DENSIFICATION OF SNOW ON THE ICE SHEET OF ELLSWORTH LAND AND THE SOUTHERN ANTARCTIC PENINSULA

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ABSTRACT. Density-depth curves constructed from seismic travel-time data observed at 25 locations, show a large variation over an essentially isothermal area ($T = -22 \pm 2 \cdot 6°$ C). This is correlated with the wide range of mean annual accumulation values ($A = 20$ to 50 g./cm.² yr.). Quantitative estimates of mean annual accumulation can be obtained from seismic refraction data in this area by the expression $A = 23 \cdot 5 + 0 \cdot 049(3500 - V_{200})$ where V_{200} is the compressional wave velocity (m./sec.) observed at 200 m. from the shot point. The density–depth curves can be used to give estimates of accumulation by the expression:

$$A = 2 \cdot 36(0 \cdot 913 - \rho_{40})$$

where ρ_{40} is the density (g./cm.³) at 40 m. depth. The internal consistency of these methods is indicated by the standard deviation of $\pm 2 \cdot 9$ g./cm.² yr. for comparisons of accumulation estimates. The theoretical basis for these results is discussed.

RÉSUMÉ. *Densification de la neige sur l'indlandsis d'Ellsworth Land et le sud de l'Antarctic Peninsula.* Les courbes des densités en fonction des profondeurs construites à partir des temps de parcours sismiques observés en 25 stations, montrent une importante variation pour une zône pratiquement isotherme ($T = -22 \pm 2{,}6°$ C). Ceci est en rapport avec le grand écart des valeurs moyennes des accumulations annuelles ($A = 20$–50 g./cm.² an). Des estimations quantitatives d'accumulation moyenne annuelle peuvent être obtenues a partir de données de réfraction sismique dans cette zone par l'expression $A = 23{,}5 + 0{,}049(3\,500 - V_{200})$ ou V_{200} (m/s) est la vitesse de l'onde de compression observée à 200 m du point de tir. La courbe des densités en fonction des profondeurs peut être utilisée pour donner des estimations d'accumulations par l'expression suivante;

$$A = 2{,}36(0{,}913 - \rho_{40})$$

dans laquelle ρ_{40} est la densité a 40 m de profondeur. La cohérence intrinsèque de ces méthodes est indiquée par un écart standard de $\pm 2{,}9$ g./cm.² an pour des comparaisons de l'estimation de l'accumulation. La base théorique concernant ces résultats est discutée.

ZUSAMMENFASSUNG. *Verdichtung des Schnees auf dem Eisschild von Ellsworth Land und im Süden der Antarctic Peninsula.* Dichte-Tiefen-Diagramme, abgeleitet aus seismischen Laufzeitbeobachtungen an 25 Stellen, zeigen starke Unterschiede innerhalb eines im wesentlichen isothermen Gebietes ($T = -22 \pm 2{,}6°$ C). Diese Tatsache steht mit der starken Streuung der Werte für die mittlere Jahresakkumulation ($A = 20$–50 g/cm² Jahr) in Zusammenhang. Zahlenmässig kann die mittlere Jahresakkumulation aus seismischen Refraktionswerten in diesem Gebiet mit der Formel $A = 23{,}5 + 0{,}049(3500 - V_{200})$ abgeschätzt werden, worin V_{200} die Geschwindigkeit (m/sec) der Druckwellen in 200 m Abstand vom Schusspunkt bedeutet. Auch die Dichte-Tiefen-Kurven können zur Abschätzung der Akkumulation herangezogen werden, und zwar nach einer der folgenden Formel:

$$A = 2{,}36(0{,}913 - \rho_{40}),$$

worin ρ_{40} die Dichte in 40 m Tiefe bedeutet. Die innere Übereinstimmung zwischen diesen Methoden kann aus dem mittleren Fehler von $\pm 2{,}9$ g/cm² Jahr beurteilt werden, der sich aus dem Vergleich von Akkumulationsabschätzungen ergibt. Die theoretischen Grundlagen dieser Ergebnisse werden diskutiert.

INTRODUCTION

During the austral summer of 1961–62 a seven-man party carried out geophysical and glaciological investigations on the 1,700 km. Antarctic Peninsula traverse in Ellsworth Land and the southern Antarctic Peninsula. Figures 1 and 2 show the area studied. Seismic measurements of ice thickness were made at 25 locations (Table I and Fig. 2) as discussed by Behrendt (1963). Velocity–depth curves were constructed by a method of numerical integration (Slichter, 1932) using the time and distance data for the refracted compressional waves traveling through the snow. A continuous velocity increase with depth was assumed to the depth where maximum velocity is reached (about 150 m. in this area).

From these velocity–depth curves, density–depth curves were constructed using the empirical expression Robin (1958, p. 88) showed to be valid below depths of 15 m.:

$$\rho = 2 \cdot 21 \times 10^{-4}(1 - 0 \cdot 00061\,T)^{-1} V_p + 0 \cdot 059 \qquad (1)$$

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where ρ is the density in g./cm.³, V_p is the compressional wave velocity in m./sec., and T is the temperature in °C. Figures 3–7 show the results of these calculations using the temperatures at 10 m. (from H. Shimizu personal communication). The variations in these curves are probably the result of the wide range in annual accumulation values (20 to 50 g./cm.² yr.) since the standard deviation of the mean annual (10 m.) temperatures is only $\pm 2 \cdot 6°$ C. from the average of $-22°$ C. This suggested a method of determining quantitative estimates of mean annual accumulation from the seismic data.

Table I. Accumulation Determined from Seismic Data and Near-Surface Glaciology

Station	A_L *from* ρ *at* $h = 40$ *m*	A_C *from* ρ *at* $h = 40$ *m*	A_V *from* V_p *at* $X = 200$ *m.*	*Mean seismic accumulation*	*Measured accumulation*
	g./cm.² yr.	g./cm.² yr.	g./cm.² yr.	g./cm.² yr.	g./cm.² yr.
224	47	48	49	48	—
256	44	42	44	43	48
288	41	39	45	42	—
320	43	41	40	41	—
352	44	43	46	44	—
382	37	34	33	32	—
404	44	43	37	31	—
432	40	42	34	39	40
464	24	28	23	24	—
496	37	34	36	36	26
528	35	32	43	37	—
604	38	35	37	37	46
636	39	37	42	39	34
668	38	36	41	38	41
700	40	38	39	39	45
732	47	49	47	48	50
764	50	57	51	53	48
796	37	34	37	39	—
840	28	28	27	28	28
864	24	27	23	24	—
908	29	28	30	29	—
940	23	27	25	25	20
976	40	37	43	40	—
1008	36	33	32	33	35
1028	35	35	24	31	—

* Personal communication from H. Shimizu.

Discussion

Pit measurements of accumulation values for twelve stations (Table I), furnished the author by Shimizu, were used with a value for "Eights" station determined from one season's observed snow accumulation. Figure 8 is a graph of accumulation A versus the density ρ at 40 m. depth. The straight line shown was fitted by least squares to the data and the standard deviation of the points is $\pm 4 \cdot 7$ g./cm.² yr. in terms of accumulation. This graph shows an inverse relation between accumulation and the density at 40 m. at this relatively constant temperature.

Another empirical study was carried out using the velocity vs. distance curves constructed directly from the travel-time curves. Figures 9–11 compare accumulation with compressional wave velocities recorded at distances of 50, 100, and 200 m. from the shot point, respectively. Assuming linear relationships for these graphs also, the standard deviations in terms of measured accumulations from the least-square straight lines are $\pm 6 \cdot 0$ g./cm.² yr., $5 \cdot 8$ g./cm.² yr., and $\pm 4 \cdot 5$ g./cm.² yr. for 50, 100, and 200 m. distances respectively. The depths h corresponding to the velocities at each of these distances X were measured and averaged from the velocity–depth curves with the following results: $X = 50$ m., $h = 11$ m., with standard deviation

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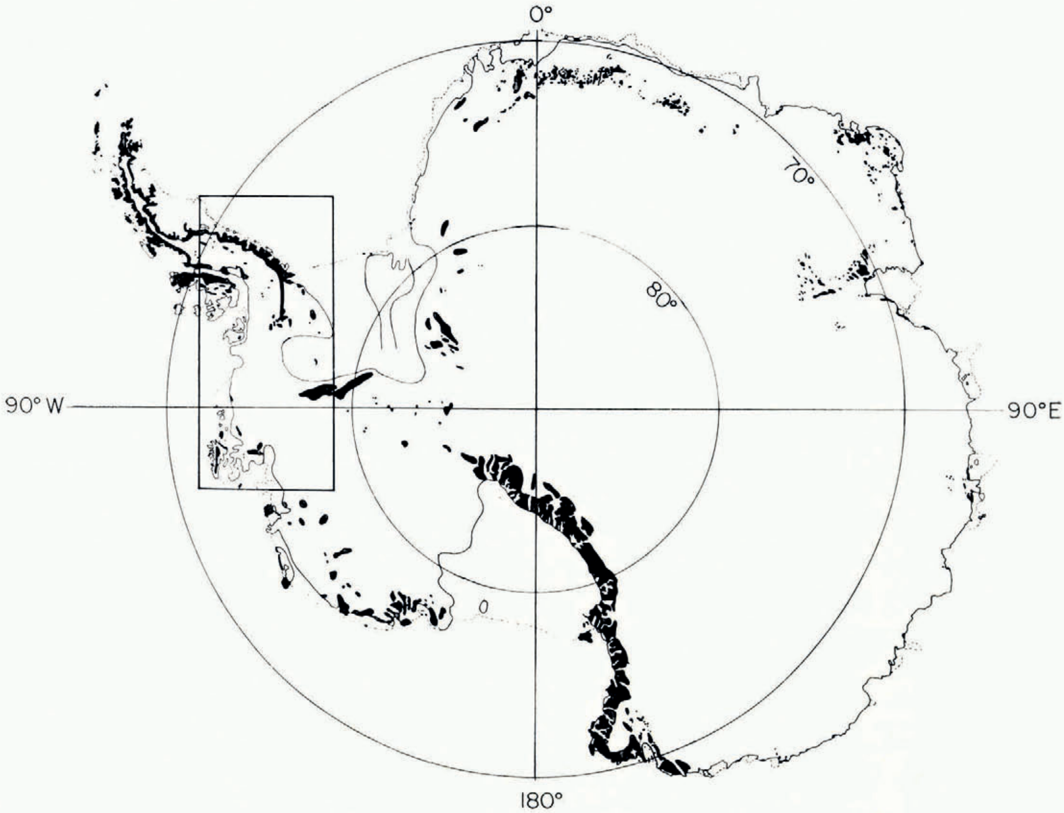

Fig. 1. Map of Antarctica showing area studied

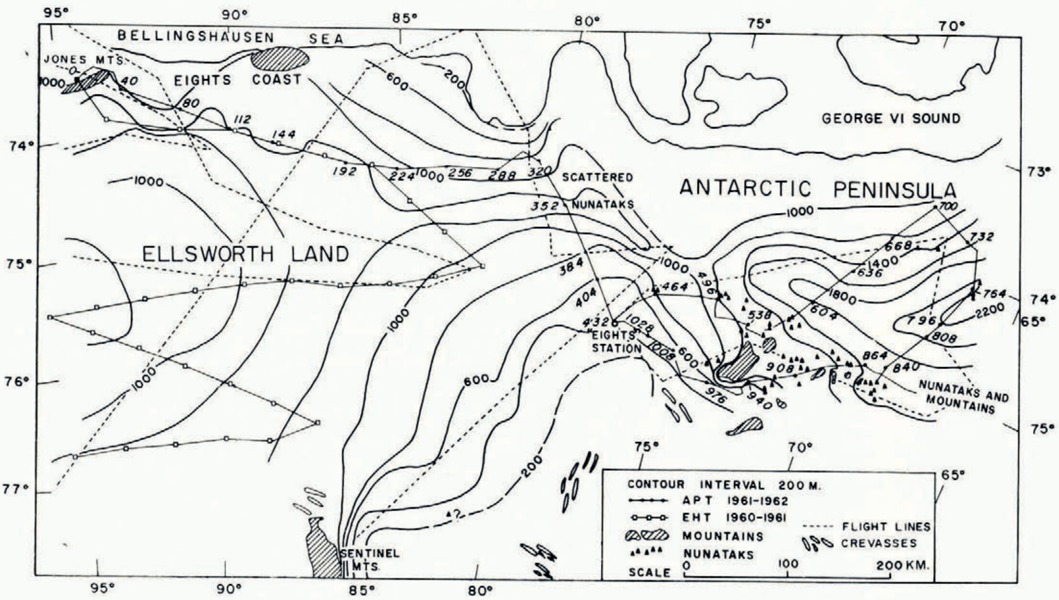

Fig. 2. Surface elevation map showing location of major stations (from Behrendt, 1963)

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$s = \pm 3 \cdot 3$ m.; $X = 100$ m., $h = 21$ m., $s = \pm 4 \cdot 2$ m.; and $X = 200$ m., $h = 41$ m., $s = \pm 2 \cdot 7$ m. The least scatter from the straight lines was for $X = 200$ m. (Fig. 11) corresponding to a mean depth of 41 m.; this is essentially equivalent to Figure 8 for accumulation versus density at 40 m. depth.

I attempted an explanation of these empirical results on a theoretical basis. Bader (1962) and Landauer (1957) gave the following relations for the rate of snow densification $(-v)$:

$$-v = \frac{1}{\rho}\frac{d\rho}{dt} = c \sinh(\sigma/\sigma_0), \tag{2}$$

$$\sigma = At \tag{3}$$

where t is the time, σ the load in g./cm.^{2} $= \bar{\rho}h$ where $\bar{\rho}$ is the average density from the

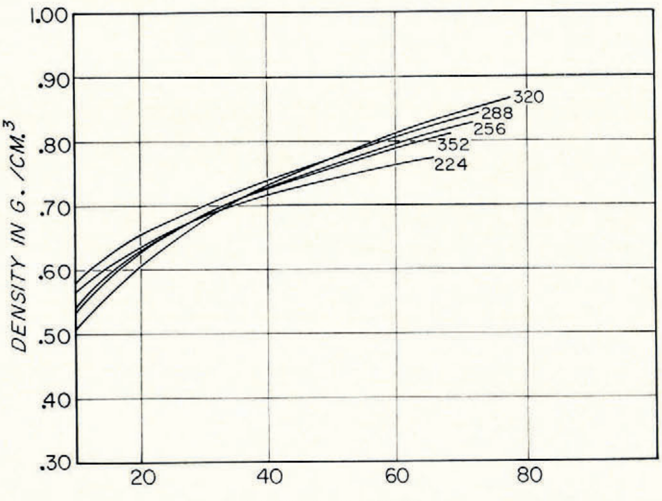

Fig. 3. Density-depth curves stations 224–320

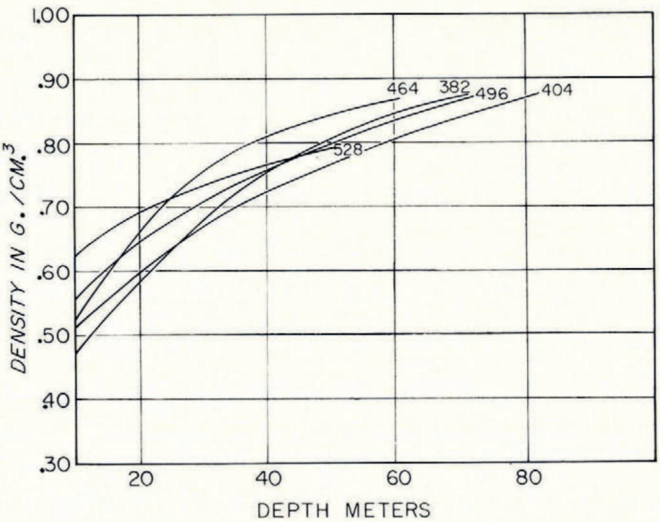

Fig. 4. Density-depth curves stations 382–528

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surface to a depth h, $\sigma_o = 700$ g./cm.²; and c is a constant depending on snow type and temperature. Thus

$$d\rho/\rho = c \sinh(At/\sigma_o)\, dt. \tag{4}$$

Integrating and rearranging gives

$$\ln \rho = c(\sigma_o/A) \cosh(\sigma/\sigma_o) + k \tag{5}$$

where k is a constant of integration. In the following discussion, I assumed k was also constant throughout the area studied. Using the 2 m. pit density data furnished by Shimizu and the density–depth curves of Figures 3–7, values of σ at 40 m. depth were calculated for each of

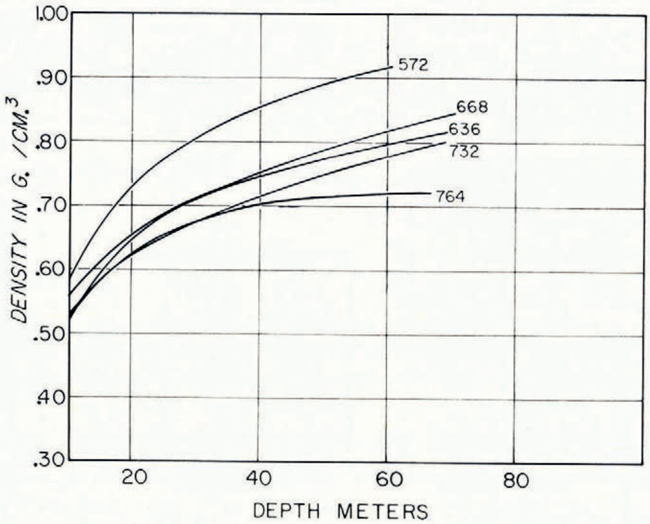

Fig. 5. Density-depth curves stations 572–764

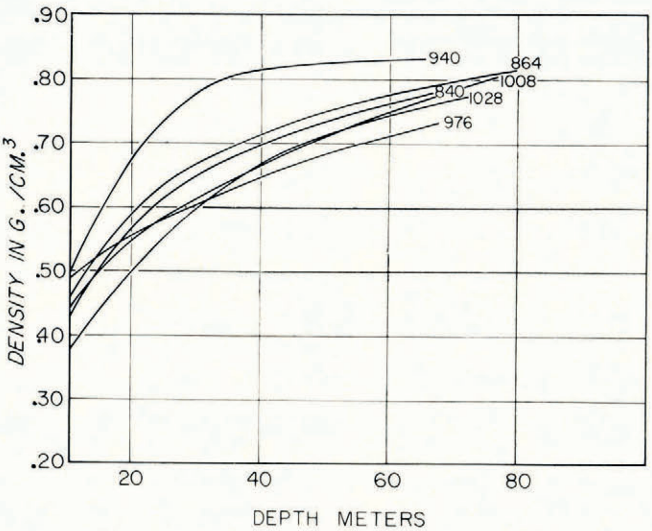

Fig. 6. Density-depth curves stations 840–1028

the thirteen stations of Figure 8 by numerical integration. These were found to show the following apparently linear relation with ρ at 40 m. depth (Fig. 12):

$$\bar{\rho} = 0{\cdot}54\rho + 0{\cdot}215. \tag{6}$$

Using this expression introduced an error of $\pm 0{\cdot}9$ per cent in $\bar{\rho}$. Within the range of densities of Figure 8

$$1/\rho = -1{\cdot}445 \ln \rho + 0{\cdot}919 \tag{7}$$

or

$$\ln \rho = 0{\cdot}636 - 0{\cdot}692/\rho$$

with an introduced error $\pm 0{\cdot}4$ per cent in $1/\rho$. (The error introduced by the assumption a linear relation between $1/\rho$ and $\ln \rho$ is less than that between ρ and $\ln \rho$ in the density range from $0{\cdot}7$ to $0{\cdot}8$ g./cm.³.) From equation (6) and the relationship $\sigma = \bar{\rho}$h, a theoretical curve of $\cosh(\sigma/\sigma_0)$ versus $\ln \rho$ was calculated and plotted on a graph of the observed data in Figure

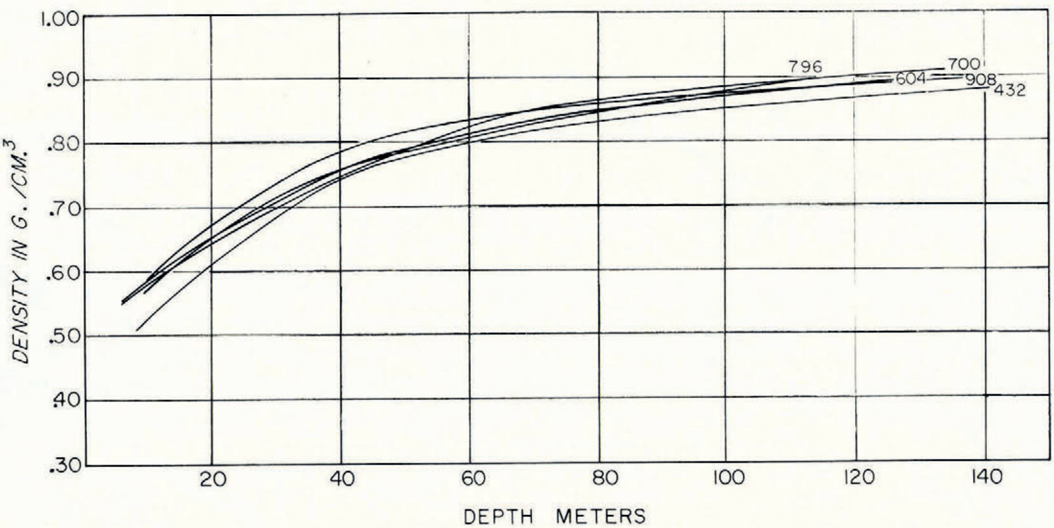

Fig. 7. Density-depth curves for 5 refraction stations extended to penetrate to depth of maximum velocity

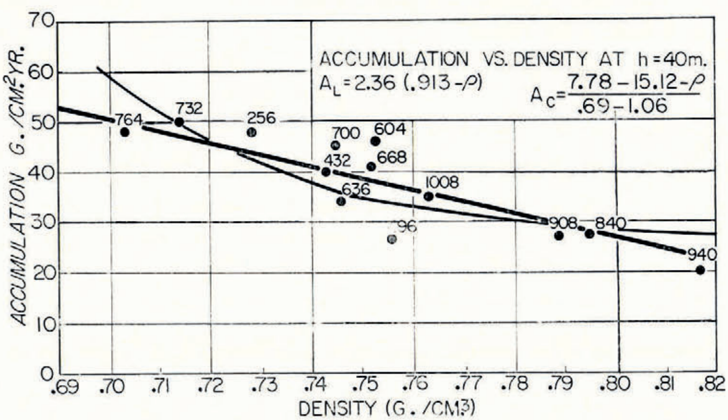

Fig. 8. Graph of accumulation vs. density at 40 m. depth. Straight line is the least-square fit through the data; curve is determined by the expression shown

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13. The standard deviation of points from the curve is ±3·0 per cent and from the least-square fit of a straight line is ±2·7 per cent. Thus the approximation of the linear relationship

$$\cosh(\sigma/\sigma_0) = 41\cdot16 \ln \rho + 29\cdot21 \tag{8}$$

has an error of ±3 per cent.

Substituting equation (7) into equation (8) and substituting both into equation (5) gives:

$$0\cdot636 - \frac{0\cdot692}{\rho} = \frac{c\sigma_0}{A}\left[55\cdot39 - \frac{28\cdot48}{\rho}\right] + k.$$

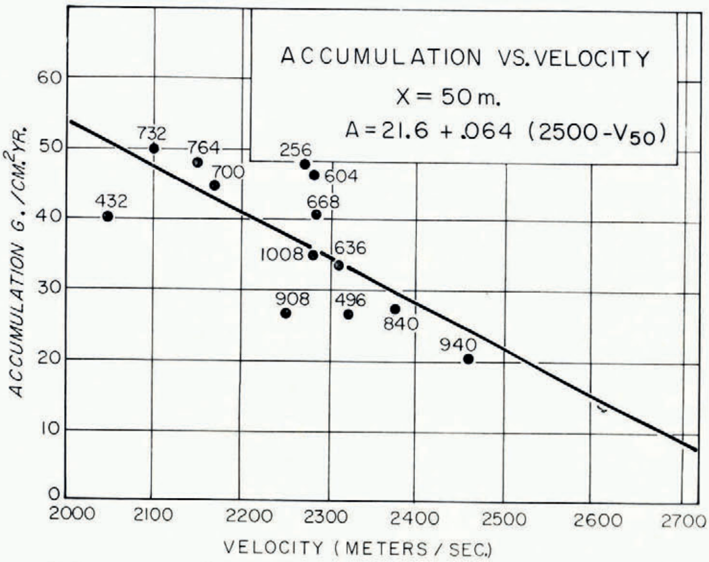

Fig. 9. Graph of accumulation vs. compressional wave velocity observed 50 m. from the shot point. The straight line is the least-square fit to the data

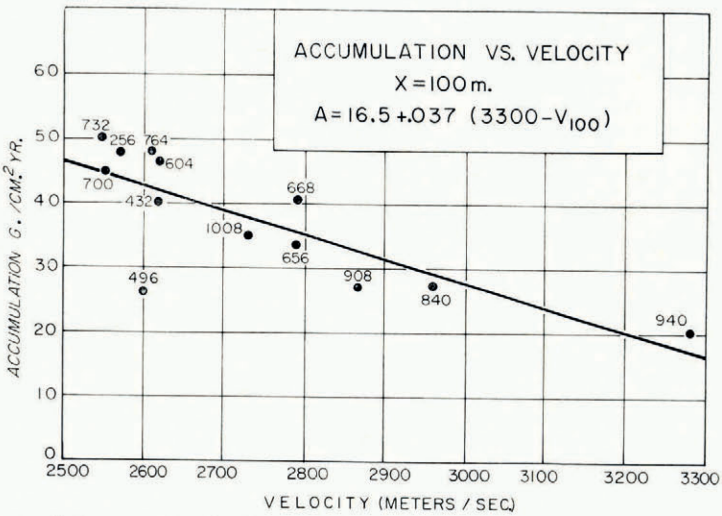

Fig. 10. Graph of accumulation vs. compressional wave velocity observed 100 m. from the shot point. The straight line is the least-square fit to the data

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Substituting the value of σ_0 and solving for ρ this reduces to

$$\rho = \frac{(1\cdot 99 \times 10^4)\ c - 0\cdot 692A}{(3\cdot 88 \times 10^4)\ c + A(k - 0\cdot 636)} \tag{9}$$

with a combined error in ρ of $\pm 3\cdot 4$ per cent where ρ is in the range $0\cdot 7$ to $0\cdot 8$ g./cm.³. The best fit to the observed data was found to be $c = 3\cdot 9 \times 10^{-4}$ yr.$^{-1}$ and $k = -0\cdot 42$. These gave the expression

$$A = \frac{7\cdot 78 - 15\cdot 12\rho}{0\cdot 69 - 1\cdot 06\rho} \tag{10}$$

from which the curve of Figure 8 was drawn. The standard deviation from the observed data

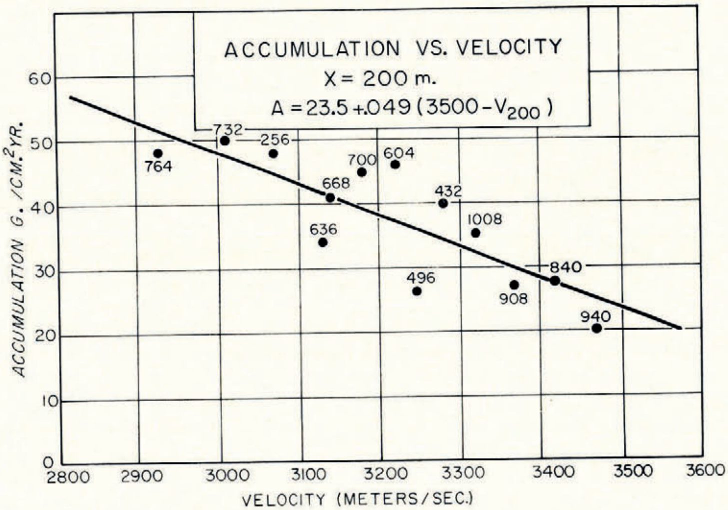

Fig. 11. Graph of accumulation vs. compressional wave velocity observed 200 m. from the shot point. The straight line is the least-square fit to the data

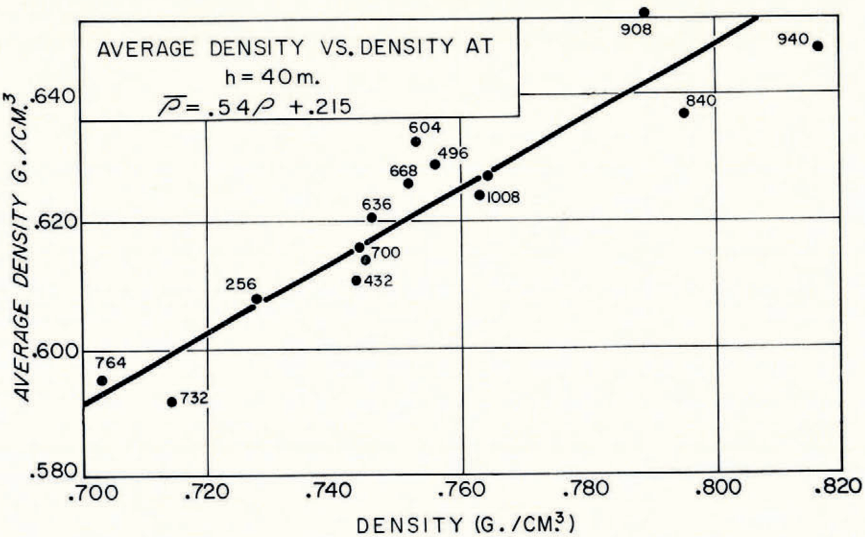

Fig. 12. Average density vs. density at 40 m. depth. The straight line is the least-square fit to the data

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is $\pm 6 \cdot 8$ g./cm.^{2} yr. compared with $\pm 4 \cdot 7$ g./cm.^{2} yr. for the linear fit and $\pm 4 \cdot 5$ g./cm.^{2} yr. from V_p at $X = 200$ m. (Fig. 11). Since a number of approximations and assumptions were made in deriving equation (10), it is not intended as a rigorous theoretical expression for the variation of density with accumulation. Rather, it allows some understanding of the empirical relations observed in Figures 8–11.

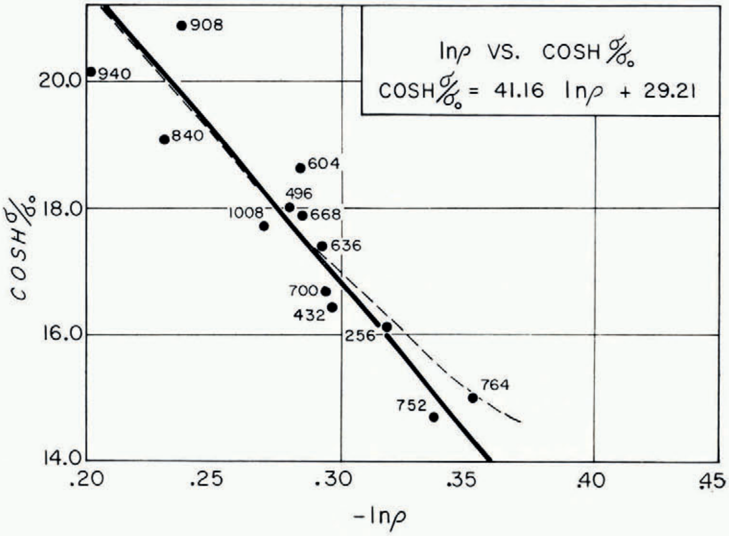

Fig. 13. Graph cosh (σ/σ_0) *vs.* ln ρ. *The straight line is the least-square fit to the data; the curve is the theoretical relationship*

This discussion leads to the conclusion that at least in the area of the Antarctic Peninsula traverse, where the scatter in mean annual temperatures is small, the compressional wave velocity at 200 m. distance from the shot point or the density at 40 m. depth can be used to obtain reasonably reliable values of accumulation. The graph of accumulation versus velocity at 200 m. is preferred over the graph of accumulation versus density because it is much less time-consuming to determine a velocity (which can be taken directly from a seismogram) at a given distance than a density at a specific depth.

Determinations of the accumulation were made for all of the seismic stations as shown in Table I using Figures 8 and 11. A_L and A_c refer to values from the straight line and curve of Figure 8; A_v refers to the values from V_p at $X = 200$ m. The standard deviation is $\pm 2 \cdot 9$ g./cm.^{2} yr. for the comparison of the three methods. This should be regarded as an indication of the internal agreement of the data rather than an indication of absolute accuracy of the accumulation values.

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