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REVIEW ARTICLE

Perspectives for extraordinary elastic-wave control in non-Hermitian meta-structures

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Abstract
Meta-structures, including metamaterials and metasurfaces, possess remarkable physical properties beyond those observed in natural materials and thus have exhibited unique wave manipulation abilities ranging from quantum to classical transports. The past decades have witnessed the explosive development and numerous implications of meta-structures in elastic-wave control under the Hermitian condition. However, more notably, a lot of recent research has been made to show that non-Hermitian meta-structures offer novel means for wave manipulation. Non-Hermiticity has enhanced both the accuracy and efficiency of wave steering capabilities. To this end, starting from electromagnetics and acoustics, we mainly review the up-to-date progress of non-Hermitian elastic meta-structures with a focus on their extraordinary elastic-wave control. A variety of promising scenarios realized by non-Hermitian elastic metamaterials and metasurfaces, such as the parity-time-symmetric system and the skin effect, are summarized. Furthermore, the perspectives and challenges of non-Hermitian elastic meta-structures for future key opportunities are outlined.

Introduction

The Hermitian system, in view of its energy conservation property and guaranteed real eigenvalues, plays a key role in both quantum physics and classical wave systems. However, the inevitable coupling with the external environment generally produces energy exchange and leads to energy gain/loss (Bender et al., 2003; Bender, 2007), which results in the breakdown of Hermiticity, thus requiring a non-Hermitian description. In recent decades, a flourishing development of non-Hermitian manipulation in open systems has been witnessed. It has been shown that non-Hermiticity not only offers suitable physical descriptions to non-conservative systems but also generates more unprecedented and exotic physics. In particular, with chiastopic fusion with another frontier field of metamaterials, non-Hermitian meta-structures have attracted ever-increasing attention and brought new perspectives to engineer architected structures with extraordinary wave-manipulation capabilities.

In a non-Hermitian system, gain/loss is treated as an imaginary part of the eigenvalue, which results in the eigenvalue ceasing to be complex. However, a unique case, known as a parity-time-symmetric (PT-symmetric) system (Hou and Assouar, 2018; Qi et al., 2019; Fang et al., 2021a;

Cao et al., 2022; Yi et al., 2022), with balanced gain and loss can possess real eigenvalues in non-Hermitian condition. In a PT-symmetric system, the exceptional point (EP) is a concept that cannot be ignored. The real and imaginary parts of eigenvalues coalesce at this critical value, and numerous particular phenomena may arise around the EP (Ding et al., 2016; Rosa et al., 2021). For example, let us direct our attention towards a scenario involving two resonators interconnected by a slender tube (Gao et al., 2020; Gu et al., 2021). The introduction of gain or loss within one or both resonators leads us into the realm of the non-Hermitian regime. Within this context of non-Hermitian manipulation, the system has the potential to traverse an EP, a juncture that interconnects the phases of PT symmetry and PT breaking. This progression is distinctly observable through the dynamic changes experienced by both the real and imaginary components of the eigenfrequency. Concomitant with the alterations in the non-Hermitian modulation, a phenomenon known as non-Hermitian degeneracy emerges. This effect becomes particularly apparent when scrutinizing the developmental trajectory of the eigenfrequency driven by the introduction of non-Hermiticity (Gao et al., 2020; Gu et al., 2021):

$$\frac{da_1}{dt} = i\omega_1 a_1 - \gamma_1 a_1 + i\kappa a_2 \tag{1}$$

$$\frac{da_2}{dt} = i\omega_2 a_2 - \gamma_2 a_2 + i\kappa a_1 \tag{2}$$

Herein, a_1 and a_2 indicate the amplitudes of the field within the two resonators, κ is the strength of reciprocal coupling, ω_1 and ω_2 are the resonant frequencies of the two resonators, and γ_1 and γ_2 indicate non-Hermitian modulation, respectively. The aforementioned developments can be succinctly elucidated through the application of coupled-mode theory (Maksimov et al., 2015; Huang et al., 2020). Given a time-dependence of the form $e^{i\omega t,}$, an expression for the eigenfrequency of the interconnected cavities can be obtained as

$$\omega_\pm = \omega_0 + i\frac{\gamma_1 + \gamma_2}{2} \pm \sqrt{\kappa^2 - \frac{(\gamma_1 - \gamma_2)^2}{4}} \tag{3}$$

Equation (3) delineates that alterations in the modulation coefficient lead to the attainment of non-Hermitian conditions. It subsequently resulted in the emergence of degeneracy within the eigenfrequency curves, which was influenced by fluctuations in the non-Hermitian coupling strength. Drawing from the foundations laid by Eqs. (1) and (2), we can additionally derive the effective Hamiltonian of the system in a matrix form:

$$H = \begin{pmatrix} \omega_0 - i\gamma_1 & -\kappa \\ -\kappa & \omega_0 - i\gamma_2 \end{pmatrix} \tag{4}$$

Through the solution of the eigenvalues inherent to this matrix, one can derive the expression for the eigenfrequencies as well. Indeed, the effective Hamiltonian has emerged as a versatile tool, assuming a pivotal role in the exploration of non-Hermitian modulation. As summarized in Figure 1, extraordinary wave phenomena in non-Hermitian meta-structures have received unprecedented attention in the fields of electromagnetic, acoustic, and elastic waves. Perfect absorption (Achilleos et al., 2017) and asymmetric diffraction (Zhong et al., 2020) can be realized by inspiring the appearance of EP. Furthermore, the coupling between wave control and non-Hermiticity can achieve intriguing wave manipulations like wave amplification (Wen et al., 2022), unidirectional transparency (Wu et al., 2019), and the skin effect (Weidemann et al., 2020). Meanwhile, the emergence of non-Hermitian wave phenomena has significantly broadened their corresponding applications and shown tremendous functionalities, such as whispering gallery (Hu et al., 2021; Pernas-Salomón et al., 2022), energy harvesting (Cai et al., 2022), holographic projection (Zhu et al., 2018b), topological structures (Mandal et al., 2022), acoustic diode (Zhang et al., 2015), single-mode laser (Feng et al., 2014), noise control (Huang et al., 2018), and tunable filter (Puri et al., 2021).

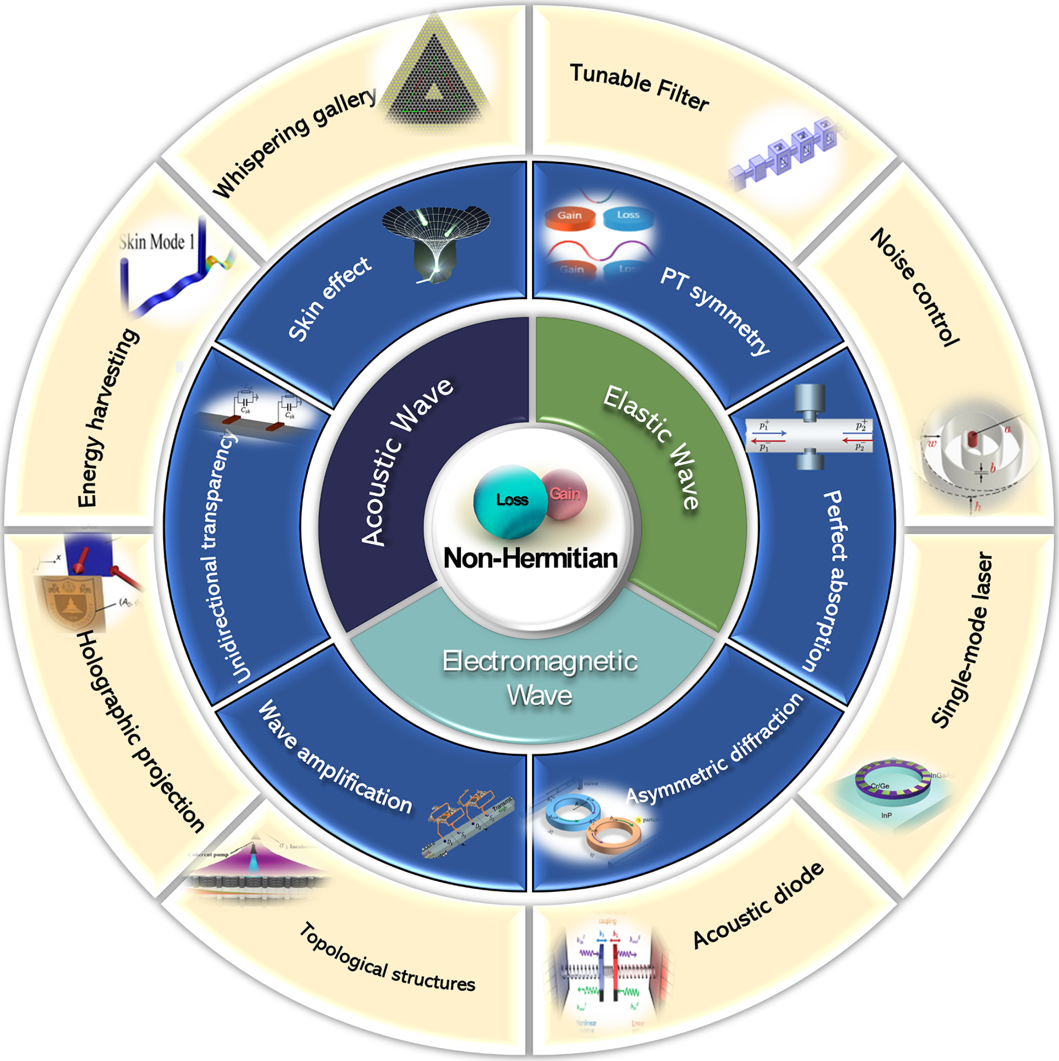

Figure 1. *Non-Hermitian physical phenomena and applications (Feng et al., 2014; Zhang et al., 2015; Fleury et al., 2015b; Achilleos et al., 2017; Huang et al., 2018; Zhu et al., 2018b; Qi et al., 2019; Wu et al., 2019; Weidemann et al., 2020; Zhong et al., 2020; Fang et al., 2021b; Hu et al., 2021; Puri et al., 2021; Cai et al., 2022; Mandal et al., 2022; Pernas-Salomón et al., 2022; Wen et al., 2022).*

Early attempts at non-Hermitian wave control can be traced back to electromagnetics. Quite a few non-Hermitian meta-structures have been proposed to manipulate electromagnetic waves by changing the amplitude, phase, and transmission properties (Khorasaninejad et al., 2016). Specifically, the physical properties of electromagnetic meta-structures are mainly delineated by effective permittivity and permeability. When non-Hermitian elements are introduced to electromagnetics, meta-structures can innovatively employ these two vital elements in a complex space and enrich striking applications (Pendry, 2000). For simplicity, we focus on two non-Hermitian features of electromagnetic meta-structures: 1) The appearance of EP (Chen et al., 2020; Dong et al., 2020; Wang et al., 2020). Figure 2(a) illustrates an electromagnetic metasurface that can realize angular asymmetry at EP. It exhibits completely suppressed reflection when light is incident from the left, whereas highly efficient retro-reflection occurs from the opposite side. The loss is introduced by adding a slit in the subunit and the angle asymmetry is obtained by accurately manipulating the phase gradient and diffraction orders. 2)

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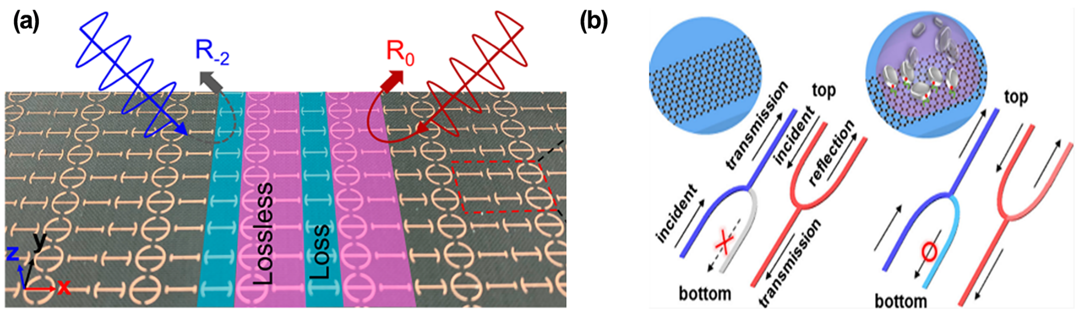

Figure 2. *(a) Unidirectional retro-reflection for non-Hermitian metasurface (Dong et al., 2020). (b) Illustration of the transmission and reflection of THz waves incident from the bottom (outlined as blue) and top (outlined as red) of a PT-symmetric sensor (Chen and Jung, 2016).*

The combination of PT symmetry and electromagnetics (Feng et al., 2014; Chen and Jung, 2016; Gu et al., 2017). In Figure 2(b), PT symmetry is realized by adjusting the resistive metallic filament and amplifying the graphene metasurface to balance the loss and gain. When THz waves are incident, the coupling between the spectral singularity and the chemical sensitivity of graphene may give rise to unidirectional reflectionlessness.

In contrast to electromagnetics, acoustics is, to a great extent, treated as a particular platform to pursue potential physical progress. In recent decades, with the dawn of non-Hermitian systems, the coupling between non-Hermitian and acoustic waves has provided a pioneering future in acoustic wave control. Compared to minimizing loss, the performance efficiency of non-Hermitian acoustic meta-structures can be greatly improved by precisely modulating the loss (Achilleos et al., 2017; Hou et al., 2018; Zhu and Assouar, 2019; Magariyachi et al., 2021). The most notable realizations are the accurate manipulation of diffraction orders (Fang et al., 2021a; Li et al., 2017; Yang et al., 2019) and the realization of asymmetric waveguides (Zhang et al., 2015; Fleury et al., 2015a; Ramezani et al., 2016; Shi et al., 2016; Lee et al., 2020; Zheng et al., 2020; Ju et al., 2021; Li et al., 2021). We first review the above outstanding phenomena by introducing reflection- and transmission-type metasurfaces. As Figure 3(a) exhibits, Li et al. (2017) proposed a lossy acoustic metasurface in 2017. After introducing an isotropic loss in the metasurface unit cells, tunable acoustic asymmetric transmission can be achieved. In 2019, Wang et al. (2019) proposed a gradient index lossy reflection-type metasurface. With an exactly tailored metasurface, extremely asymmetrical reflection was realized at the EP by harnessing the loss. As illustrated in Figure 3(b), the loss changes in the subunit can control the diffraction orders. When the incidence angle $\theta_i = 45^{\circ}$, the specular reflection and the extraordinary reflection are strongly suppressed. However, for $\theta_i = -45^{\circ}$, only the specular reflection is suppressed. Extraordinary reflection still exists in the form of an extremely asymmetrical reflection.

Meanwhile, along with systematic non-Hermitian manipulations that have been introduced into acoustics (Gerard and Jing, 2020; Gu et al., 2021), non-Hermitian acoustic meta-structures are capable of numerous rich-wave phenomena (Popa et al., 2015; Liu et al., 2018; Zhu et al., 2018a; Lan et al., 2020), including perfect absorption (Achilleos et al., 2017; Huang et al., 2018), wave amplification (Poshakinskiy et al., 2016; Hu et al., 2021), holographic projection (Zhu et al., 2018b; Zhu and Assouar, 2019), and unidirectionally invisible cloak (Zhu et al., 2014). Typically, a perfect acoustic absorber via a spiral metasurface composed of coiled channels and embedded apertures has been proposed (Huang et al., 2018) (Figure 3(c)). The embedded aperture offers additional freedom and hence can tune the impedance independently. In Figure 3(d), by modulating the ratio between the resistance and reactance, this structure is able to realize perfect absorption with tunable frequencies. This non-Hermitian design may contribute tremendously to relevant applications in low-frequency acoustic absorption. In addition, by combining acoustic resonances with non-Hermitian ingredients, Hu et al. proposed a triangular crystal made of a topological lattice that can generate the topological whispering gallery (Hu et al., 2021). Here, the topological lattice is realized by adding gain elements,

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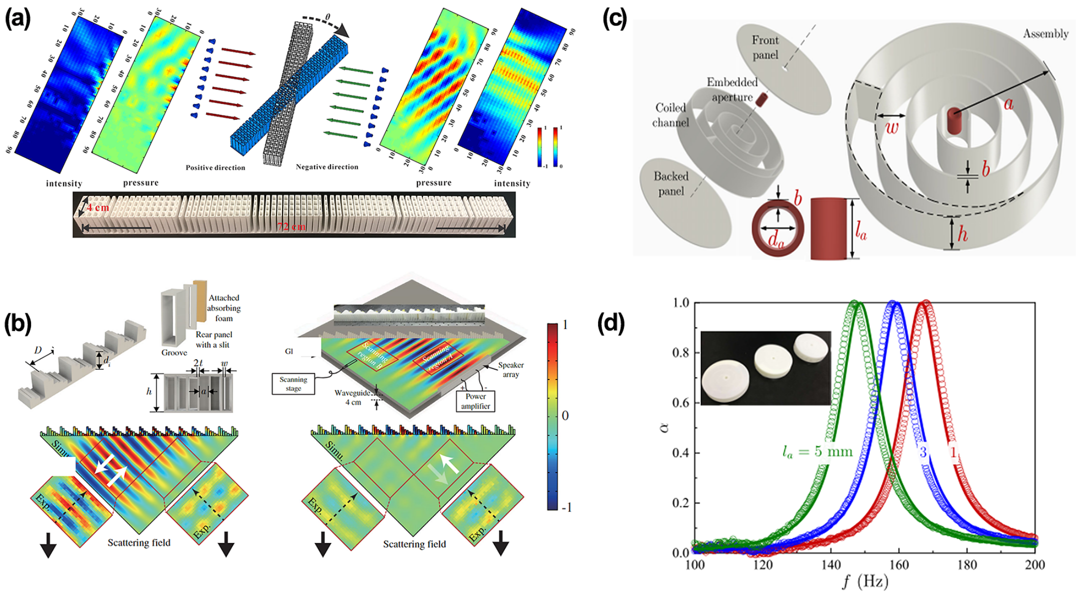

Figure 3. *(a) Asymmetric transmission via lossy metasurface (Li et al., 2017). (b) Gradient index lossy metasurface and asymmetric wave phenomenon (Wang et al., 2019). (c) Spiral metasurface (Huang et al., 2018). (d) The perfect absorptions when the lengths of embedded apertures are 5 mm, 3 mm, and 1 mm (Hu et al., 2021).*

namely pasting carbon nanotubes on the insulator rods. This system breaks the chiral symmetry of the whispering-gallery modes and achieves simplified acoustic waveguides in topologically robust environments. Furthermore, this design may contribute to nondestructive testing and acoustic sensing.

Following the development of non-Hermitian modulation, extensive acoustic and electromagnetic applications of non-Hermitian systems have been introduced in multiple reviews (Wang et al., 2021; Ding et al., 2022; Zhang et al., 2022; Lin et al., 2023). Specifically, Nikhil et al. treated the loss as more of a friend than a foe and systematically introduced corresponding applications of non-Hermitian acoustic metasurfaces in 2019 (Gerard and Jing, 2020). In 2021, Gu et al. analysed the mechanisms of non-Hermitian metasurfaces using basic concepts and mathematical tools (Gu et al., 2021). Apart from acoustics and electromagnetics, elastic waves play a vital role in nontrivial physics and reality. Compared with electromagnetics and acoustics, elastic waves can produce more complex multi-mode coupling, conversion, and dispersion characteristics. Moreover, recent developments in non-Hermitian control have prompted us to see loss and gain in a new light: as ubiquitous elements in environment, precise manipulation of non-Hermiticity can open new avenues for unconventional strategies over elastic-wave control. Due to the fact that few existing reviews trace systematic research into elastic waves with non-Hermiticity, in this perspective, we focus on the coupling between non-Hermitian and elastic waves and summarize the corresponding investigations. The contents of this paper are organized as follows: firstly, we introduce the non-Hermitian system of elastic waves. In particular, we focus on non-Hermitian subwavelength structures including non-Hermitian metamaterials and metasurfaces. Secondly, we discuss the bulk-boundary correspondence (BBC) under the Hermitian condition. Furthermore, by extending the BBC to non-Hermitian systems, we mainly summarize a peculiarly interesting phenomenon, the non-Hermitian skin effect in elastic waves. Finally, we outline the perspectives and challenges of non-Hermitian elastic meta-structures for future key opportunities.

Non-Hermitian in elastodynamics

Non-Hermitian phenomena expand the degrees of freedom of wave manipulations and inspire potential applications in elastic waves (Liu et al., 2000; Jung et al., 2019). Therefore, to investigate the internal

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mechanisms of non-Hermitian elastic systems, we subdivide them into two sections: non-Hermitian elastic metamaterials and non-Hermitian elastic metasurfaces. Recently, accompanied by the rise of non-Hermitian modulation, various applications have been innovatively realized in metamaterials. Thus, to summarize the significant properties of non-Hermitian elastic metamaterials, we first discuss the following three aspects: 1) PT-symmetric metamaterials; 2) inspiring the realization of EP; and 3) topological non-Hermitian metamaterials.

Non-Hermitian elastic metamaterials

Parity-time-symmetric metamaterials

PT-symmetric metamaterials exhibit balanced loss and gain, thus maintaining energy conservation. An attractive phenomenon shows that these structures also possess real eigenvalues and hence exhibit unique physical characteristics (Hou and Assouar, 2018; Fang et al., 2021a; Cao et al., 2022; Yi et al., 2022). Active control offers a highly efficient method to designing PT-symmetric elastic metamaterials (Hou et al., 2018). As illustrated in Figure 4(a), a PT-symmetric piezoelectric metamaterial was proposed with external circuits containing positive and negative resistances. This tunable device, manipulated by an external shunted circuit, can run at the expected frequency without any geometric change (Hou and Assouar, 2018). As such, the outstanding tunability of these active control systems provides comprehensive versatility and high performance for non-Hermitian elastic metamaterials. Alternatively, upon further exploration, a periodic-like coherent perfect absorber was demonstrated in a PT-symmetric metamaterial beam (Yi et al., 2022). As Figure 4(b) illustrates, the loss/gain element is supplied by the positive/negative resistance of an external circuit. It is noteworthy that this particular periodicity can be predicted by observing the related frequency of the maximum Bloch wavenumber.

On the other hand, an increasing number of investigations have shown that energy cannot keep balance in the broken phase of PT. Eigenfrequencies become complex under non-Hermitian conditions, thus generating extraordinary wave phenomena, such as non-reciprocal wave transmission (Ghatak et al., 2013; Peng et al., 2014; Gear et al., 2017; Song et al., 2021). In fact, most of these extraordinary elastic-wave manipulations in non-Hermitian metamaterials are closely related to EP. Qian Wu et al. designed a piezoelectric PT-symmetrical metabeam that can realize unidirectional reflectionlessness (Wu et al., 2019). Due to the fact that effective mass density is assumed to be real, the PT-symmetric condition in this structure is illustrated by the effective bending stiffness. Thus, to realize a complex effective bending stiffness, this structure induces non-Hermiticity by the piezoelectric system. As shown in Figure 4(c), two piezoelectric patches are bonded to the beam. Each patch is connected to an external circuit that includes a negative/positive resistor and a negative capacitor in parallel. Here, the gain component is composed of negative resistance, the loss component is provided by positive resistance, and the negative capacitance can amplify the gain/loss elements.

Furthermore, as depicted in Figure 4(d), the reflection coefficients pertaining to two distinct incident directions are presented. The markers of unidirectional reflectionlessness, delineated by green lines, manifest at frequencies of 0.706 kHz and 0.805 kHz. The phenomenon of unidirectional reflectionlessness in this context can also be analysed by scrutinizing the locus of EPs. A notable observation is that, at unidirectional reflectionless points, both the amplitudes and phases of the two eigenvalues are identical. Besides, the phase transition points are also exactly the unidirectional reflectionlessness points. These findings definitively establish a clear alignment between the unidirectional reflectionless points observed within the context of the non-Hermitian PT-symmetric beam and the conceptual framework of EPs. This assertion gains further substance from the demonstration of tunable EPs achieved by manipulating the parameters of the shunting circuit.

In addition, numerous recent studies have utilized PT symmetry and identified EPs in non-Hermitian metamaterials to explore increasingly peculiar elastic waves phenomena. Figure 4(e) depicts the design of a PT-symmetric metaplate utilizing piezoelectric elements (Cai et al., 2023). Through the manipulation of these piezoelectric parameters, a striking phenomenon of unidirectional reflectionlessness becomes manifest. Expanding upon this concept, a metaplate featuring a perforation is introduced

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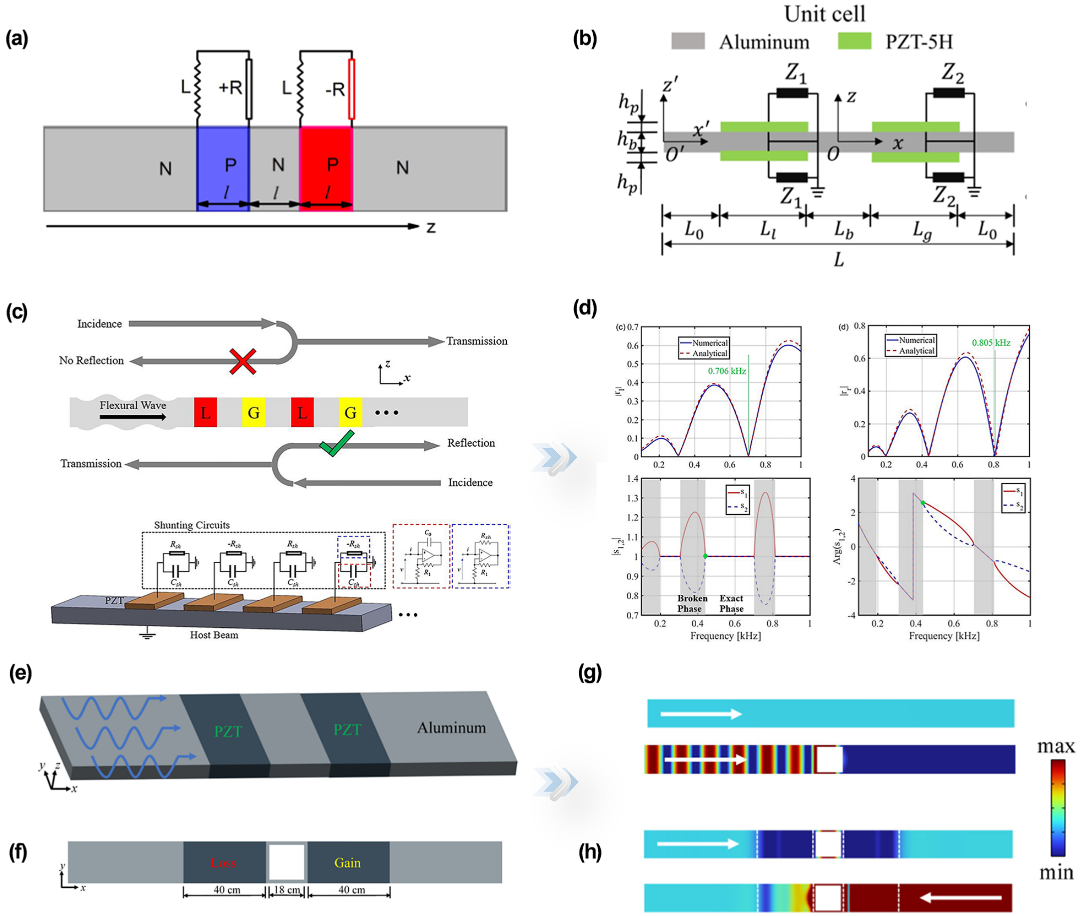

Figure 4. *(a) The PT-symmetric metabeam can actively be tuned without alterations to its geometric configuration (Hou and Assouar, 2018). (b) The piezoelectric PT-symmetric metabeam (Yi et al., 2022). (c) Unidirectional reflectionlessness of PT-symmetric beam. (d) Calculated reflection coefficients, amplitude, and phase of eigenvalues in PT-symmetric metabeam (Wu et al., 2019). (e) Schematic illustration of a metaplate. (f) Schematic illustration of a non-Hermitian metaplate with a hole. (g) Displacement fields of the metaplate and metaplate with hole respectively. (h) Displacement fields of the metaplate with piezoelectric materials (Cai et al., 2023).*

in Figure 4(f) to illustrate the concept of unidirectional cloaking. Given that the aperture inherently generates scattering waves and consequently influences the scattered field, the PT-symmetric system endeavours to counteract these scattering effects through the employment of EPs. Figure 4(g) visually portrays the displacement field across the metaplate. Notably, the wave field within the perforated metaplate exhibits disturbances when compared to an undisturbed solid plate. With the integration of piezoelectric materials, as demonstrated in Figure 4(h), the displacement field external to the piezoelectric region can be selectively restored in a unidirectional manner. It is imperative to underline that this cloaking effect finds its origins in the phenomenon of unidirectional reflectionlessness facilitated by EPs. In summary, the domain of non-Hermitian metamaterials, augmented by active control, offers unparalleled prospects for exploring the intricacies of wave amplification, energy harvesting, and the meticulous refinement of highly sensitive sensors.

Realization of the exceptional point

Analogous to electromagnetics and acoustics, EP plays an essential role in non-Hermitian elastic metamaterials (Shmuel and Moiseyev, 2020; Gupta and Thevamaran, 2022; Wu et al., 2022; Gupta

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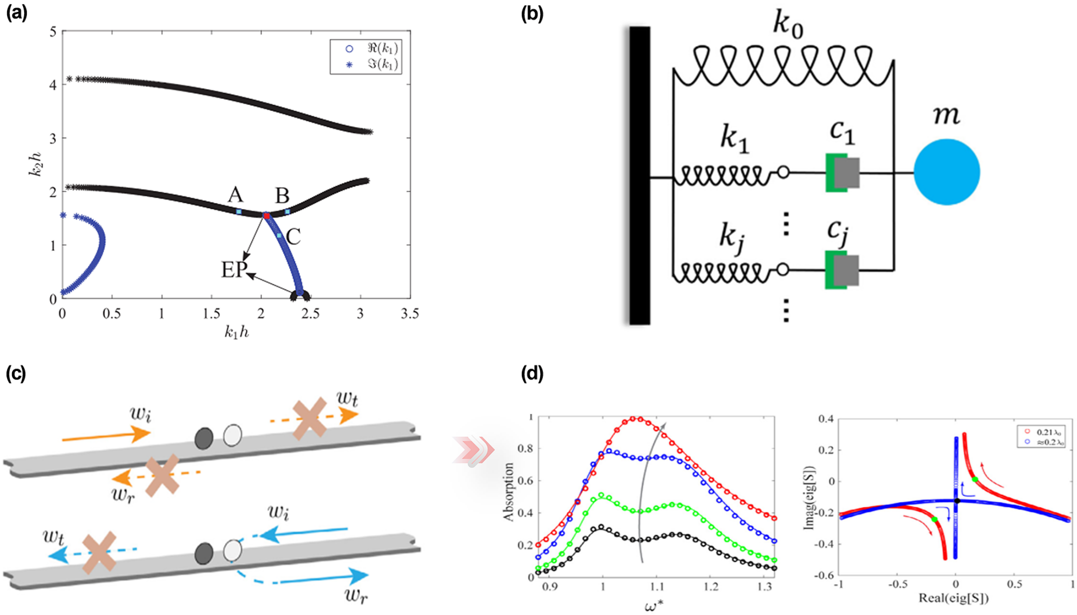

Figure 5. *(a) EP occurs at the intersection of real branches and the branch with complex values (Mokhtari et al., 2020). (b) The viscoelastic generalized Maxwell model (Yi et al., 2022). (c) Asymmetric transmission on a non-Hermitian metabeam. (d) Schematic of the EP at perfect absorption (Li et al., 2022).*

et al., 2023). The scattering properties of elastic waves for non-Hermitian metamaterials with sharp changes around the EP have been verified in Figure 5(a) (Mokhtari et al., 2020). In this section, we focus on the realization of unidirectional zero reflection by constructing the EP in the non-Hermitian system. In Figure 5(b), a viscoelastic generalized Maxwell model consisting of different resonators is proposed. The coupling between the viscoelasticity and resonators can widen the bandgap (Alamri et al., 2018; 2019). Besides, the asymmetry of the system is significantly affected by the mass of the oscillators. Notably, by controlling oscillator masses, this viscoelastic system also exhibits unidirectional zero reflection at the EP (Yi et al., 2022).

More recently, an extremely asymmetric perfect absorption and reflection resonant scatterer has been proposed (Li et al., 2022). As exhibited in Figure 5(c), this structure consists of a pair of loss-induced asymmetrical resonant scatterers. Unidirectional reflection can be observed in two opposite incident directions. What is distinctive about the perfect absorption of this coupled resonator is the critical coupling condition, which means that the inherent losses in resonators equal the energy loss, which can be obtained by manipulating the intrinsic mechanical damping. As expected, interesting transmission characteristics can also be illustrated in eigenvalue curves. In Figure 5(d), double absorption peaks arise at smaller damping coefficients, whereas the two peaks start combining into a single peak at the optimum damping. Besides, when changing the distance of two resonators, eigenvalues coalesce at the optimum spacing distance and the system realizes perfect absorption.

Topological non-Hermitian metamaterials

In the field of elastic waves, topology has been employed to characterize different types of elastic waves based on their unique properties. The existing literature offers available methodologies to realize wave modulation in topological fields, including topological phase transitions and nontrivial topological states. At this point, the influence of topological phase control has been closely examined (Ghatak et al., 2013; Peng et al., 2014; Gear et al., 2017; Song et al., 2022; Gupta et al., 2023). A soft elastic metamaterial shown in Figure 6(a) can realize dynamic tunability of topological states (Li et al., 2018).

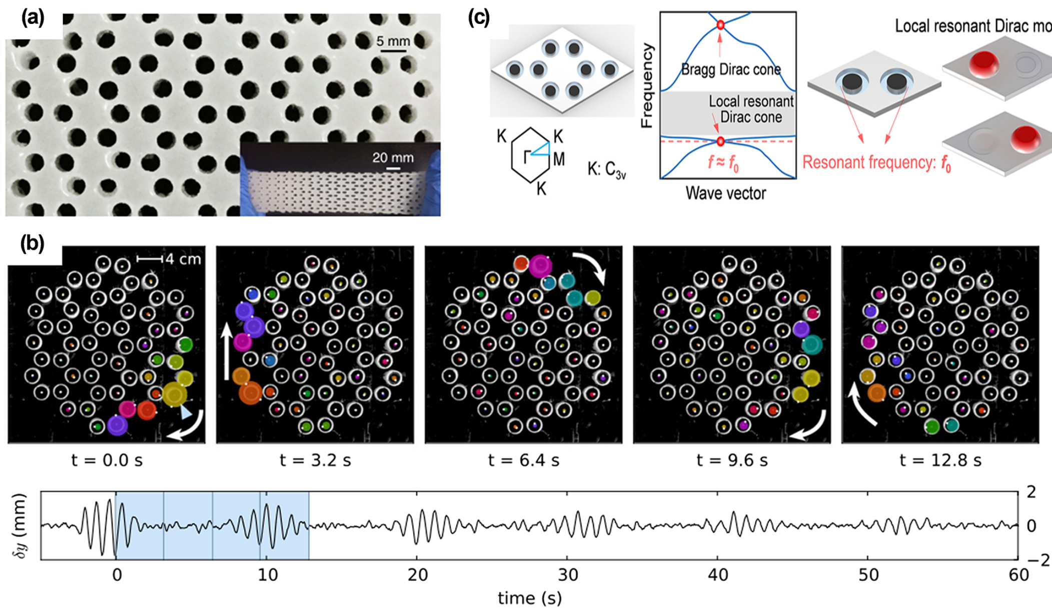

***Figure 6.** (a) Soft elastic metamaterials (Li et al., 2018). (b) Local resonant valley Hall insulator (Zhang et al., 2020). (c) Unidirectional waveguide modes excited by a single-edge gyroscope (Nash et al., 2015).*

Moreover, it is worth noting that topologically protected wave propagation possesses robust defect-immune transport properties. One interesting aspect is that breaking reciprocity to mimic the quantum Hall effect (QHE) (Huckestein, 1995; Zhang et al., 2005) is a broad path to realize topologically protected wave propagation. For example, the broken space/time reversal symmetry provides a new prospect for obtaining the QHE. Figure 6(b) elucidates the nontrivial state, and it can realize the topologically protected elastic-wave edge mode when the time-reversal symmetry is broken (Nash et al., 2015). In recent years, researchers have explored the possibility of topological protection in the absence of an applied magnetic field. In this field, the quantum spin Hall effect (QSHE) (Kane and Mele, 2005b; Qi and Zhang, 2011) and the quantum valley Hall effect (QVHE) (Xiao et al., 2007; Pan et al., 2014) exhibit fascinating applications. Most remarkably, the QHE can be obtained by breaking the space-reversal symmetry, and it can be characterized by the generation of counter-propagating chiral edge states localized at the opposite edges of the material. In Figure 6(c), an elastic metamaterial plate generates a new local resonant bandgap that supports topological edge modes and illustrates the QVHE (Zhang et al., 2020).

A further attractive concept that has expanded from quantum systems to classical waves is topological non-Hermiticity. More recently, along with the fusion of non-Hermitian and elastic waves, there has been an extensive focus on elastic topological structures within non-Hermiticity. The idea of combining the topology and non-Hermiticity in elastic waves has not only been obtained but also adapted in both theories and experiments (Zhang et al., 2019; Tang et al., 2020; Riva et al., 2021; Wu et al., 2021). For example, a freestanding metabeam with non-Hermitian modulation can achieve unidirectional amplification or attenuation of wave propagation (Figure 7(a)). Furthermore, the spectrum shows that the elastic modulus is related to the topological index, and this phenomenon results in the localization of vibrational modes (Figure 7(b)) (Chen et al., 2021). On the basis of these related achievements, we set out to clarify the non-Hermitian manipulation of the topological edge modes. Figure 7(c) shows the design of a 1D trimerized model consisting of a resonant plate and linking beam (Fan et al., 2022), where the topological edge modes can be caused by non-Hermiticity when damping layers are added to the structure. This may excite more explorations about the plentiful mechanisms

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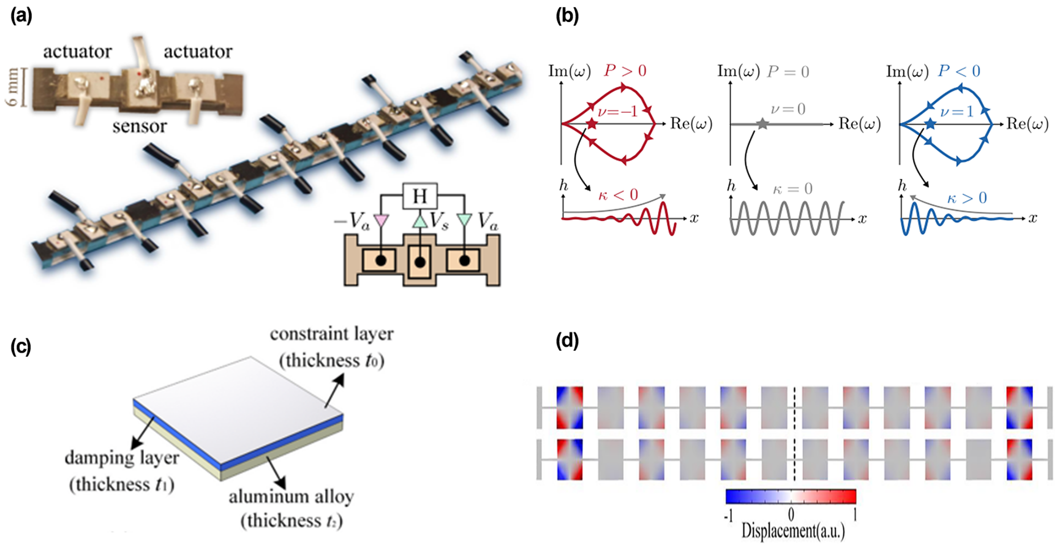

Figure 7. *(a) A non-Hermitian metabeam with piezoelectric elements and electronic feed-forward control. (b) The presence of a localized mode (Chen et al., 2021). (c) Schematic of the composite plate from top to bottom: the tin foil constraint layer, the rubber damping layer, and the aluminium alloy host plate (Fan et al., 2022). (d) Out-of-plane displacement of the topological edge state (Fan et al., 2022).*

between topology and non-Hermiticity. Moreover, other research has investigated the realization of topological edge modes. It explores the topological edge states under the Hermitian and non-Hermitian conditions, respectively (Fan et al., 2022). This system is able to maintain unanimous hopping strengths in unit cells and modulate nontrivial topologies by adjusting additional damping layers. The out-of-plane displacement field presented in Figure 7(d) illustrates the two edge modes localized at the two edges. On the other hand, a recent thesis evaluated a topological coupler with four ports with non-Hermiticity (Meng et al., 2022). Combining the gain and loss control by the non-Hermiticity, highly efficient elastic-wave guiding with topological robustness was realized. In summary, the coupling between topology and non-Hermitian may be extended to further fields and offers more useful findings in sensing and signal processing.

Non-Hermitian elastic metasurfaces

In recent decades, the domain of metamaterials comprising subwavelength microstructures has witnessed remarkable and swift progress within the realm of elastic waves, facilitating the emergence of sophisticated wave control phenomena. However, due to the typically periodic characteristics, metamaterials are normally limited in bulky structures. Thus, aiming at structure simplification and application enrichment, a lightweight two-dimensional planar structure named metasurface is proposed. As shown in Figure 8(a), this structure possesses phase discontinuity and is hence able to manipulate wave propagation (Yu et al., 2011). As shown in Figure 8(b), Liu et al. introduced a source illusion device aimed at the manipulation of flexural Lamb waves through the utilization of elastic metasurfaces (Liu et al., 2017). They have devised a comprehensive framework for the creation of source illusions targeting the A_0 mode Lamb waves. This framework showcased a range of versatile elastic illusion effects from all angles. Assessed through the lens of Huygens–Fresnel diffraction theory, this effect exhibited broadband characteristics and remained resilient even in the face of shifts in source positions.

A great deal of investigations provide plentiful applications in high-order diffractions (Liu and Jiang, 2018; Hou et al., 2019; Hu et al., 2023; Wang et al., 2023; Li et al., 2024), negative refraction (Xie et al., 2014; Willis, 2016; Yang et al., 2018), and asymmetric propagation (Cao et al., 2018;

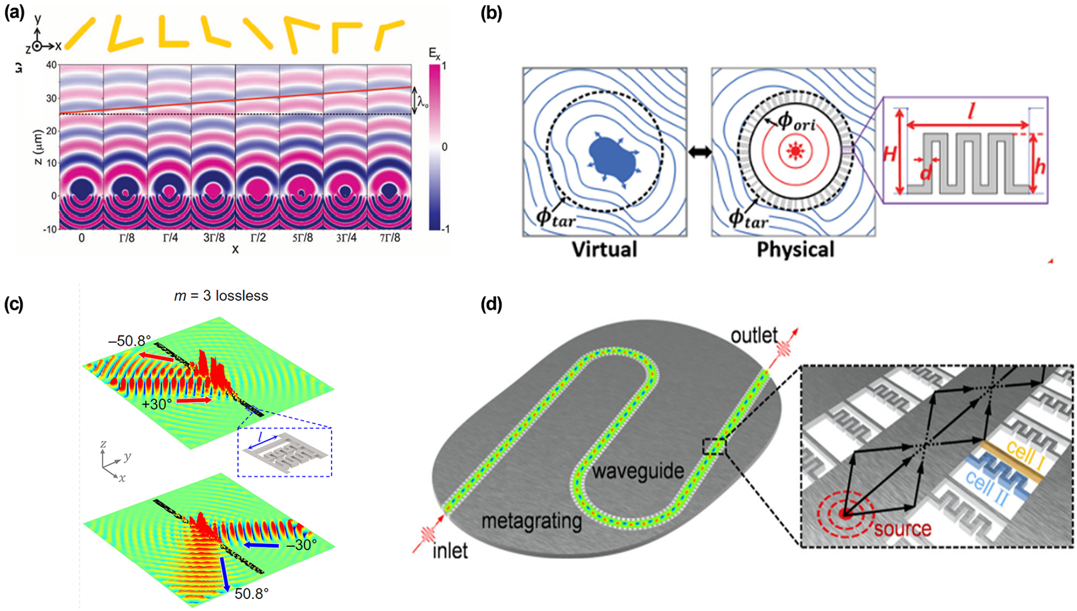

Figure 8. *(a) Schematic of eight antennas and the transmission fields of the crosspolarized (Yu et al., 2011). (b) Schematic of making illusions using metasurfaces (Liu et al., 2017). (c) Efficient asymmetric transmission in lossless metasurfaces (Li et al., 2020). (d) An ultrathin waveguide routing along an arbitrary path in a plate and corresponding two layers of thin elastic metagratings (Hu et al., 2022).*

Li et al., 2020). For example, Figure 8(c) demonstrates a lossless metasurface that can achieve asymmetric transmission. It stems from the parity of multiple reflections against different diffraction orders (Li et al., 2020). In addition, based on subwavelength metagratings, an ultrathin waveguide framework for omnidirectional wave trapping and efficient routing was proposed, as illustrated in Figure 8(d). Compared with metamaterial-based waveguides, this strategy exhibits absolute advantages in compact size and broadband wave routing (Hu et al., 2022).

Owing to its unique phase modulation, the challenges seem more attractive for combining non-Hermiticity and metasurface. Some factors affecting non-Hermitian metasurfaces have been explored. Similar to non-Hermitian elastic metamaterials, the related approaches are generalized in elastic metasurfaces. In stark contrast to the prevailing research paradigm rooted in Hermitian physics, a recent work embarks on an exploration of the uncharted territory of non-Hermitian elastic metamaterials, thus ushering in new frontiers within open quantum and classical systems characterized by gain or loss. As Figure 9(a) shows (Cheng et al., 2024), this paradigm shift hinges on the introduction of non-Hermitian modulation, as embodied by the creation of the non-Hermitian metagrating (NHMG). The NHMG, characterized by its single type of subwavelength unit cell, exhibits remarkable characteristics under specific low-loss conditions, epitomizing the realization of perfect wave absorption (Figure 9(b)) and culminating in the establishment of a versatile design framework for NHMG materials, accommodating irregular and arbitrary shapes. This framework attests to the NHMG's robust performance, underscoring its capacity to achieve omnidirectional and perfect absorption capabilities regardless of the boundary shape, rotation angle, and wave source location (Figure 9(c,d)). Consequently, this study not only diversifies the landscape of elastic-wave manipulation but also advances the fundamental understanding of non-Hermitian materials.

Furthermore, investigations of EP on non-Hermitian elastic metasurfaces have provided design paradigms for elastic structural implementations. A recent work discusses the transmission characteristics of flexural waves in an asymmetric piezoelectric metasurface. The structure of this elastic metasurface is shown in Figure 9(e) (Stojanoska and Shen, 2022). The non-Hermitian degree can

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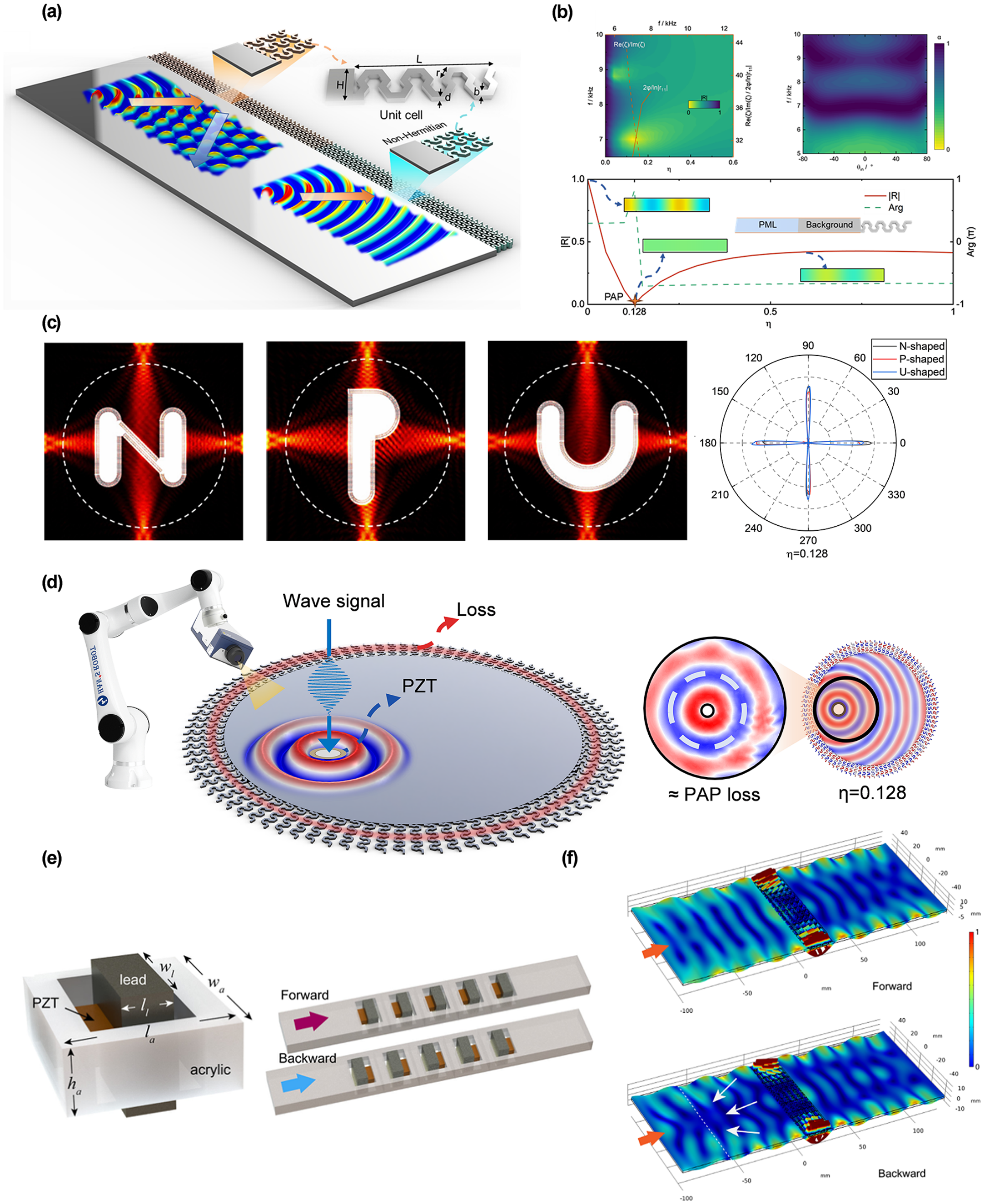

Figure 9. (a) Schematic diagram of the structure and function of NHMG. (b) Theoretical verification of perfect absorption in the NHMG and a perfectly absorbing NHMG unit cell. (c) The energy field distributions corresponding to NPU-shaped defects. (d) Experimental setup and experimental model (Cheng et al., 2024). (e) Schematic of the piezoelectric non-Hermitian metasurface. (f) The realization of unidirectional focusing (Stojanoska and Shen, 2022).

be adjusted by modulating the shunting resistance and the negative capacitance of the piezoelectric system. Under a lossless condition, the reflection and transmission amplitudes are equal in the opposite wave propagation directions, while under non-Hermitian conditions, the unidirectional zero reflection can be observed. This arises from the meticulous engineering of each unit cell to manifest an EP at the designated operational frequency. As a consequence, the metasurface attains a state of complete reflection suppression (Figure 9(f)). The real and imaginary parts of the eigenvalues coalesce at this critical value (i.e. EP). By accessing the focal length, the focusing of reflected waves can be obtained.

Although a few pioneering studies have been conducted, given that investigations of non-Hermitian metasurfaces are still in their infancy, existing analytical studies offer a new strategy for the design of non-Hermitian elastic meta-structures and structural wave control. Of equal significance, the aforementioned designs lay a foundation for potential research endeavours aimed at harnessing wave-related applications, including but not limited to structural health monitoring and nondestructive testing. Furthermore, this paves a natural path towards delving into the realm of non-Hermitian systems and their implications for these applications.

Skin effect in non-Hermitian meta-structures

Bulk-boundary correspondence in a non-Hermitian system

Ever since the development of condensed matter physics has brought cutting-edge research hotspots, a growing body of researchers has found that the electrons and atoms in quantum physics are able to form particular structures and exhibit unique properties. Hence, based on the novel phenomena of quantum states, numerous fundamental theories such as topological state, density matrix, and wave function have been proposed (Landau and Lifshitz, 1980; Anderson, 2018; Bouwmeester et al., 1997; Kane and Mele, 2005a, 2005b; Bernevig et al., 2006; Bernevig and Zhang, 2006; Konig et al., 2007; Hsieh et al., 2008; Breuer et al., 2009; Yao et al., 2012; Broadbent and Schaffner, 2016; Streltsov et al., 2017). In the quantum world, topology centres on the quantized model and its physical quantities, including topological structure, topological quantum circuit, and topology of porous material. Among these, topological meta-structures have garnered significant attention in the field of topological state research, primarily because they offer more efficient control over wave propagation. Moreover, topological band theory has achieved tremendous success with extensive literature, particularly on its boundary state and topological invariants.

The Bloch theorem plays a crucial role (Hasan and Kane, 2010; Qi and Zhang, 2011; Chiu et al., 2016) in Hermitian topological meta-structure. As elaborated below, the Bloch wave functions give a description of the meta-structure band. Actually, in order to accurately describe their quantized behaviours, the concept of topological invariant is proposed to show topology properties. The topological invariant describes the overall space of structures, and structures with similar "genus" are topologically equivalent (Essin and Gurarie, 2011). Besides, the topology represents the state of the substance. Thus, the topological phase of a system is delimited by topological invariants, like the Chern number (Hatsugai, 1993) and Winding number (Yin et al., 2018). The Chern number is treated as the integral of berry curvature over the Brillouin zone, and it can illustrate the QHE, while the Winding number is capable of determining how many times a curve wraps around a specific point. Specifically, the curve should be closed and denoted as $C(t)$, where t ranges from 0 to 2π. Choose a point z_0 in the complex plane and the Winding number can be calculated as: $W(C, z_0) = (1/2\pi i) \oint C \, dz/(z - z_0)$, where $\oiint$ represents the contour integral, which means integrating around the closed curve C, dz is the differential element along the curve, and z is the parameter representing a point on the curve C.

As shown in Figure 10, the BBC in the Hermitian system is an essential principle under open boundary conditions (OBCs). A significant milestone in the research of boundary states is that topologically protected boundary states become more stable and possess quantized features. As Figure 11(a) shows, in Hermitian systems, the boundary states under OBCs can be described by the topological invariants in periodic boundary conditions (PBCs). In non-Hermitian systems, open boundary states are completely different from periodic boundary states, which means the failure of BBC (Deng and Yi, 2019; Jin and Song, 2019; Kunst and Dwivedi, 2019; Lee et al., 2019; Yang et al., 2022). In Hermitian systems, the eigenstates under the open boundary represent the linear superpositions of Bloch waves

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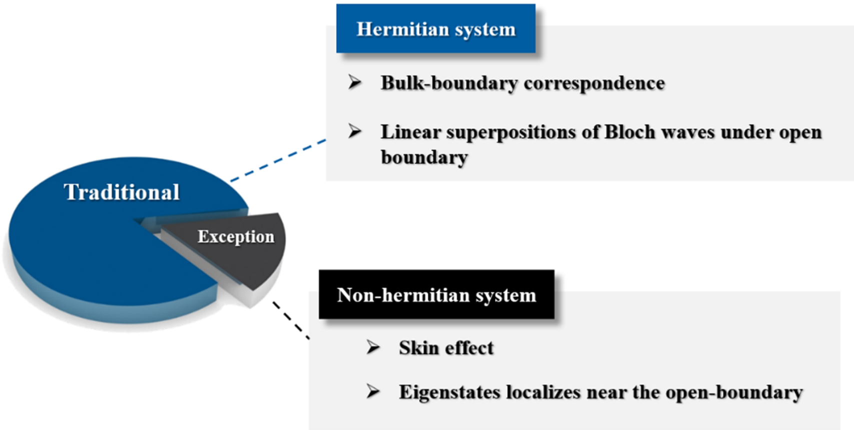

Figure 10. *Different bulk-boundary states of Hermitian and non-Hermitian systems.*

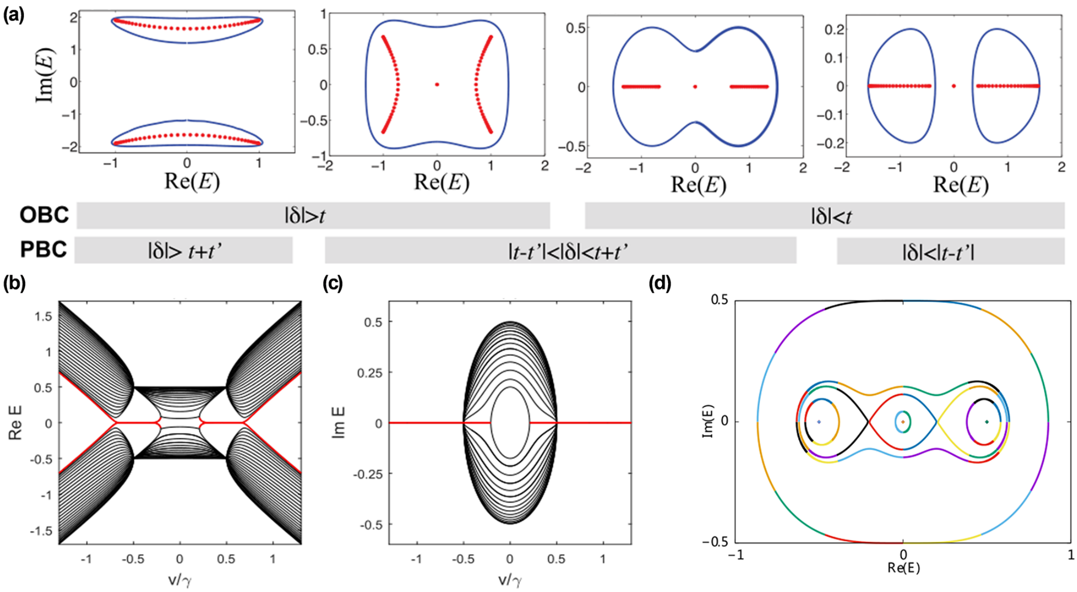

Figure 11. *(a) Eigenspectra in different boundary conditions and topological phases (Longhi, 2019). (b) Real and (c) imaginary parts of the spectrum for an open chain (Lee, 2016). (d) The traces of the eigenvalues (Xiong, 2018): the Hamiltonian encounters EPs and the alternation of eigenvalues splits into three unconnected loops.*

under the periodic boundary, whereas in non-Hermitian systems, eigenstates decay exponentially and localize near the boundary. This phenomenon in the non-Hermitian system is called the "non-Hermitian skin effect" (Liang and Huang, 2013; Lee, 2016; Leykam et al., 2017; Alvarez et al., 2018; Kunst et al., 2018; Yao et al., 2018; Lee et al., 2019; Lee and Thomale, 2019; Scheibner et al., 2020; Zhang et al., 2021; Mandal et al., 2022; Xiao and Chan, 2022).

Due to the disparity of boundary states in non-Hermitian systems, an increasing number of literature recognizes the crucial role of non-Bloch BBC in regulating non-Hermitian meta-structures. The breakdown of BBC has been demonstrated in a series of research (Lee, 2016; Alvarez et al., 2018; Xiong, 2018). Lee (2016) proposed that the non-Hermiticity breaks the usual BBC that can be modified in a non-Hermitian system. Figure 11(b) and (c) show the real and imaginary parts of the spectrum for an open chain. Here the zero-energy state originates from non-Hermiticity. Further, in order to observe

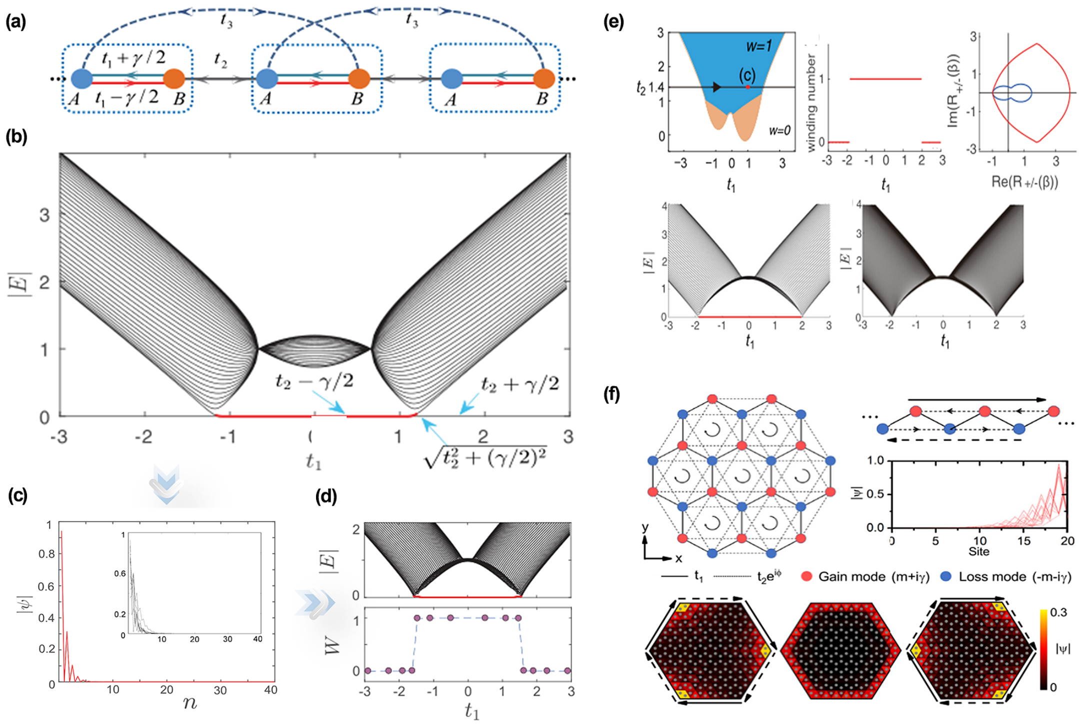

Figure 12. *(a) Schematic of non-Hermitian SSH model. (b) Numerical spectra E of an open chain model. (c) The eigenstates are localized near the boundary. (d) Spectrum of open chain corresponding to topological invariants (Shunyu and Wang, 2018). (e) Phase diagram and bulk-edge correspondence in a non-Hermitian SSH model (Yokomizo and Murakami, 2019). (f) Hybrid skin topological modes and non-Hermitian Haldane model (Li et al., 2022).*

special phenomena of boundary states, Xiong analytically solved the non-Hermitian Hamiltonian of the Hatano–Nelson model (Xiong, 2018). This model can obtain the OBC by eliminating the hopping strength between the two ends of the chain. In this tight-binding model, the topological invariant generally represents the Winding numbers, and the topological invariant is encoded in the traces of the eigenvalues. In Figure 11(d), as the length of the chain is increased and the hopping strength ***r*** is changed, the Hamiltonian will encounter EPs and change its topological traces. Thus, this physical characteristic makes the spectrum topology different for chains with and without boundaries.

Moreover, there is an urgent need to address the challenge that the "non-Hermitian skin effect" necessarily redefines topological invariants in a generalized Brillouin zone (Shen et al., 2018; Yao et al., 2018; Kawabata et al., 2020). One work starts with constructing and solving the non-Hermitian Su–Schrieffer–Heeger (SSH) model (Zeuner et al., 2015; Weimann et al., 2017; Lieu, 2018). As Figure 12(a) shows (Shunyu and Wang, 2018), the Bloch Hamiltonian of the model is

$$H(k) = d_x\sigma_x + \left(d_y + i\frac{\gamma}{2}\right)\sigma_y \tag{5}$$

Here, $d_x = t_1 + (t_2 + t_3)\cos k$, $d_y = (t_2 - t_3)\sin k$, hopping parameters given by different $t$and the $\sigma_{x,y}$ are Pauli matrices. Besides, based on the chiral symmetry of the SSH model, it can ensure the eigenvalues appear in $(E;-E)$ pairs:

$$E_\pm(k) = \pm\sqrt{d_x^2 + \left(d_y + \frac{i\gamma}{2}\right)^2} \tag{6}$$

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First, taking $t_3 = 0$ to simplify the analysis, as Figure 12(b) shows, the EPs close at $(d_x, d_y) = (\pm\gamma/2,0)$ in PBCs, which means $t_1 = t_2 \pm \gamma/2\ (k = \pi)$ or $t_1 = -t_2 \pm \gamma/2\ (k = 0)$. However, in OBCs, the spectrum is noticeably different from that of the periodic boundary. Not only do the EPs close when $t_1 = \sqrt{t_2{}^2 + (\gamma/2)^2}$, but also the topology of H(k) cannot determine the corresponding boundary states. In order to gain familiarity with the Hermitian phenomenon in this model, a similarity transformation method is proposed when taking the equation as

$$\overline{H} = S^{-1}HS \tag{7}$$

As shown in Figure 12(c), a bulk eigenstate of Hermitian $\overline{H}$ is extended, and eigenstates of H are localized at the end of the chain. Since this method is applicable only to $t_3 = 0$ case, a generalizable solution is proposed when $t_3 \neq 0$. Referring to the similarity transformation, the Bloch Hamiltonian is extended to the β complex plane that names the generalized Brillouin zone. Here, the Hamiltonian H becomes the function of β. In Figure 12(d), when β varies in a generalized Brillouin zone, $H(\beta)$ is able to describe the open boundary spectra and realize the BBC in non-Hermitian systems. To date, several conclusive studies of the skin effect and corresponding BBC in non-Hermitian meta-structures have been proposed. In Figure 12(e), a research (Yokomizo and Murakami, 2019) analyses and reveals the characteristics of a 1D SSH model. Apart from this, in Figure 12(f), the gain-/loss-induced hybrid skin topological effect and the PT phase transition in skin topological modes are shown (Li et al., 2022).

Skin effect of non-Hermitian system with elastic-wave modulation

The above research is the fundamental theoretical physics of non-Hermitian skin effects. More spectacularly, an increasing number of researchers find that the investigations into skin effects have wider relevance for elastic-wave control in meta-structures. Thus, several works (Scheibner et al., 2020; Gao and Wang, 2022; Zhou et al., 2023) have started to explore a wealth of strategies for the coupling between the skin effect and meta-structures and suggest potential possibilities for specific applications.

Rosa and Ruzzene proposed non-Hermitian elastic meta-lattice that are characterized by non-local feedback interactions (Rosa and Ruzzene, 2020). They aimed to describe the relative skin effect through the analysis of a 1D meta-lattice. The meta-lattice includes equal masses $\boldsymbol{m}$ and is connected by springs of equal stiffness $\boldsymbol{k}$. As Figure 13(a) shows, this model is derived by an external force and that interaction is proportional to the elongation of a spring at location $\boldsymbol{n}$. This force can be expressed as

$$f_n = k_c\left(u_{n-a} - u_{n-(a+1)}\right) \tag{8}$$

Here k_c denotes the proportional control gain and u_n means the displacement of mass n along the x axis. For a lattice of N masses, the equations of motion can be written as follows:

$$\boldsymbol{M\ddot{u}} + \boldsymbol{ku} = 0 \tag{9}$$

where $\boldsymbol{u} = [u_1, u_2, \ldots, u_n]^T$, and $\boldsymbol{M}$ and $\boldsymbol{K}$ denote the mass and stiffness matrices, respectively. Then, a Bloch-wave solution is proposed by $Ue^{i(\omega t - \mu n)}$, where ω and μ, respectively, represent angular frequency and non-dimensional wavenumber, to substitute the equation that can yield the dispersion relation:

$$\Omega^2 = 2(1 - \cos\mu) - \gamma_c\left(1 - e^{i\mu}\right)e^{i\mu a} \tag{10}$$

Here, $\Omega = \omega/\omega_0, \omega_0 = \sqrt{k/m}$ and $\gamma_c = k_c/k$. In addition, it is necessary to clarify exactly what is meant by the dispersion feature in the imaginary component. Actually, its positive wavenumbers are related to the loss (pink areas in Figure 13(b)), while the negative wavenumbers correspond to gain (green areas

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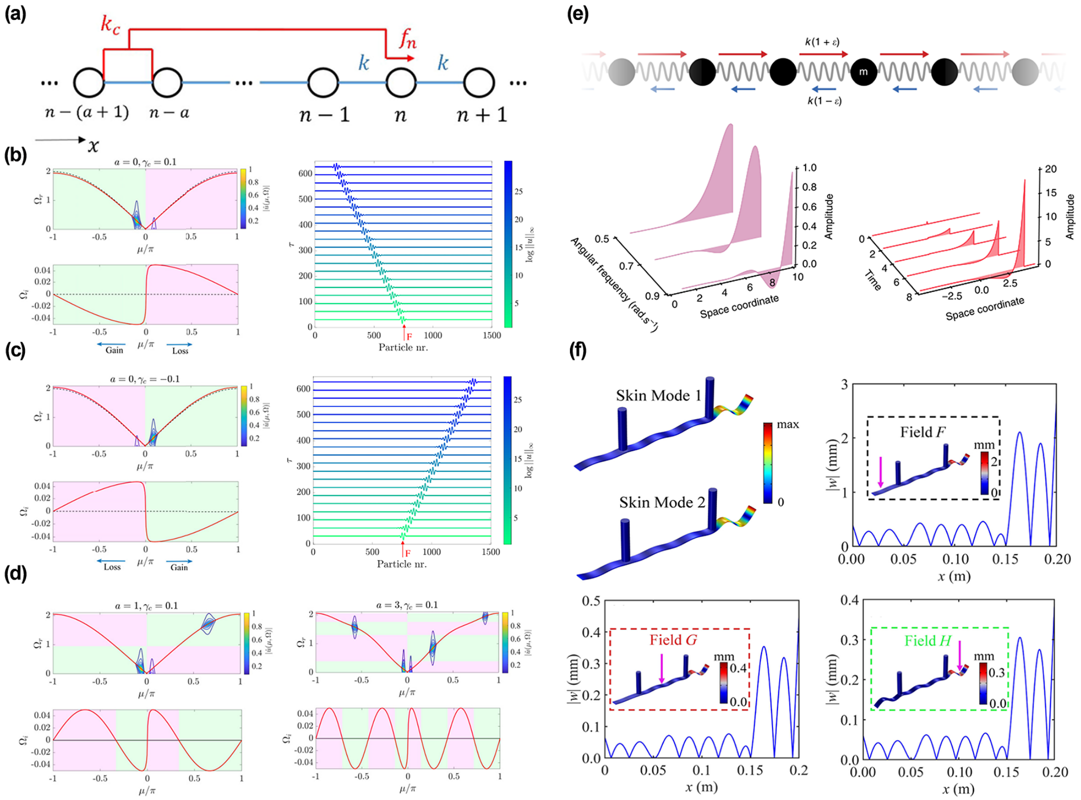

Figure 13. *(a) One-dimensional lattice with feedback control interactions. (b)–(d) Non-reciprocal amplification and attenuation of waves in lattices with local feedback interactions (a = 0/1/3) (Rosa and Ruzzene, 2020). (e) Analysing unidirectional wave amplification in a non-reciprocal mass-and-spring model (Brandenbourger et al., 2019). (f) The skin mode with non-reciprocity feedback and the realization of skin modes in different excitation locations (Cai et al., 2022).*

in Figure 13(b)). First, this work discusses the characteristics of a local control case ($a = 0$). Based on the non-reciprocity associated, the lattice with $\gamma_c = 0.1$ amplifies the left waves and attenuates the right waves (Figure 13(b)). In Figure 13(c), an opposite phenomenon can be observed with $\gamma_c = -0.1$. In Figure 13(d), when $a > 0$, the transmission characteristics of the lattice are defined by non-local interactions. There will be two distinct regions of gain or loss for each wave propagation direction. In general, the number of non-reciprocal bands increases along with the higher value of a.

In view of the above research, non-reciprocity is achieved by combining the broken spatial symmetries and nonlinearities (Sounas and Alu, 2017; Wang et al., 2018; Lee et al., 2019). Meanwhile, breaking reciprocity inspires a wealth of nontrivial physics and significant applications. Thus, the obvious idea arises that non-reciprocal feedback plays a crucial role in the non-Hermitian skin effect. As Figure 13(e) shows, Brandenbourger et al. (2019) designed a non-reciprocal robotic metamaterial. In non-reciprocal conditions, the solution shows exponentially localized standing waves or localized oscillatory standing waves with an exponential envelope. By virtue of non-reciprocal effects, the initial pulse is amplified along one propagation direction but attenuated along the opposite propagation direction. These phenomena realize the mechanical analogue of the non-Hermitian skin effect. Such a non-reciprocal case shows extreme asymmetry of angular displacement and asymmetric modes. In addition, a non-Hermitian metabeam proposed by Jin et al. is presented in Figure 13(f). This metabeam is enforced by a pair of external forces (Cai et al., 2022). In this metabeam, one interesting phenomenon

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is the skin mode of elastic waves. Through imposing external forces (whose values are related to the velocities of the neighbouring pillar) on both sides of the metabeam, non-reciprocity feedback can be obtained. To be specific, the metabeam only allows the elastic waves to propagate in a single direction, and the responses are localized at one end. More notably, as Figure 13(f) shows, this skin mode, based on the non-reciprocity feedback, is immune to the excitation location.

In the context of the skin mode for the elastic-wave device reviewed here, the exploitation of the skin effect in elastic waves has received unprecedented interest. The above pioneering strategies provide promising avenues to achieve diverse applications such as broadband energy harvesting, communication, and sensing with higher sensitivity. Hence, we can envision that further developments of skin modes in elastic meta-structures may generate more extraordinary wave manipulation fashions and offer appealing potential applications.

Summary and outlook

Beyond the realm of Hermitian physics, non-Hermiticity has recently enabled, a surge of nontrivial phenomena and captured remarkable attention in both quantum and classical systems. Excitingly, taking into account non-Hermiticity in meta-structures has not only improved the wave-control accuracy but also provided new paradigms for extraordinary and intriguing wave manipulations. In this perspective, starting from electromagnetics and acoustics, we mainly focused on the up-to-date progress of non-Hermitian elastic meta-structures with a review of their extraordinary elastic-wave manipulations. The corresponding studies also highlighted this crucial component of non-Hermitian modulation, such as the phase break of PT-symmetric (Fang et al., 2021a; Cao et al., 2022), non-reciprocal feedback (Lau and Clerk, 2018; Clerk, 2022; Gao and Wang, 2022), etc. Furthermore, non-Hermitian elastic meta-structures offer an ideal platform for the exploration of innovative wave phenomena within the domains of disorder, nonlinearity, and active control.

Up to now, the presence of non-Hermitian effects (loss/gain) has significantly impacted the mechanical behaviour of elastic meta-structures and their wave manipulation abilities. However, current research on gain design is relatively sparse. There is a lack of a systematic modulation scheme for structural design to provide gain factors, and experimental feasibility verification of non-Hermitian meta-structures is also deficient. On the other hand, in non-Hermitian metasurfaces, higher-order diffraction waves are particularly sensitive to the effects of gain/loss. Existing literature on controlling high-order diffraction waves under non-Hermitian conditions is relatively insufficient in terms of wave theory, analytical methods, and experimental approaches. In response to the aforementioned issues and challenges, we believe that embracing active methods holds the potential to broaden the scope of design and applications for non-Hermitian meta-structures. In particular, the quantitative introduction of gain/loss through piezoelectric and intelligent control to achieve precise manipulation of phase and amplitude is anticipated to offer new experimental avenues for non-Hermitian metasurfaces. This method may provide fresh design concepts for more efficient elastic guided wave control, thereby expanding the phenomenon of non-Hermitian elastic guided wave control.

Here, as a further step, based on the previous literature, we surmise the outlook for non-Hermitian elastic waves. While non-Hermitian meta-structures have been explored in the context of elastic waves, achieving accurate modulation of loss and flexible wave control is rather challenging. Considering future works in the exploration of non-Hermitian meta-structures, we expect that the coupling between non-Hermitian and elastic waves can be more beneficial in the case of transmission, reflection and absorption applications. As Figure 14 shows, from the perspective of physics, the exploration of PT-symmetric, wave amplification, non-reciprocity, nonlinearity, and skin effects may become research hotspots in non-Hermitian fields. In addition, the practice of these phenomena will enrich the applications of non-Hermitian meta-structures, such as single-mode oscillation, asymmetric reflection, unidirectional amplification, broadband non-reciprocity, signal power amplification, and transient skin effect. Furthermore, hybrid couplings between phase-broken PT-symmetric structures, non-local feedback mechanisms, and the utilization of high-order diffraction hold significant promise

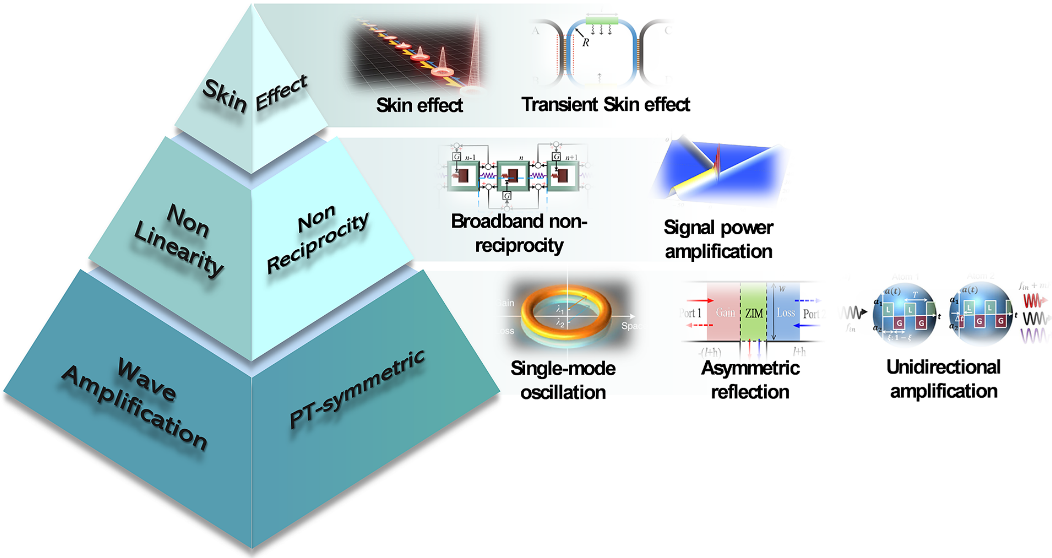

Figure 14. *Vision for non-Hermitian system in elastic waves (Fu and Xu, 2017; Nunes et al., 2017; Lau and Clerk, 2018; Zhang et al., 2020; Gao and Wang, 2022; Gu et al., 2022; Liu et al., 2022; Zhang et al., 2023).*

for enabling a wide array of efficient wave-manipulation applications in non-Hermitian meta-structures. In conclusion, further exploration in these fields holds the potential to broaden the horizons of non-Hermitian meta-structures, offering valuable strategies for advancing research in the realm of non-Hermitian investigations.

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