch (1926) and Alt ([1936] 1971) as their chapters 19 and 20, as successive steps in the axiomatic approach to utility theory.

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index number that would satisfy Fisher's tests, Frisch prompted the Danish mathematician Georg Rasch "to formalize a separability theorem that continues today to serve as the basis of a wide range of theoretical and applied developments in psychological and social measurement" (Fisher 2021, p. 21). Rasch's separability theorem has been sufficiently influential that forums at which William Fisher presented his paper include the Rasch Measurement Special Interest Group of the American Educational Research Association and the Copenhagen conference marking the fiftieth anniversary of Rasch's book (Rasch 1960).

## VI. CONCLUSION

Ragnar Frisch (1947, p. 3) spoke of "the crucial contribution" and "monumental importance" of Irving Fisher's *Mathematical Investigations in the Theory of Value and Price* (1892):

> I remember the intensity with which, in my younger days, I dug into Fisher's dissertation, and the same can undoubtedly be said about many other economists of our generation. Later, this work was followed up by his paper on the *Measurement of Marginal Utility.* When we are speaking not about the ideas that cause the shorter swings, or even the sub-secular swings of the thinking in economics, but about those that are responsible for the really long-time trend of our science, then it will be hard to find any single work that has been more influential than Fisher's dissertation. It will be standing there as a mile-stone long after our great-grandchildren are dead and forgotten.

Just as Frisch (1926) has received scanty notice in accounts of the history of axiomatic approaches to economic theory, Frisch's recognition of the "monumental importance" of Fisher (1892), notably its presentation of a behaviorist approach to ordinal utility and choice before Pareto, has not been widely shared: the sole mention of Fisher (1892) in Roy Weintraub (1985, p. 37) states only that "Irving Fisher has a photograph of a complicated piece of hydraulic machinery as a model in his book on value theory," without indicating that Fisher's book was about general equilibrium (or citing the book). But there can be no doubt of the strength of the book's impact on Frisch, or of the importance of the collaboration between Fisher and Frisch for Frisch's career and for the establishment of the Econometric Society, *Econometrica,* and the Cowles Commission.

William Barber reported that "Fisher [1927] was breaking new ground here. He subsequently learned, however, that Ragnar Frisch had anticipated him in an essay published in French in 1926. When distributing reprints of his own article, Fisher thereafter enclosed a mimeographed statement acknowledging Frisch's priority in utility measurement" (Fisher 1997, vol. 12, p. 2n).[18] Fisher (1937, p. 31), in an *Econometrica* article on income taxation that had been a series of four lectures to the 1936 Cowles Commission summer conference, held that "we need not wholly despair of someday seeing econometric science conquer, to some extent, the problem of measuring even the

[18] Allowing for the publication lag of the volume honoring Clark, Fisher likely wrote his essay before Frisch published (Bjerkholt and Qin 2010, p. 21, suggest that Fisher 1927 was written in 1925) but after Frisch wrote the first version of his 1926 study in Paris in 1923.

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Y's [psychological satisfactions translated into money]. In fact, an attempt has been made by me to take a first step in that direction; and Professor Ragnar Frisch had independently done the same in a different way. He has since made still more notable contributions to this elusive subject" with three footnotes citing Fisher (1927), Frisch (1926), and Frisch (1932). Going beyond Fisher's acknowledgment of Frisch having published first, Bjerkholt and Qin (2010, p. 21) state that Fisher came "to recognize that Frisch [1926] had chosen a better approach," which is somewhat stronger than Fisher's acknowledgment of priority or the praise given in Fisher (1937). Frisch (1947, p. 3) did not mention anything about Fisher recognizing Frisch's approach as better when he told the American Statistical Association and Econometric Society dinner honoring Fisher's eightieth birthday:

> Fisher and I had quite an argument over the way in which he presented his theory in that paper on utility measurement. I said that he should have condensed it to a few pages by a free use of a compact mathematical symbolism. That, I claimed, would have saved many of us a lot of time. But he would not give in to this argument. Then, some years later, I gave a series of lectures in Oslo, on utility axiomatics and the application of utility theory to the theory of demand. As the weeks passed by, I was greatly grieved by discovering that the audience dwindled down to nothing, when it finally became apparent that the students didn't understand a word of what I had been trying to tell them. Then one day one of the students came along and told me that in my collection of reprints—which is always at the disposal of the students—he had picked up Fisher's paper on utility measurement, and then it had suddenly dawned upon him what I had been trying to say. You may imagine the joy I got out of writing to friend Fisher and telling him of this story.

Frisch added, "I think that this is much more than a good story. It is symbolic of Fisher's ability to see clearly, and to stick to it in the midst of both friendly and adverse criticism." Frisch's telling of this story in his tribute to Fisher was also symbolic of his generous appreciation of a fellow scientist and of the long-standing influence on Frisch's thought of what Frisch termed Fisher's 1892 development of "the choice point of view in the analysis of utility." In addition to leading Frisch to a pioneering axiomatization of utility theory, the shared interest of Fisher and Frisch in utility measurement had consequences for Frisch, as the University of Oslo matched Yale's offer of a full professorship very early in Frisch's career, and for the Econometric Society being founded during Frisch's year visiting Fisher, just in time to benefit from Alfred Cowles's disillusionment with stock forecasters.

## COMPETING INTERESTS

The author declares no competing interests exist.

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