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ON THE CALKIN ALGEBRA AND THE COVERING HOMOTOPY PROPERTY, II

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For a separable Hilbert space $\mathscr{H}$, $\mathscr{B}(\mathscr{H})$ is the algebra of bounded linear operators on $\mathscr{H}$, $\mathscr{C} = \mathscr{C}(\mathscr{H})$ is the ideal of compact operators, and π is the natural map of $\mathscr{B}(\mathscr{H})$ onto the Calkin algebra $\mathscr{B}(\mathscr{H})/\mathscr{C}$. If $\mathscr{A}$ and $\mathscr{D}$ are any C^*-algebras then a *-homotopy of $\mathscr{A}$ into $\mathscr{D}$ is a continuous map $\Phi : \mathscr{A} \times I \to \mathscr{D}$ such that for each t in $I = [0, 1]$, $\Phi_t(\cdot) = \Phi(\cdot, t)$ is a *-homomorphism of $\mathscr{A}$ into $\mathscr{D}$. If $\mathscr{D} = \mathscr{B}(\mathscr{H})/\mathscr{C}$ and $\Phi : \mathscr{A} \times I \to \mathscr{B}(\mathscr{H})/\mathscr{C}$ is a *-homotopy then an initial lifting of Φ is a *-homomorphism $\varphi : \mathscr{A} \to \mathscr{B}(\mathscr{H})$ such that $\pi \circ \varphi = \Phi_0$. A C^*-algebra $\mathscr{A}$ has the C^*-covering homotopy property if for every *-homotopy $\Phi : \mathscr{A} \times I \to \mathscr{B}(\mathscr{H})/\mathscr{C}$ with initial lifting $\varphi : \mathscr{A} \to \mathscr{B}(\mathscr{H})$ there is a *-homotopy $\Psi : \mathscr{A} \times I \to \mathscr{B}(\mathscr{H})$ such that $\Psi_0 = \varphi$ and $\pi \circ \Psi = \Phi$. In [**4**] it was shown that $\mathscr{A} = C(X)$ has the C^*-covering homotopy property whever X is an interval, a circle, or a totally disconnected space. In this note it is proved that every approximately finite (AF) C^*-algebra has the covering homotopy property. Since the abelian AF C^*-algebras are precisely those $C(X)$ for X totally disconnected, this will generalize the principal result of [**4**].

As stated in [**4**], the original motivation for studying this property was its connection to the work of Brown, Douglas, and Fillmore [**2**]. Since the publication of [**2**] the problem of studying extensions of arbitrary C^*-algebras has been undertaken, and by the work of Voiculescu [**7**] and Choi and Effros [**3**] it is now known that the set of equivalence classes of extensions of a nuclear C^*-algebra forms a group. Approximately finite C^*-algebras are nuclear.

A C^*-algebra $\mathscr{A}$ is approximately finite if there is an ascending sequence $\mathscr{A}_1 \subseteq \mathscr{A}_2 \subseteq \mathscr{A}_3 \subseteq \ldots$ of C^*-subalgebras of $\mathscr{A}$ such that each $\mathscr{A}_n$ is finite dimensional and $\bigcup_{n=1}^{\infty} \mathscr{A}_n$ is dense in $\mathscr{A}$ ([**1**] and [**6**]). If $\Phi : \mathscr{A} \times I \to \mathscr{B}(\mathscr{H})/\mathscr{C}$ is a *-homotopy with initial lifting $\varphi : \mathscr{A} \to \mathscr{B}(\mathscr{H})$ then $\Phi^{(n)} = \Phi|\mathscr{A}_n \times I$ is a *-homotopy with initial lifting $\varphi_n = \varphi|\mathscr{A}_n$. If it can be shown that for each n there is a *-homotopy $\Psi^{(n)} : \mathscr{A}_n \times I \to \mathscr{B}(\mathscr{H})$ such that $\pi\Psi^{(n)} = \Phi^{(n)}$, $\Psi_0^{(n)} = \varphi_n$ and $\Psi^{(n+1)}|\mathscr{A}_n \times I = \Psi^{(n)}$ then it follows that $\mathscr{A}$ has the C^*-covering homotopy property. It is precisely this that will be proved.

Now if $\mathscr{A}$ is a finite dimensional C^*-algebra then the Artin-Wedderburn Theorem implies that $\mathscr{A}$ is the direct sum of full matrix rings. Hence the case $\mathscr{A} = M_n$, the $n \times n$ matrices, is the subject of the first effort.

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Define a collection of elements $\{e_{ij} : 1 \leqq i, j \leqq n\}$ in a C^*-algebra $\mathscr{A}$ to be a *collection of matrix units* if

a) $e_{ij}e_{kl} = 0$ for $k \neq j$,

b) $e_{ij}e_{jl} = e_{il}$,

c) $e_{ij}^* = e_{ji}$.

Notice that if $\mathscr{A} = M_n$ and e_{ij} is the matrix with 1 in the (i, j) place and zeros elsewhere then $\{e_{ij}\}$ is a system of matrix units for M_n. Also, if $\{e_{ij} : 1 \leqq i, j \leqq n\}$ is a system of matrix units in $\mathscr{A}$ then there is a homomorphism of M_n into $\mathscr{A}$. Moreover, it is easy to see that M_n has the C^*-covering homotopy property precisely if paths of matrix units in $\mathscr{B}(\mathscr{H})/\mathscr{C}$ can be lifted to paths of matrix units in $\mathscr{B}(\mathscr{H})$.

LEMMA 1. *Let $u : I \to \mathscr{B}(\mathscr{H})/\mathscr{C}$ be a path of partial isometries with U_0 a partial isometry in $\mathscr{B}(\mathscr{H})$ such that $\pi U_0 = u(0)$, and suppose $p = u^*u$, $q = uu^*$, and $q \leqq p^\perp \equiv 1 - p$. If $P, Q : I \to \mathscr{B}(\mathscr{H})$ are paths of projections such that $\pi P = p$, $\pi Q = q$, $Q \leqq P^\perp$, $P(0) = U_0^*U_0$, and $Q(0) = U_0U_0^*$, then there is a path $U : I \to \mathscr{B}(\mathscr{H})$ of partial isometries such that:*

$$\pi U = u, \quad U^*U = P,$$
$$U(0) = U_0, \quad UU^* = Q.$$

Proof. Using the Bartle-Graves Theorem [**5**], there is a path $A : I \to \mathscr{B}(\mathscr{H})$ such that $\pi A = u$ and $A(0) = U_0$. By replacing A with QAP we may assume that $A = QAP$. Disregarding the trivial case, we may assume that $p(0) \neq 0$; that is P_0, and hence Q_0, are infinite rank projections. If it is the case that $A(t)$ is invertible for all t (so U_0 is a unitary operator) then by taking $A = U(A^*A)^{1/2}$ to be the polar decomposition of A, U would be the required path of unitaries. The remainder of the proof is devoted to overcoming this helpful but over restrictive hypothesis.

Using Lemma 7 of [**4**], there are paths of partial isometries R, S with $R^*R = Q$, $RR^* = Q(0)$, $R(0) = Q(0)$, $S^*S = P(0)$, $SS^* = P$, $S(0) = P(0)$. If $B = SU_0^*R$ then B is a path of partial isometries with $B^*B = Q$, $BB^* = P$, and $B(0) = U_0^*$.

If $C = A + B + (P^\perp - Q)$ then $C^*C = A^*A + P^\perp$. Since $P(A^*A)P = A^*A$, it follows that $|C| \equiv (C^*C)^{1/2} = |A| + P^\perp$. Let $A = U|A|$ be the minimal polar decomposition of A. Note, $A(t) = U(t)|A(t)|$ and $t \to |A(t)| = [A^*A(t)]^{1/2}$ is continuous, but $t \to U(t)$ is not necessarily continuous. (Indeed, if U were a path the proof would be finished.) Now for each time t, $U^*U(t) \leqq P(t)$ and $UU^*(t) \leqq Q(t)$ since $QAP = A$. Hence $U + B + (P^\perp - Q)$ is a partial isometry and

$$C = [U + B + (P^\perp - Q)]|C|.$$

So if $C = W|C|$ is the minimal polar decomposition of C then

$$W^*W \leqq [U + B + (P^\perp - Q)]^*[U + B + (P^\perp - Q)]$$
$$WW^* \leqq [U + B + (P^\perp - Q)][U + B + (P^\perp - Q)]^*$$

and $U + B + (P^\perp - Q)$ extends W.

Let $\delta > 0$ be such that for $|s - t| < \delta$,

$$||A(t) - A(s)||, ||B(t) - B(s)||, ||P(t) - P(s)||, ||Q(t) - Q(s)|| < 1/12.$$

Now $C(0) = A(0) + B(0) + P(0)^{\perp} - Q(0) = U_0 + U_0^* + P^{\perp}(0) - Q(0)$ is hermitian and it is easy to check that $C(0)^2 = 1$; so $C(0)$ is a self adjoint unitary. If $0 \leqq t \leqq \delta$ then $||C(t) - C(0)|| < \frac{1}{3} < 1$, so $C(t)$ is invertible. Hence $W(t)$ is unitary and $W = U + B + (P^{\perp} - Q)$ on $[0, \delta]$. But $P^{\perp}$, Q, B, and $W = C|C^{-1}$ are continuous on $[0, \delta]$, so U must be continuous there. Also $A(0) = U_0$ implies that $U(0) = U_0$ and $u = \pi(A) = \pi(U|A|) = \pi(U)\pi(|A|) = \pi(U)p = \pi(U)\pi(P) = \pi(UP) = \pi(U)$. Also, on $[0, \delta]$

$$1 = W^*W = U^*U + P^{\perp}$$
$$1 = WW^* = UU^* + 1 - Q.$$

So $U^*U = P$ and $UU^* = Q$. This defines the path $U : [0, \delta] \rightarrow \mathscr{B}(\mathscr{H})$ with all the required properties.

If $K = U(\delta) - A(\delta)$ then $K \in \mathscr{C}$. Put $A_1 = Q[A + K]P$ and $C_1 = A_1 + B + (P^{\perp} - Q)$. Now $A_1(\delta) = U(\delta)$ and $C_1(\delta) = W(\delta)$, a unitary operator. If $\delta \leqq t \leqq 2\delta$ then $||A_1(t) - A_1(\delta)|| = ||Q(t)[A(t) - A(\delta)]P(t) + [Q(t) - Q(\delta)]U(\delta)P(t) + Q(\delta)U(\delta)[P(t) - P(\delta)]|| < \frac{1}{4}$. Hence $||C_1(t) - W(\delta)|| = ||C_1(t) - C_1(\delta)|| < 1$, and $C_1(t)$ is invertible for $\delta \leqq t \leqq 2\delta$. Let $C_1 = W_1|C_1|$, $A_1 = U_1|A_1|$ be the minimal polar decompositions; then $U_1(\delta) = U(\delta)$, $W_1(\delta) = W(\delta)$, $U_1^*U_1 \leqq P$, $U_1U_1^* \leqq Q$. As before $W_1 = U_1 + B + (P^{\perp} - Q)$ on $[\delta, 2\delta]$ and so $U_1^*U = P$, $U_1U_1^* = Q$. Define $U(t) = U_1(t)$ on $[\delta, 2\delta]$ and this gives a continuous path $U : [0, 2\delta] \rightarrow \mathscr{B}(\mathscr{H})$ of partial isometries having the required properties.

If $K_2 = U(2\delta) - A(2\delta)$, $A_2 = Q[A + K_2]P$, and $C_2 = A_2 + B + (P^{\perp} - Q)$ then the argument given above will result in an extension of U to $[0, 3\delta]$. After a finite number of such arguments, U will be defined on I and have all the required properties. This completes the proof.

Identify M_n with the subalgebra of M_{n+1} consisting of all matrices (a_{ij}) such that

$$a_{n+1,j} = 0 = a_{i,n+1} \quad \text{for } 1 \leqq i, j \leqq n + 1.$$

LEMMA 2. *Let* $\Phi : M_{n+1} \times I \rightarrow \mathscr{B}(\mathscr{H})/\mathscr{C}$ *be a *-homotopy with initial lifting* $\varphi : M_{n+1} \rightarrow \mathscr{B}(\mathscr{H})$. *If* $\Theta : M_n \times I \rightarrow \mathscr{B}(\mathscr{H})$ *is a *-homotopy such that* $\Theta_0 = \varphi|M_n$ *and* $\pi\Theta = \Phi|M_n \times I$ *then there is a *-homotopy* $\Psi : M_{n+1} \times I \rightarrow \mathscr{B}(\mathscr{H})$ *such that*

$$\Psi_0 = \varphi, \quad \pi\Psi = \Phi, \quad \textit{and} \quad \Psi|M_n \times I = \Theta.$$

Furthermore, if $P : I \rightarrow \mathscr{B}(\mathscr{H})$ *is a path of projections such that* $P(0) = \varphi(1)$, $\pi P(t) = \Phi(1, t)$, *and* $P(t) \geqq \Theta(1, t)$ *then* Ψ *may be chosen such that* $\Psi(1, t) = P(t)$ *for* $0 \leqq t \leqq 1$.

Proof. Let $\{e_{ij} : 1 \leqq i, j \leqq n+1\}$ be the system of matrix units for M_{n+1} and put $E_{ij}(t) = \Theta(e_{ij}, t)$ for $1 \leqq i, j \leqq n$, $0 \leqq t \leqq 1$. If $P : I \to \mathscr{B}(\mathscr{H})$ is given as in the statement of this lemma let $P_{n+1} = P - \sum_{i=1}^{n} E_{ii}$. Notice that

$$P_{n+1}(0) = \varphi(e_{n+1,n+1}),$$
$$\pi P_{n+1}(t) = \Phi(e_{n+1,n+1}, t),$$
$$P_{n+1} \leqq 1 - \sum_{i=1}^{n} E_{ii}.$$

If P is not given, then Lemma 5 of [**4**] says that there exists a path of projections $P_{n+1} : I \to \mathscr{B}(\mathscr{H})$ having the above properties.

Now Lemma 1 implies there is a path $U : I \to \mathscr{B}(\mathscr{H})$ of partial isometries such that $U(0) = \varphi(e_{1,n+1})$, $\pi U(t) = \Phi(e_{1,n+1}, t)$, $U^*U = P_{n+1}$, and $UU^* = E_{11}$. For $1 \leqq k \leqq n$ let $E_{k,n+1} = E_{k,1}U$; $E_{n+1,k} = U^*E_{1k}$; and $E_{n+1,n+1} = P_{n+1}$. It is easy to check that $\{E_{ij} : 1 \leqq i,j \leqq n+1\}$ is a collection of paths such that at each time t they form a system of matrix units. At $t = 0$ they equal $\{\varphi(e_{ij})\}$ and $\pi E_{ij}(t) = \Phi(e_{ij}, t)$. If $\Psi((\alpha_{ij}), t) = \sum_{i,j} \alpha_{ij} E_{ij}(t)$ then Ψ has all the required properties.

COROLLARY 3. *M_n has the C^*-covering homotopy property.*

Proof. For $n = 2$ this amounts to combining Lemma 1 and Lemma 5 of [**4**]. The proof is completed by using the previous lemma to furnish the induction step.

If $n, m \geqq 1$ then $M_n \oplus M_m$ will be identified with the subalgebra of M_{n+m} consisting of all matrices (α_{ij}) such that $\alpha_{ij} = 0$ except possibly when $1 \leqq i, j \leqq n$ or $n+1 \leqq i, j \leqq n+m$.

LEMMA 4. *Let $\Phi : M_{n+m} \times I \to \mathscr{B}(\mathscr{H})/\mathscr{C}$ be a *-homotopy with initial lifting $\varphi : M_{n+m} \to \mathscr{B}(\mathscr{H})$. If $\mathscr{A} = M_n \oplus M_m$ and there is a *-homotopy $\Theta : \mathscr{A} \times I \to \mathscr{B}(\mathscr{H})$ such that $\Theta_0 = \varphi|\mathscr{A}$ and $\pi\Theta = \Phi|\mathscr{A} \times I$ then there is a *-homotopy $\Psi : M_{n+m} \times I \to \mathscr{B}(\mathscr{H})$ such that*

$$\Psi_0 = \varphi, \quad \pi\Psi = \Phi, \quad \textit{and} \quad \Psi|\mathscr{A} \times I = \Theta.$$

Proof. Let $\{e_{ij} : 1 \leqq i, j \leqq n+m\}$ be the standard matrix units for M_{n+m}; so

$$\mathscr{A} = \left\{ \sum_{i,j=1}^{n} \alpha_{ij} e_{ij} + \sum_{i,j=n+1}^{n+m} \alpha_{ij} e_{ij} : \ \alpha_{ij} \in C \right\}.$$

For $1 \leqq i, j \leqq n$ and $n+1 \leqq i, j \leqq n+m$ let $E_{ij}(t) = \Theta(e_{ij}, t)$.

Lemma 1 implies there is a path $U : I \to \mathscr{B}(\mathscr{H})$ of partial isometries such that $U(0) = \varphi(e_{n+1,1})$, $U^*U = E_{11}$, $UU^* = E_{n+1,n+1}$, and $\pi U(t) = \Phi(e_{n+1,1}, t)$ for $0 \leqq t \leqq 1$. For $n+1 \leqq i \leqq n+m$ and $1 \leqq j \leqq n$ define

$$E_{ij} \equiv E_{i,n+1} U E_{1j};$$
$$E_{ji} \equiv E_{j1} U^* E_{n+1,i}.$$

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It is an exercise to show that for each t, $\{E_{ij}(t) : 1 \leqq i, j \leqq n + m\}$ is a system of matrix units in $\mathscr{B}(\mathscr{H})$ and $\Psi((\alpha_{ij}), t) = \sum \alpha_{ij}E_{ij}(t)$ has all the required properties.

Let J_m denote an $m \times m$ identity matrix. $M_n \otimes J_m$ is identified with the subalgebra of M_{nm} consisting of all matrices (α_{ij}) where

$$\alpha_{ij} = \alpha_{kn+i,kn+j}$$

for $1 \leqq i, j \leqq n$ and $0 \leqq k \leqq m - 1$.

LEMMA 5. *Let $\Phi : M_{nm} \times I \to \mathscr{B}(\mathscr{H})/\mathscr{C}$ be a *-homotopy with initial lifting $\varphi : M_{nm} \to \mathscr{B}(\mathscr{H})$. If $\mathscr{A} = M_n \otimes J_m$ and $\Theta : \mathscr{A} \times I \to \mathscr{B}(\mathscr{H})$ is a *-homotopy such that $\Theta_0 = \varphi|\mathscr{A}$ and $\pi\Theta = \Phi|\mathscr{A} \times I$ then there exists a *-homotopy $\Psi : M_{nm} \times I \to \mathscr{B}(\mathscr{H})$ such that*

$$\Psi_0 = \varphi, \quad \pi\Psi = \Phi, \quad \text{and} \quad \Psi|\mathscr{A} \times I = \Theta.$$

Proof. Let $\{e_{ij} : 1 \leqq i, j \leqq nm\}$ be the standard matrix units for M_{nm}. If $1 \leqq i, j \leqq n$ and

$$f_{ij} = \sum_{k=0}^{m-1} e_{kn+i,kn+j},$$

then $\{f_{ij} : 1 \leqq i, j \leqq n\}$ is a system of matrix units for $\mathscr{A}$ that spans $\mathscr{A}$. Let $e_k = e_{(k-1)n+1,(k-1)n+1}$ for $1 \leqq k \leqq m$; so $e_1 + \ldots + e_m = f_{11}$. By Lemma 5 of [**4**] there are paths $E_1, \ldots, E_m$ of projections in $\mathscr{B}(\mathscr{H})$ such that $E_k \perp E_l$ for $k \neq l$, $E_k(0) = \varphi(e_k)$, $\pi E_k(t) = \Phi(e_k, t)$, and $(E_1 + \ldots + E_m)(t) = \Theta(f_{11}, t)$.

Let $F_{ij}(t) = \Theta(f_{ij}, t)$ for $1 \leqq i, j \leqq n$. If $1 \leqq k \leqq m$ and $(k - 1)n + 1 \leqq p, q \leqq kn$ then define $E_{pq} : I \to \mathscr{B}(\mathscr{H})$ by

$$E_{pq} = F_{p-(k-1)n,1}E_kF_{1,q-(k-1)n}.$$

It is easy to check that $\{E_{pq}(t) : (k - 1)n + 1 \leqq p, q \leqq kn, 1 \leqq k \leqq m\}$ is a system of matrix units at each time t, and each $F_{ij}(t)$ is the sum of some collection of $E_{pq}(t)$. In this way a *-homotopy $\Theta' : \mathscr{E} \times I \to \mathscr{B}(\mathscr{H})$ is obtained where $\mathscr{E} = M_n \oplus \ldots \oplus M_n \equiv$ *all* (α_{ij}) in M_{nm} such that $\alpha_{ij} = 0$ except possibly when $(k - 1)n + 1 \leqq i, j \leqq kn$ for some k, $1 \leqq k \leqq m$. Moreover, $\Theta'|\mathscr{A} \times I = \Theta$, $\Theta_0' = \varphi|\mathscr{E}$, and $\pi\Theta' = \Phi|\mathscr{E} \times I$. By applying Lemma 4, the desired *-homotopy $\Psi : M_{nm} \times I \to \mathscr{B}(\mathscr{H})$ is obtained.

If $\mathscr{E}$ is a C^*-subalgebra of M_n then a little thought and the fact that $\mathscr{E}$ must be isomorphic to the direct sum of full matrix rings yields the following. Representing M_n with a fixed orthonormal basis,

$$\mathscr{E} = \bigoplus_{i=1}^{p} M_{k_i} \oplus \bigoplus_{j=1}^{q} (M_{l_j} \otimes J_{s_j}) \oplus 0$$

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where the zero has dimension

$$n - \sum_{i=1}^{p} k_i - \sum_{j=1}^{q} l_j s_j.$$

Thus, if $\Phi : M_n \times I \to \mathscr{B}(\mathscr{H})/\mathscr{C}$ is a *-homotopy with initial lifting $\varphi : M_n \to \mathscr{B}(\mathscr{H})$ and $\Theta : \mathscr{E} \times I \to \mathscr{B}(\mathscr{H})$ is a *-homotopy such that $\Theta_0 = \varphi|\mathscr{E}$ and $\pi\Theta = \Phi|\mathscr{E} \times I$ then the preceding lemmas imply that there is a *-homotopy $\Psi : M_n \times I \to \mathscr{B}(\mathscr{H})$ such that $\Psi_0 = \varphi$, $\pi\Psi = \Phi$, and $\Psi|\mathscr{E} \times I = \Theta$.

Now suppose $\mathscr{A}_1$ is a C^*-subalgebra of the finite dimensional C^*-algebra $\mathscr{A}_2$ and $\Phi : \mathscr{A}_2 \times I \to \mathscr{B}(\mathscr{H})/\mathscr{C}$ is a *-homotopy with initial lifting $\varphi : \mathscr{A}_2 \to \mathscr{B}(\mathscr{H})$. Let $\Theta : \mathscr{A}_1 \times I \to \mathscr{B}(\mathscr{H})$ be a *-homotopy such that $\Theta_0 = \varphi|\mathscr{A}_1$, $\pi\Theta = \Phi|\mathscr{A}_1 \times I$. Now $\mathscr{A}_2 \approx M_{n_1} \oplus \ldots \oplus M_{n_m}$ and under this same isomorphism, $\mathscr{A}_1 \approx \mathscr{E}_1 \oplus \ldots \oplus \mathscr{E}_m$ where $\mathscr{E}_j$ is a *-subalgebra of M_{n_j}. The preceding paragraph shows that Θ can be extended to a *-homotopy $\Psi : \mathscr{A}_2 \times I \to \mathscr{B}(\mathscr{H})$ such that $\pi\Psi = \Phi$ and $\Psi_0 = \varphi$. This proves the main result of this note.

THEOREM. *An approximately finite C^*-algebra has the C^* covering homotopy property.*

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