In Chapter IX, real Hilbert space is defined axiomatically, and after some discussion of subspaces and of bilinear functionals, the concept of a self-adjoint operator is introduced. The representation of a completely continuous self-adjoint operator in terms of its eigenvalues and eigenvectors is established, and, finally, the usefulness of this is illustrated by a brief consideration of integral equations with symmetric L_2 kernels.

It seems odd that there is no mention of complex Hilbert space (even though there is a reference, at the end of Chapter VIII, to quantum theory); but as with Volume 1, it is surprising to find so much material in so few pages, without undue compression. The authors have again achieved a nice blend of the abstract and the concrete. The Graylock translation benefits from the inclusion of a number of exercises, prepared by H. Kamel, which are distributed through the book and usefully amplify the text. An appendix, translated from the Russian edition of this volume, corrects some minor errors in Volume 1.

Here, in the Graylock translation, is a very good book: an excellent companion to the first volume, and also to Kolmogorov's well-known little book on probability (of which an up-to-date English edition would now be welcome).

J. D. WESTON

AHLFORS, L. V., AND OTHERS, *Analytic Functions* (Princeton University Press, 1960), vii+197 pp., 40s.

This book contains the principal addresses delivered at a Conference on Analytic Functions held at the Institute for Advanced Study, Princeton, in September, 1957. As one would expect, the papers are of a specialised character and they are as follows: *On differentiable mappings* by R. Nevanlinna; *Analysis in non-compact complex spaces* by H. Behnke and H. Grauert; *The complex analytic structure of the space of closed Riemann surfaces* by L. V. Ahlfors; *Some remarks on perturbation of structure* by D. C. Spencer; *Quasiconformal mappings and Teichmüller's theorem* by L. Bers; *On compact analytic surfaces* by K. Kodaira; *The conformal mapping of Riemann surfaces* by M. Heins and *On certain coefficients of univalent functions* by J. A. Jenkins.

The value placed by the reader on any particular article will naturally depend on his knowledge and predilections but, in the opinion of the reviewer, the book is well worth possessing if only for the article by Behnke and Grauert. This, when taken in conjunction with a lecture given by Behnke at the Amsterdam congress (*Funktionentheorie auf Komplexer Mannigfaltigkeiten*, Proceedings of the International Congress of Mathematicians 1954 (Amsterdam), 3 pp. 45-57) provides an excellent survey of the work done on complex manifolds by H. Cartan, Serre, Stein and Ahlfors since 1950. The value of this paper is enhanced, too, by four pages of references at the end.

The printing and layout of the book are first class.

D. MARTIN

AHLFORS, L. V., AND SARIO, L., *Riemann Surfaces* (Princeton University Press, 1960), xi+382 pp., 80s.

With the appearance of this book a comprehensive and modern treatment of the subject in English has become available for the first time. The first chapter gives a thorough and extensive treatment of the topology of surfaces. Particular attention is paid to bordered surfaces, open polyhedra (triangulated surfaces) and to compactification. The terminology is in a few instances non-standard. Thus an unlimited covering surface is called regular and a regular covering surface is called normal; the latter change of usage certainly seems preferable. Riemann surfaces appear first in the second chapter. The problem of constructing harmonic functions with given singularities on an open Riemann surface is solved in the third chapter by means of Sario's

https://doi.org/10.1017/S0013091500014590 Published online by Cambridge University Press

theory of normal operators and specialised to apply to a number of particular cases such as planar surfaces. The authors' special interests are treated in the fourth chapter, which deals with the classification of open Riemann surfaces of different classes and the derivation of inclusion relations between them. The final chapter on differentials contains the classical results, obtained by Hilbert space theory and Weyl's lemma, and culminating in the Riemann-Roch theorem and the theory of Weierstrass points. There is a very comprehensive bibliography and index.

The treatment is almost completely self-contained. Possibly because of the great wealth of material included, study of the book demands considerable concentration; a fair amount of work is also required of the reader since arguments are not always given in complete detail. For this reason the student beginning the study of Riemann surfaces may find it advantageous to read first a less comprehensive and sophisticated account, such as is given, for example, by G. Springer's *Introduction to Riemann Surfaces* (Addison-Wesley, 1957). For the research worker, however, the book by Ahlfors and Sario is indispensable and is likely to remain the standard text for some time.

R. A. RANKIN

N. BOURBAKI, *Éléments de Mathématique XXVI. Groupes et Algèbres de Lie: Chapitre* 1. Algèbres de Lie (Hermann et cie), 148 pp., 21 NF.

Chevalley completed two volumes of his well known work on Lie groups before he embarked on a systematic account of Lie algebras. In this latest volume of Bourbaki, which is the first in a series on Lie groups and algebras, we are first introduced to the theory of Lie algebras. A hundred pages of theory is balanced by over thirty pages of exercises. The theory in the case of a Lie algebra over a ring of prime characteristic is sketched in a number of the exercises. For the moment we confine our attention to those Lie algebras that are finite dimensional vector spaces over a field of characteristic zero.

A number of criteria are obtained for a Lie algebra to be one of the following types: soluble, nilpotent, semi-simple. Varying soluble and nilpotent radicals play an important role. The soluble radical has a complement—a Levi subalgebra—which is semi-simple; two Levi subalgebras are transformed into each other by a special automorphism (Theorem of Levi-Mal'cev). The theory reaches its climax in the Theorem of Ado. Every Lie algebra has a faithful representation of finite dimension such that the image of the nilpotent radical consists entirely of nilpotent elements.

The enveloping algebra of a Lie algebra over a commutative ring with unit element is considered in detail and the general form of the Poincaré-Birkhoff-Witt Theorem on the isomorphism between the symmetric algebra and the graded algebra associated with the filtered enveloping algebra is obtained.

The well known theory of Lazard on the connection between groups and Lie algebras is confined to a sketchy mention in the exercises and the following topics receive no attention: free Lie algebras, the Campbell-Hausdorff formula and basic monomials.

The volume is almost self contained and can be recommended as an introduction to the theory of Lie algebras.

S. MORAN

BORSUK, KAROL, AND SZMIELEV, WANDA, *Foundations of Geometry* (North Holland Publishing Co., Amsterdam, 1960), 400 pp., 90s.

This book is concerned with the foundations of Euclidean, Bolyai-Lobachevskian (hyperbolic) and real projective geometries and carries the development of each to the point at which the system of axioms can be shown to be categorical as well as consistent.