Macroeconomic Dynamics (2025), **29**, e105, pp. 1–24
doi:10.1017/S1365100525000252

ARTICLE

Mitigation or adaptation to climate change? The role of fiscal policy

Mouez Fodha[1,2] and Hiroaki Yamagami[3]

[1]Paris School of Economics, University Paris 1 Panthéon-Sorbonne, Paris, France
[2]Aix Marseille Univ, CNRS, AMSE, Marseille, France
[3]Faculty of Economics, Seikei University, Musashino-shi, Tokyo, Japan
Corresponding author: Hiroaki Yamagami; Email: yamagami@econ.seikei.ac.jp

Abstract

This article examines the interplay between fiscal policy and investments in climate change mitigation and adaptation. Adaptation is funded by public revenues from taxation and public bonds, whereas households can invest in mitigation and receive subsidies. We show that adaptation and mitigation are substitutes or complements, depending on the level of economic development and fiscal policy decisions. If the capital stock is initially low, adaptation and mitigation are complements (resp. substitutes) if the mitigation subsidy is low (resp. high). When the government is in debt, we show that increasing public spending to finance adaptation and/or mitigation could be beneficial if the capital stock is high enough but could be detrimental for countries with low capital stock. Thus, we add a new argument to the debate on the optimal mix between adaptation and mitigation, namely fiscal policy and the funding schemes of these investments. Finally, we propose extensions that consider a level of adaptation proportional to pollution flow, debt financing of public investment, and public mitigation investment alongside private adaptation investment.

Keywords: Environmental taxation; mitigation; adaptation; public debt; overlapping generations

JEL classifications: H23; H63; Q56

1. Introduction

Climate change poses major challenges that climate policy has not yet been able to address. In addition, technological solutions have not yet reached the scale needed to be sufficiently effective and to reduce polluting emissions to the extent required. Sustained and coordinated efforts by all stakeholders such as public and private sectors are needed to significantly reverse the trend.

This study develops a theoretical dynamic general equilibrium model to explore the interplay between public finance, public investment in adaptation infrastructure, and public support for private mitigation. The environmental tax will raise public revenues, but public adaptation and subsidies for private mitigation expenditure require revenue. We are interested in the issue of the efficient mix between adaptation and mitigation, which has been extensively analyzed in the literature, but also in the role of fiscal policy in this mix. Pollution taxation encourages the reduction of polluting emissions and provides public revenue. This revenue can be used to finance both mitigation and adaptation. In addition, while adaptation can have undesirable effects on the environment and increase pollution-related tax revenues, mitigation reduces pollution and tax revenues (cf. Enríquez-de-Salamanca et al. 2017). The fiscal dimension has been neglected in economic literature, with the exception of Barrage (2020), Catalano et al. (2020a), and Habla and Roeder (2017). In the latter, the mitigation policy consists of taxation and adaptation is a public expenditure while Catalano et al. (2020a) focus only on adaptation financing. We add to this literature by considering a mitigation sector financed by private expenditure in abatement activities (which is different from the pollution tax assumed in Habla and Roeder 2017). By contrast, adaptation is a public policy, comprising public investment in infrastructure such as dams, bridges and roads. We assume investment in adaptation infrastructure as a public decision, because these investments require good knowledge of the long term.

To analyze the interactions between environmental taxation, adaptation, and mitigation investments, this study considers an overlapping generations (OLG) model with an environmental externality. We extend the model developed by John and Pecchenino (1994) to consider public adaptation and private mitigation. We study the interactions between private decisions regarding pollution mitigation and government intervention in adaptation. We focus on an economy in which households are involved in environmental abatement through a trade-off between consumption and mitigation alongside public adaptation to protect them from the consequences of pollution. Pollution emissions occur through consumption, which harms the welfare of future generations. Public expenditure on adaptation investments is financed by taxation, and debt financing is also considered. Fiscal policy has a twofold effect on households' budget constraints through taxation for financing public adaptation and through subsidies for private mitigation. We show that adaptation and mitigation may be substitutes or complements depending on the level of economic development, but also on the level of mitigation subsidy. Finally, when the government is indebted, we show that financing public adaptation and/or subsidizing private mitigation through an increase in public debt could be beneficial for countries with a high capital stock, but is detrimental when its capital stock is too low.

Our study contributes to the literature in two ways. First, it is related to the literature on the intergenerational issues of environmental policies (e.g., Bovenberg and Heijdra 1998, 2002; Chiroleu-Assouline and Fodha 2006; Mariani et al. 2010; Karp and Rezai 2014). In these studies, intergenerational conflicts arise because of the distributional effects of environmental policies on the welfare of current and future generations. In Habla and Roeder (2017), climate policies have heterogeneous welfare consequences mainly through the impacts on the budget constraints of current and future generations. Fodha and Seegmuller (2012, 2014) and Fodha et al. (2018) analyze debt financing schemes of mitigation. They show that a poverty trap may exist, but efficient environmental tax reform may be designed under specific conditions on abatement technologies and initial level of capital stock. In these studies, public and private actions to mitigate pollution rely on the same technologies; they are thus supposed to be perfectly substitutable. In our study, we add adaptation policies and consider a vulnerability function that translates pollution into welfare losses. We show that public and private decisions may also be complements.

Second, our study adds to the theoretical literature on mitigation and adaptation, which focuses on the conditions under which adaptation and mitigation are substitutes or complements. Ayong and Pommeret (2017) consider a pollution threshold above which adaptation is no longer efficient. Bréchet et al. (2013) study optimal mitigation and adaptation investments at the macroeconomic level and show that the issue of substitutability between the two instruments depends on the stage of development. Ingham et al. (2013) develop a range of economic models to explore the relationship between mitigation and adaptation. They show that mitigation and adaptation are substitutes in almost all economic models of climate change. However, they also find that complementarity is possible when adaptation costs depend on the level of mitigation. Schumacher (2019) assumes that mitigation is a public good, whereas adaptation is a private good, and concludes that mitigation must be preferred to adaptation. Regarding the public finance issue, the articles take into account the taxation of pollution to finance either adaptation expenditure (Barrage, 2020; Bachner et al. 2019; Habla and Roeder 2017), or an abatement sector (Jondeau et al. 2022). Barrage (2020) shows that distortive taxation affects the efficiency of adaptation and increases the welfare costs of climate change. Conversely, Bachner et al. (2019) suggest that a well-designed adaptation policy has a positive effect on economic growth and welfare. Habla and Roeder (2017) argue that the

https://doi.org/10.1017/S1365100525000252 Published online by Cambridge University Press

redistributive characteristics of the tax structure plays an important role for the acceptability of the adaptation policies. Finally, Jondeau et al. (2022) conclude that public subsidies to the abatement sector, when financed by pollution taxes, is an efficient policy. In our article, private mitigation and public adaptation decisions are considered simultaneously. This is important because the two investments interact with each other, affecting pollution emissions and public budget. We show that the financing scheme is an important argument in the debate on the optimal mix of adaptation and mitigation.

The remainder of this paper is organized as follows. Section 2 describes the OLG model. Section 3 defines the intertemporal equilibria and examines the stability of the steady states. Section 4 presents the welfare analysis. Section 5 considers three extensions: (i) adaptation investment proportional to pollution flow, (ii) adaptation financed by public debt, and (iii) mitigation by government and adaptation by private agents. The final section concludes.

2. An OLG model

We consider an OLG model with three agents: individuals (young and old), firms, and a government. In a model à la John and Pecchenino (1994), we extend the results of Fodha and Seegmuller (2012) by considering public adaptation alongside private mitigation.

2.1. The environment

We suppose that a stock of pollution, E_{t+1}, is increased by consumption, c_t, but reduced by mitigation, m_t:

$$E_{t+1} = (1 - \delta_E)E_t + \epsilon c_t - \gamma m_t, \tag{1}$$

where $\delta_E \in (0, 1)$ measures a natural rate of pollution absorption. $\epsilon, \gamma > 0$ represent the rate of pollution from consumption and the efficiency of pollution abatement from mitigation, respectively.[1]

Pollution stock is a source of damage, resulting in climate change, frequent flooding, rising sea levels, and an increase in infectious diseases. However, damage can be alleviated through adaptation H_{t+1}, such as dams, sea- or river-side banks, and medical knowledge. This could be public infrastructure or human capital. The adaptation stock is increased by public investment h_t:

$$H_{t+1} = \bar{H} + (1 - \delta_H)H_t + h_t \tag{2}$$

where $\bar{H} > 0$ is a basic (natural) adaptation level that nature provides and $\delta_H \in (0, 1)$ denotes the rate of adaptation stock depreciation. Natural adaptation ($\bar{H}$) is a long-term exogenous adaptation capacity that can be interpreted as the natural capacity of an ecosystem to protect humans. The higher the $\bar{H}$, the more resistant the human being is to the hazards of nature, reflecting the low impact of pollution on well-being. $\bar{H}$ represents a natural long-term equilibrium (water, air, biodiversity, etc.) in the absence of any human activity.[2]

Adaptation and mitigation investments play distinct roles. On the one hand, adaptation h is an investment that increases a stock of public good, H, financed by the government. On the other hand, mitigation m is a privately provided good that decreases the pollution stock E. With regard to climate change, h is investment in infrastructure, education and R&D. m is investment in GHG emission abatement services, such as CCS or renewable energy, and in the insulation of buildings.

2.2. Households

We consider an OLG model with 2-period-lived agents. The population size of each generation is assumed constant and normalized to unity. An individual born at time t inelastically supplies one unit of labor when young, shares her wage between savings and investment in mitigation,

m_t, and consumes when old, c_{t+1}. In addition, she suffers from a stock of pollution when old, E_{t+1}, through climate change, sea level rise, or infectious diseases. However, vulnerability to environmental degradation is alleviated by a stock of adaptation, H_{t+1}.

We define a sub-utility function, $q_{t+1} = Q(E_{t+1}, H_{t+1})$, which measures the quality of life related to the pollution stock. This quality is assumed to decrease with the pollution stock but increases with the adaptation stock. Together with consumption, we suppose that the utility of an individual born at time t is

$$u_t = c_{t+1}^{\beta} q_{t+1}^{1-\beta}, \tag{3}$$

$$q_{t+1} = E_{t+1}^{-\phi} H_{t+1}^{\mu}, \tag{4}$$

where $\beta \in (0, 1)$ is a preference parameter for consumption, $\phi > 0$ measures the sensitivity of quality of life to the pollution stock, and $\mu > 0$ is an efficiency parameter of the adaptation stock. ϕ can be interpreted as a parameter that determines the willingness to pay to reduce pollution or as a factor determining the share of mitigation expenditure in household income. μ can be interpreted as a vulnerability parameter, and H^{μ} represents the environmental resilience or the efficiency of adaptation to protect people from adverse climate change effects.

An individual born at time t earns disposable income when young as the difference between wage w_t, and lump-sum tax T_t. She allocates this to mitigation m_t and savings s_t. When old, the agent consumes (c_{t+1}) her remunerated savings, with R_{t+1} the real interest rate. The mitigation subsidy is v, and the rate of consumption tax is τ_c. Thus, the individual maximizes (3) subject to (4) and (1) and the following budget constraints:

$$(1 - v)m_t + s_t = w_t - T_t, \tag{5}$$

$$(1 + \tau^c)c_{t+1} = R_{t+1}s_t. \tag{6}$$

Households have to pay a tax τ^c on polluting consumption (i.e., Pigovian tax). Their life-cycle income is also affected by a transfer T_t (positive or negative) that balances the government budget. Households bear double taxation over their lifecycles (T and τ_c). These two instruments are complementary, and allow us to define a complete system of instruments to decentralize the social optimum. The lump-sum tax has an impact on the level of life-cycle net income (scale effect), while the tax on consumption influences the pollution externalities from consumption.

Combining (5) and (6) leads to the intertemporal budget constraint.

$$(1 - v)m_t + \frac{1 + \tau^c}{R_{t+1}} c_{t+1} = w_t - T_t.$$

H_{t+1} in (2) is given for agents. We derive the demand for mitigation when young, and consumption when old.

$$m_t = \frac{w_t - T_t}{1 - v} - \frac{\beta}{1 - \beta} \frac{E_{t+1}}{\gamma\phi}, \tag{7}$$

$$c_{t+1} = \frac{R_{t+1}}{1 + \tau^c} \frac{\beta}{1 - \beta} \frac{1 - v}{\gamma\phi} E_{t+1}. \tag{8}$$

The consequences of mitigation on the pollution stock are twofold. First, mitigation m_t directly decreases the pollution stock E_{t+1}. Second, higher investment in mitigation implies a decrease in individual savings and lower consumption in period $t + 1$. As adaptation is given, the vulnerability indicator does not influence agents' trade-offs. Public adaptation has no direct consequence for the mitigation private decision.

2.3. Firms

The firm produces an aggregate output using a Cobb–Douglas technology with capital stock K_t and labor L_t: $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$ with $A > 0$ and $\alpha \in (0, 1)$. Labor input equals one. We rewrite the production function as per capita $y_t = Ak_t$, where y_t and k_t are the output and capital stock per young capita, respectively. The tax rate on production is τ^y. This will affect the dynamics of capital accumulation.

The first-order conditions of profit maximization are given by

$$R_t = (1 - \tau^y)\alpha A k_t^{\alpha-1}, \tag{9}$$

$$w_t = (1 - \tau^y)(1 - \alpha)A k_t^{\alpha} \tag{10}$$

We assume a full depreciation of capital in one period.

2.4. Government

The government's budget is balanced at each period:

$$T_t = \nu m_t + h_t - \tau^c c_t - \tau^y y_t. \tag{11}$$

We first assume that investment in adaptation is constant over time, $h_t = \bar{h}, \forall t$. This investment is an environmental policy instrument. We therefore assume that adaptation is a choice made by independent local authorities and taken as given by central authorities in charge of taxation. We will relax this assumption later.

The distributive effects of adaptation and mitigation differ. The costs of environmental policies at time t (investment in adaptation and mitigation subsidies) are borne by all agents living at time t including old agents and firms, whereas the benefits will take place at time $t + 1$. Mitigation is privately financed by the young at time t for their own benefit when old at time $t + 1$.

3. Intertemporal equilibrium

Capital stock equals total savings: $k_{t+1} = s_t$. By substituting (6) and (8), we obtain

$$k_{t+1} = \frac{\beta}{1-\beta} \frac{1-\nu}{\gamma\phi} E_{t+1}. \tag{12}$$

With (1), the dynamics of capital stock in (12) is rearranged as

$$k_{t+1} = (1 - \delta_E)k_t + \frac{\beta}{1-\beta} \frac{1-\nu}{\gamma\phi}(\epsilon c_t - \gamma m_t). \tag{13}$$

We also rewrite m_t in (7) with (10) – (12) and c_t in (8) with (9) and (12):

$$m_t = \left\{(1 - \tau^y)(1 - \alpha) + \tau^y\right\} A k_t^{\alpha} - \bar{h} + \tau^c c_t - k_{t+1}. \tag{14}$$

$$c_t = \frac{1 - \tau^y}{1 + \tau^c} \alpha A k_t^{\alpha}. \tag{15}$$

Substituting (14) and (15) into (13) yields the following dynamics of capital stock:

$$\begin{aligned} k_{t+1} &= \Gamma(k_t) \\ &= \frac{\left\{1 - \alpha \frac{1-\tau^y}{1+\tau^c}(1 + \epsilon/\gamma)\right\}(1 - \nu)\beta A k_t^{\alpha} - \phi(1 - \beta)(1 - \delta_E)k_t - \beta(1 - \nu)\bar{h}}{\beta(1 - \nu) - (1 - \beta)\phi}. \end{aligned} \tag{16}$$

https://doi.org/10.1017/S1365100525000252 Published online by Cambridge University Press

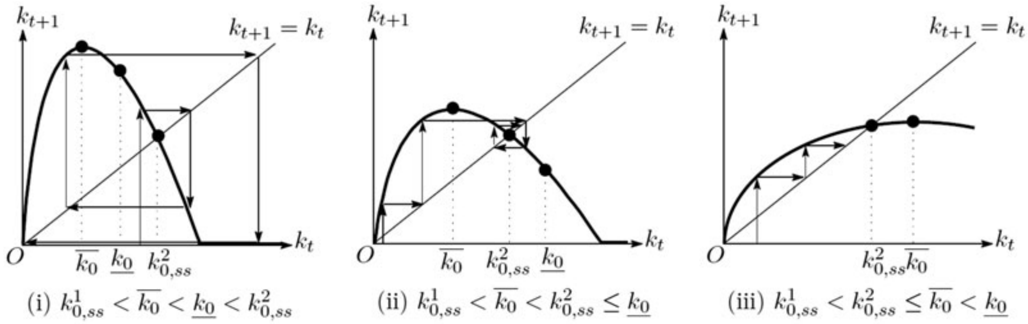

Figure 1. Capital stock dynamics in the laissez-faire economy.

3.1. Laissez-faire equilibrium

We first consider the laissez-faire case ($\tau^c=\tau^y=\nu=\bar{h}=0$). Dynamics defined in eq. (16) reduces to

$$k_{t+1}\equiv\Gamma_0(k_t)=\frac{\{1-\alpha(1+\epsilon/\gamma)\}\beta Ak_t^{\alpha}-\phi(1-\beta)(1-\delta_E)k_t}{\beta-(1-\beta)\phi}. \tag{17}$$

We make the following assumptions to obtain the nontrivial steady state in the laissez-faire economy:

Assumption 1. *(a)* $\frac{\beta}{1-\beta}>\phi$ *and (b)* $\frac{\gamma}{\epsilon}>\frac{\alpha}{1-\alpha}$.

Assumption 1 (a) compares the sensitivity of the quality of life to pollution (ϕ) to the sensitivity of utility to consumption (β). Assumption 1 (b) is standard and not too restrictive. This implies that $\gamma(1-\alpha)Ak^{\alpha}>\epsilon\alpha Ak^{\alpha}$. In the absence of taxes, we have $c=Rs=Rk=\alpha Ak^{\alpha}$ and $w=(1-\alpha)Ak^{\alpha}$. Thus, the assumption is rewritten as $\gamma(1-\alpha)w>\epsilon\alpha c$, meaning that if an agent uses all her wages for mitigation ($m=w$), it efficiently fights the flow of pollution from consumption, and then ensures that the flow of net pollution is negative.

Consequently, we have a positive denominator in (17) from assumption 1 (a). Additionally, $\Gamma_0(0)=0$, $\lim_{k_t\to 0}\Gamma_0'(k_t)=+\infty$, and $\lim_{k_t\to+\infty}\Gamma_0'(k_t)<0$.[3] Therefore, we obtain an inverted U-shaped curve in the dynamics of k with an intercept at $k_{t+1}=0$. Then, two steady states exist, such that $k=\Gamma_0(k)$. Letting $k^1_{0,ss}$ and $k^2_{0,ss}$ be the capital stock levels at steady state, we have

$$k^1_{0,ss}=0;\ \text{and}\ k^2_{0,ss}=\left[\frac{\{1-\alpha(1+\epsilon/\gamma)\}\beta A}{\beta-(1-\beta)\phi\delta_E}\right]^{1/(1-\alpha)}.$$

To examine the stability of each steady state, we define $\overline{k_0}$ as the capital stock that satisfies $\Gamma_0'(\overline{k_0})=0$ and $\underline{k_0}$ as the stock satisfying $\Gamma_0'(\underline{k_0})=-1$. These are rewritten as

$$\Gamma_0'(\overline{k_0})=0\quad\Leftrightarrow\overline{k_0}\equiv\left[\frac{\{1-\alpha(1+\epsilon/\gamma)\}\alpha\beta A}{(1-\beta)(1-\delta_E)\phi}\right]^{1/(1-\alpha)},$$

$$\Gamma_0'(\underline{k_0})=-1\quad\Leftrightarrow\underline{k_0}\equiv\left[\frac{\{1-\alpha(1+\epsilon/\gamma)\}\alpha\beta A}{(1-\beta)(2-\delta_E)\phi-\beta}\right]^{1/(1-\alpha)}.$$

From Assumption 1, we obtain $k^1_{0,ss}<\overline{k_0}<\underline{k_0}$. Then, we present three cases of dynamics: (i) $k^1_{0,ss}<\overline{k_0}<\underline{k_0}\leq k^2_{0,ss}$; (ii) $k^1_{0,ss}<\overline{k_0}<k^2_{0,ss}<\underline{k_0}$; and (iii) $k^1_{0,ss}<k^2_{0,ss}\leq\overline{k_0}<\underline{k_0}$, which are depicted in Figure 1. As shown in Figure 1, the positive steady state $k^2_{0,ss}$ is unstable in case (i) and stable in cases (ii) and (iii).

https://doi.org/10.1017/S1365100525000252 Published online by Cambridge University Press

We add the following assumption to ensure the existence of a nontrivial stable steady state in the laissez-faire cases, such as in cases (ii) and (iii). This is achieved by assuming $k^2_{0,ss} < \underline{k}_0$, which is rearranged as follows:

Assumption 2. $1 > \beta > \frac{\phi\{2-\delta_E(1-\alpha)\}}{(1+\alpha)+\phi\{2-\delta_E(1-\alpha)\}} > 0$.

This assumption also implies that utility is sufficiently sensitive to consumption.

3.2. Steady states with environmental policies

While holding Assumptions 1 and 2, we study the dynamics of k with policies, as in (13). We define k_{ss} as the capital stock per young person if the steady state is unique, whereas k^i_{ss} with $i = \{1, 2, \cdots, n\}$ are those with $k^i_{ss} < k^j_{ss}$ for $i < j$ if there are multiple steady states.

Let us examine the steady-state capital stock. For this purpose, we define $\bar{k}$, $\underline{k}$, and $\hat{k}$ that satisfy $\Gamma'(\bar{k}) = 0$, $\Gamma'(\underline{k}) = -1$, $\Gamma'(\hat{k}) = 1$, respectively.

$$\Gamma'(\bar{k}) = 0 \quad \Leftrightarrow \bar{k} \equiv \left[\frac{(1-\nu)\left\{1-\alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma)\right\}\alpha\beta A}{\phi(1-\beta)(1-\delta_E)}\right]^{1/(1-\alpha)}, \tag{18}$$

$$\Gamma'(\underline{k}) = -1 \Leftrightarrow \underline{k} \equiv \left[\frac{(1-\nu)\left\{1-\alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma)\right\}\alpha\beta A}{\phi(1-\beta)(1-\delta_E)+\{(1-\beta)\phi-(1-\nu)\beta\}}\right]^{1/(1-\alpha)}, \tag{19}$$

$$\Gamma'(\hat{k}) = 1 \quad \Leftrightarrow \hat{k} \equiv \left[\frac{(1-\nu)\left\{1-\alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma)\right\}\alpha\beta A}{\phi(1-\beta)(1-\delta_E)-\{(1-\beta)\phi-(1-\nu)\beta\}}\right]^{1/(1-\alpha)} \tag{20}$$

We define two values for the mitigation subsidy, $\underline{\nu}$ and $\bar{\nu}$, and summarize the existence and stability of the steady states as follows:

$$\underline{\nu} \equiv 1 - \frac{1-\beta}{\beta}\phi \ ; \qquad \bar{\nu} \equiv 1 - \frac{1-\beta}{\beta}\phi\delta_E. \tag{21}$$

Note that $\bar{\nu} > \underline{\nu}$ because $\delta_E \in (0, 1)$.

Additionally, from (16), we assume the following to exclude extreme cases:

Assumption 3. $(1-\beta)\phi - (1-\nu)\beta \neq 0 \Leftrightarrow \nu \neq \underline{\nu}$.

Assuming that the subsidy rate is sufficiently high or low, we can characterize the steady-state equilibrium conditions. This highlights the importance of environmental policy tools. The level of mitigation subsidy determines the properties of the steady-state equilibrium and will also help determine the consequences of environmental tax and public finance reforms.

PROPOSITION 1. *Under Assumptions 1, 2, and 3, the steady-state equilibria are characterized by the following nine cases.*

(i) There is a unique steady state, $k_{ss} > 0$, if $\Gamma(\bar{k}) \leq \bar{k}$ and $\nu \geq \bar{\nu}$. Then, the steady state is stable if $k_{ss} \in (\underline{k}, \bar{k})$ and unstable if $k_{ss} \leq \underline{k}$.

(ii) There exist two steady states, k^1_{ss} and k^2_{ss}, such that $0 < k^1_{ss} < \bar{k} < k^2_{ss}$ if $\Gamma(\bar{k}) \leq \bar{k}$ and $\underline{\nu} < \nu \leq \bar{\nu}$. Then, k^2_{ss} is unstable. k^1_{ss} is stable if $k^1_{ss} \in (\underline{k}, \bar{k})$ and unstable if $k^1_{ss} \leq \underline{k}$.

(iii) *There is a unique steady state, k_{ss}, which is strictly greater than $\bar{k}$, if $\Gamma(\bar{k}) > \bar{k}$ and $\nu \geq \bar{\nu}$. Then, the steady state is stable.*

(iv) *There exist two steady states, k_{ss}^1 and k_{ss}^2, such that $0 < k_{ss}^1 < \bar{k} < k_{ss}^2$ if $\Gamma(\bar{k}) > \bar{k}$, $\underline{\nu} < \nu < \bar{\nu}$, and $\Gamma(\hat{k}) < \hat{k}$. Then, k_{ss}^1 is stable and k_{ss}^2 is unstable.*

(v) *There is a unique steady state, $k_{ss} > 0$, if $\Gamma(\bar{k}) > \bar{k}$, $\underline{\nu} < \nu < \bar{\nu}$, and $\Gamma(\hat{k}) = \hat{k}$. Then, the steady state, k_{ss}, is unstable.*

(vi) *There is no steady state if $\Gamma(\bar{k}) > \bar{k}$, $\underline{\nu} < \nu < \bar{\nu}$, and $\Gamma(\hat{k}) > \hat{k}$.*

(vii) *There is a unique steady state, k_{ss}, if $\Gamma(\hat{k}) < \hat{k}$ and $\nu < \underline{\nu}$. Then, the steady state, k_{ss}, is zero and stable.*

(viii) *There exist two steady states, k_{ss}^1 and k_{ss}^2 such that $k_{ss}^1 < k_{ss}^2$, if $\Gamma(\hat{k}) = \hat{k}$ and $\nu < \underline{\nu}$. Then, k_{ss}^1 is zero and stable. k_{ss}^2 is unstable.*

(ix) *There are three steady states, k_{ss}^1, k_{ss}^2, and k_{ss}^3 with $k_{ss}^1 < k_{ss}^2 < k_{ss}^3$, if $\Gamma(\hat{k}) > \hat{k}$ and $\nu < \underline{\nu}$. Then, k_{ss}^1 is zero and stable, k_{ss}^2 is unstable, and k_{ss}^3 is stable if $k_{ss}^3 < \underline{k}$ and unstable if $k_{ss}^3 \geq \underline{k}$.*

Proof. *See Appendix A.* □

In the following section, we focus on the stable steady state.

3.3. Comparative statics

We evaluate the capital stock dynamics shown in (16) at the steady state by setting $k_{t+1} = k_t = k_{ss}$. We then define the implicit function I as

$$I \equiv \left\{1 - \alpha \frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma)\right\} A k_{ss}^{\alpha} + \left\{\frac{(1-\beta)\phi\delta_E}{(1-\nu)\beta} - 1\right\} k_{ss} - \bar{h} = 0. \tag{22}$$

The implicit function theorem shows that

$$\frac{dk_{ss}}{d\bar{h}} = -\frac{\partial I/\partial \bar{h}}{\partial I/\partial k_{ss}} = \left[\left\{1 - \alpha \frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma)\right\} \alpha A k_{ss}^{\alpha-1} + \left\{\frac{(1-\beta)\phi\delta_E}{(1-\nu)\beta} - 1\right\}\right]^{-1}. \tag{23}$$

From (23), $dk_{ss}/d\bar{h}$ is positive if

$$k_{ss} < \left\{(1-\nu)\alpha\beta A \cdot \frac{1 - \alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma)}{(1-\nu)\beta - (1-\beta)\phi\delta_E}\right\}^{1/(1-\alpha)} \equiv k^m. \tag{24}$$

The numerator of k^m is positive according to Assumption 1(b). However, the denominator sign remains ambiguous. If $\nu \geq \bar{\nu}$, then the set of capital stocks satisfying (24) is empty. That is, $dk_{ss}/d\bar{h} < 0$ if $\nu \geq \bar{\nu}$. There is a range $[0, k^m)$ that is not empty if $\nu < \bar{\nu}$. Then, we have $dk_{ss}/d\bar{h} > 0$ for $k_{ss} \in [0, k^m)$, whereas $dk_{ss}/d\bar{h} \leq 0$ for $k_{ss} \geq k^m$. The discussion thus far is summarized in Proposition 2.

PROPOSITION 2. *Suppose Assumptions 1, 2, and 3 are satisfied. Then, the capital stock in the steady-state equilibrium increases with public investment in adaptation if the steady-state capital stock is ex-ante sufficiently small, such that $k_{ss} < k^m$ and the mitigation subsidy is sufficiently low, such that $\nu < \bar{\nu}$. Conversely, capital stock decreases if $\nu \geq \bar{\nu}$ or $k_{ss} \geq k^m$ and $\nu < \bar{\nu}$.*

k_{ss} increases because the rise in public adaptation investment conditionally crowds out the private provision of mitigation services m. Then, it allows for more savings when young, more consumption when old, and thereby more pollution, but with a lower effect on agent's welfare since adaptation has also increased. Thus, when ν and k_{ss} are weak, more adaptation is beneficial.

We also compute the effects of variations in the environmental regeneration rate. The effect is not so obvious *a priori* because if pollution absorption increases, the environment improves, and agents will lower their abatement and increase their consumption by saving more when young. However, these effects remain ambiguous.

$$\frac{dk_{ss}}{d\delta_E} = -\frac{\partial I/\partial \delta_E}{\partial I/\partial k_{ss}} = -\frac{(1-\beta)\phi k_{ss}}{(1-\nu)\beta} \cdot \left(\frac{\partial I}{\partial k_{ss}}\right)^{-1}$$

From (22) and (23), we have $\text{sign}(dk_{ss}/d\delta_E) = \text{sign}(-dk_{ss}/d\bar{h})$. Alternatively, we have $dlk_{ss}/d\delta_E > 0$ if $k_{ss} > k^m$. Similarly, the effects of the *mitigation rate* (abatement rate to pollution rate, ϵ/γ) can be derived as

$$\frac{dk_{ss}}{d(\epsilon/\gamma)} = -\frac{\partial I/\partial(\epsilon/\gamma)}{\partial I/\partial k_{ss}} = \frac{1-\tau^{\nu}}{1+\tau^c}\alpha A k_{ss}^{\alpha} \cdot \left(\frac{\partial I}{\partial k_{ss}}\right)^{-1}.$$

We have $dk_{ss}/d(\epsilon/\gamma) > 0$ if $k_{ss} < k^m$. If the ratio increases, then the conclusions are the same as those for $\bar{h}$. That is, (ϵ/γ) increases steady-state capital stock if k_{ss} is sufficiently low.

The effects of other policies on the steady-state capital stock level can also be derived.

$$\frac{dk_{ss}}{d\nu} = -\frac{1-\beta}{\beta}\frac{\phi\delta_E}{(1-\nu)^2}k_{ss} \cdot \left(\frac{\partial I}{\partial k_{ss}}\right)^{-1},$$

$$\frac{dk_{ss}}{d\tau^y} = -\frac{1+\epsilon/\gamma}{1+\tau^c}\alpha A k_{ss}^{\alpha} \cdot \left(\frac{\partial I}{\partial k_{ss}}\right)^{-1},$$

$$\frac{dk_{ss}}{d\tau^c} = -\frac{1-\tau^y}{(1+\tau^c)^2}(1+\epsilon/\gamma)\alpha A k_{ss}^{\alpha} \cdot \left(\frac{\partial I}{\partial k_{ss}}\right)^{-1}$$

These three derivatives are negative if $k_{ss} < k^m$. The results are summarized below.

COROLLARY 1. *Suppose Assumptions 1, 2, and 3 are satisfied. Then, the capital stock at steady-state equilibrium becomes higher when δ_E and ϵ/γ are higher if $k_{ss} < k^m$ and $\nu < \bar{\nu}$. Furthermore, the mitigation subsidy, output tax, and consumption tax decrease steady-state capital stock if $k_{ss} < k^m$ and $\nu < \bar{\nu}$.*

This result is interesting when we consider the heterogeneity between countries in terms of development and exposure to pollution. Indeed, the net effect of mitigation is all the more beneficial and significant the more developed the country (i.e., high per capita capital stock). Symmetrically, the lower the capital stock, the greater the risk that a low net mitigation efficiency rate will lead the country into decline. Indeed, the capital stock decreases in this ratio if the level of development is not sufficiently high.

From these results, we can assess the effect on consumption and mitigation at the steady state. Indeed, we have $c_{ss} = \frac{1-\tau^y}{1+\tau^c}\alpha A k_{ss}^{\alpha}$ from (15). This implies that c_{ss} evolves with k_{ss}. This also depends on the tax variables.

The same results apply for m_{ss} from (14), given as

$$m_{ss} = \left(1 - \alpha\frac{1-\tau^y}{1+\tau^c}\right)Ak_{ss}^{\alpha} - \bar{h} - k_{ss}. \tag{25}$$

We show that the effects of public adaptation $\bar{h}$ on private mitigation m_{ss} are ambiguous.

$$\frac{dm_{ss}}{d\bar{h}} = \underbrace{\left[\left(1-\alpha\frac{1-\tau^y}{1+\tau^c}\right)\alpha A k_{ss}^{\alpha-1} - 1\right]}_{\Delta m}\left(\frac{dk_{ss}}{d\bar{h}}\right) - 1. \tag{26}$$

The last term of (26), that is, -1, represents the resource-constraint effect of public adaptation. If the adaptation is increased, the income available to spend on mitigation reduces by the same

https://doi.org/10.1017/S1365100525000252 Published online by Cambridge University Press

proportion. By contrast, when adaptation increases the steady-state capital stock (i.e., the two terms in square brackets, Δm), it increases the current output, which leads to higher mitigation, whereas the increased savings lead to lower mitigation. Consequently, if the first term, $\Delta m \cdot \frac{dk_{ss}}{d\bar{h}}$, is greater than unity, mitigation increases with adaptation.

However, because the sign of Δm is ambiguous, we must examine it more closely. The term Δm becomes positive if the increase in output is greater than the increase in savings. Δm is positive if the capital stock in the steady state is sufficiently small such that

$$k_{ss} < \left\{ \alpha A \left(1 - \alpha \frac{1-\tau^y}{1+\tau^c} \right) \right\}^{1/(1-\alpha)} \equiv k^n. \tag{27}$$

k^n is positive for all ν and is greater than k^m if $\nu < 1 - \frac{1-\alpha\frac{1-\tau^y}{1+\tau^c}}{\alpha\frac{1-\tau^y}{1+\tau^c}\frac{\epsilon}{\gamma}} \frac{1-\beta}{\beta} \phi\delta_E \equiv \hat{\nu}$. We have $\hat{\nu} < \bar{\nu}$ from Assumption 1 (b).

As a result, mitigation and adaptation increase at the same time under certain conditions on the mitigation subsidy and capital stock level. Then, adaptation and mitigation are complements. The effect of $\bar{h}$ on k_{ss} is very important in the transmission channels. The greater the effect, the stronger the complementarity. Otherwise, they are considered substitutes. Proposition 3 summarizes this discussion.

PROPOSITION 3. *Suppose Assumptions 1, 2, and 3 are satisfied. Then, mitigation and adaptation are complements if the steady-state capital stock is sufficiently small such that* $k_{ss} < k^m$ *and* $k_{ss} < k^n$*, or large such that* $k_{ss} > k^m$ *and* $k_{ss} > k_n$*, so that* $\Delta m \frac{dk_{ss}}{d\bar{h}} > 1$.

Proof. *See Appendix B* □

Whether mitigation and adaptation are complementary or substitutionary depends, as already shown in the literature,[4] on the level of economic development (k), but also on the accompanying environmental policy, namely, pollution taxation and mitigation subsidies. This last result is in line with the results of the work of Habla and Roeder (2017).

4. Welfare analysis

We determine the optimal levels of investment for (i) mitigation of and (ii) adaptation to climate change. The objective is to define the optimal policies to be implemented.

4.1. The social optimum

First, we derive the optimal solution, which is the benchmark. Following Ono (1996), we suppose that the benevolent social planner treats all generations symmetrically. There is no intergenerational discounting and we assume that there exists a unique and stable long-term optimal allocation. The latter should guide public policy over the very long term, and thus help to determine the optimal instruments. Then, the problem is to maximize $u = c^{\beta} \cdot (E^{-\phi} \cdot H^{\mu})^{1-\beta}$ subject to $H = (\bar{H} + h)/\delta_H$, $E = (\epsilon c - \gamma m)/\delta_E$, and $Ak^{\alpha} = c + k + m + h$.

By defining $A_0 \equiv (\alpha A)^{1/1-\alpha} \frac{1-\alpha}{\alpha}$, we can derive the optimal allocation as follows:

$$k^* = (\alpha A)^{\frac{1}{1-\alpha}}, \tag{28}$$

$$E^* = \frac{\gamma}{\delta_E} \cdot \frac{\phi(1-\beta)}{\beta - (1-\beta)(\phi - \mu)} (A_0 + \bar{H}), \tag{29}$$

$$m^* = \frac{\epsilon\beta - \phi(1-\beta)(\gamma + \epsilon)}{\gamma + \epsilon} \cdot \frac{A_0 + \bar{H}}{\beta - (1-\beta)(\phi - \mu)}, \tag{30}$$

https://doi.org/10.1017/S1365100525000252 Published online by Cambridge University Press

$$h^* = \frac{\mu(1-\beta)A_0 - \{\beta - \phi(1-\beta)\}\bar{H}}{\beta - (1-\beta)(\phi - \mu)}, \tag{31}$$

$$H^* = \frac{\mu(1-\beta)}{\delta_H} \cdot \frac{A_0 + \bar{H}}{\beta - (1-\beta)(\phi - \mu)}, \tag{32}$$

$$c^* = \frac{\gamma\beta}{\gamma + \epsilon} \cdot \frac{A_0 + \bar{H}}{\beta - (1-\beta)(\phi - \mu)}. \tag{33}$$

To rule out trivial results, we assume the following parameter conditions:

Assumption 4. $\frac{\mu(1-\beta)A_0}{\beta-(1-\beta)\phi} > \bar{H}$.

This assumption implies a not too high natural stock of adaptation $\bar{H}$. Under Assumption 4, the optimal investment in adaptation h^* in (31) decreases with the natural adaptation stock $\bar{H}$. This is because natural adaptation increases the adaptation stock H^* in (32) and decreases the marginal utility from the investment in adaptation. Conversely, natural adaptation, $\bar{H}$, increases investment in mitigation, m^* in (30), and consumption, c^* in (33). The mechanism is as follows: Natural adaptation, $\bar{H}$, decreases h^*, whereas it has no effect on the capital stock level, k^* in (28). The saved investment in h^* thus increases c^* and m^* via the resource constraint. We also note that any increase in natural adaptation increase the optimal level of the pollution stock, E^* in (29), as increased natural adaptation decreases the effect on utility from pollution emissions. In addition, following the increase in natural adaptation, the increase in emissions due to increased consumption is greater than the decrease in emissions due to the increase in mitigation; the stock of pollutants increases, that is, $\epsilon \cdot (\partial c^*/\partial \bar{H}) > \gamma \cdot (\partial m^*/\partial \bar{H})$ hence, $(\partial E^*/\partial \bar{H}) > 0$.

The optimal mix between mitigation and adaptation can be summarized by the ratio (m^*/h^*):

$$\frac{m^*}{h^*} = \frac{\frac{\epsilon\beta}{\gamma+\epsilon} - \phi(1-\beta)}{\mu(1-\beta)A_0 - \beta\bar{H} + (1-\beta)\phi\bar{H}}(A_0 + \bar{H}). \tag{34}$$

We can easily observe that this ratio decreases with μ, ϕ, and γ/ϵ. That is, when the effects on utility from adaptation and pollution become greater (μ and ϕ), adaptation should be increased, rather than mitigation. Furthermore, even when mitigation efficiency is higher, adaptation is increased more than mitigation in the optimal policy plan.

4.2. Optimal policy schemes

The economy is characterized by four inefficiencies: dynamic inefficiency, pollution externality, public provision of adaptation, and private contribution to mitigation (Ono, 1996). We need to determine the optimal values of the policy instruments, that is, τ^c, $\bar{h}$, τ^y, and ν: (i) $c_{ss}/E_{ss} = c^*/E^*$, (ii) $\bar{h} = h^*$, (iii) $m_{ss} = m^*$, and (iv) $k_{ss} = k^*$.

First, we derive $\tau^c(\tau^y, \nu)$ from (i), which satisfies:

$$\frac{R}{1+\tau^c} \cdot \frac{1-\nu}{\gamma} = \frac{\delta_E}{\gamma + \epsilon}. \tag{35}$$

Second, we set the adaptation $\bar{h}$ directly from (ii) as $\bar{h} = h^*$ in (31). Third, mitigation satisfies (iii) by setting $\nu(\tau^y, \tau^c)$ which equalizes the following:

$$\{(1-\tau^y)(1-\alpha) + \tau^y\}Ak_{ss}^{\alpha} - \bar{h} + \tau^c c_{ss} - \frac{\beta(1-\nu)}{\gamma\phi(1-\beta)}E_{ss}$$
$$= \frac{\epsilon\beta - \phi(1-\beta)}{\beta - (1-\beta)(\phi - \mu)}(A_0 + \bar{H}). \tag{36}$$

Table 1. Parameter values

β	ϕ	γ	ϵ	α	δ_E	δ_H	A	$\bar{H}$	μ
0.9	2	3	2	0.3	0.8	0.8	1	0.01	0.9

Finally, τ_y is set to derive (iv). Given that $(1+\tau^c)c_{ss} = R_{ss}s_{ss}$ from (6), $R_s s = (1-\tau^y)\alpha k_{ss}^{\alpha-1}$ from (9), and the law of motion in the capital market is $k_{ss} = s_{ss}$, we can describe τ^y such that

$$\tau^y = 1 - \frac{(1+\tau^c)c_{ss}}{\alpha A k_{ss}^{\alpha}}. \tag{37}$$

From the discussion in this subsection, we obtain the following proposition.

PROPOSITION 4. *An environmental policy combining optimal taxes on output and consumption, a mitigation subsidy, and constant public investment in adaptation can achieve optimal allocation.*

4.3. Numerical simulation

To illustrate how the model performs, this section reports the results from some simple numerical examples. We show that the economy can converge towards a unique and stable long-run steady-state, and that a stable social optimum exists. The objective is also to show that assumptions 1 to 4 for the existence and stability of steady-states can be satisfied simultaneously, for reasonable parameter values. We limit our simulations on cases where a non-zero steady-state exists (case (*iii*) with a sufficiently high mitigation subsidy and case (*ix*) with a low one).

We set the values of the parameters $\{\beta, \alpha, \phi, \gamma, \epsilon, \delta_E\}$ using the estimates usually found in the literature on applied models, and in order to comply with the conditions imposed by assumptions 1 to 4. To select the environmental parameters we used Bonen et al. (2016) for guidance. We also followed Catalano et al. (2020a, b) and Fried et al. (2018) in order to maintain comparability with other life-cycle studies. The rest of the parameters are reasonably standard within the life-cycle literature. We choose the values for the exogenous parameters as shown in Table 1.

Concerning assumption about the consequences of adaptation, it is difficult to make comparisons with assumptions and results from other models.

Indeed, we assume that adaptation helps limit the effects of pollution on welfare, which is a subjective consequence. Empirical studies assessing the consequences of adaptation generally focus on objective physical indicators such as mortality risk (Carleton et al. 2022), agricultural yield (Hultgren et al. 2022), factor productivity (Catalano et al. 2020a, b) or factor mobility (like migration). On the other hand, Bonen et al. (2016) considers transmission channels for adaptation expenditure relatively close to our assumptions. Bonen et al. (2016) introduces adaptation expenditure into the utility function, but as a flow of public expenditure and not as a stock of infrastructure or knowledge for adaptation. They assume a value for the elasticity of public capital used for adaptation in utility equal to 0.05, whereas we consider a value of 0.09, but for the impact of the adaptation stock.

First, we calculate the social optimal values, which depend only on technological (A, α) and environmental $(\gamma, \epsilon, \bar{H}, \delta_H, \delta_E)$ parameters and preferences (β, ϕ, μ). As a benchmark, we obtain the optimal level of adaptation expenditure (h) equal to 0.04. Then, we evaluate steady-states under different scenarios relating to budget spending. We present two alternative scenarios: in Table 2, the subsidy to mitigation ν is low (equal to 0), while in Table 3, the subsidy is high $(\nu = 0.9)$. We add to these two policies decisions where the government engages in over- (resp. under-) spending on adaptation ($\bar{h} = 0.15 > h^*$ *resp.* $\bar{h} = 0.029 < h^*$). These policies are combined with two tax schemes: *laissez-faire* $\tau^y = \tau^c = 0$ and taxation with $\tau^y = 0.2$ and $\tau^c = 0.1$.

https://doi.org/10.1017/S1365100525000252 Published online by Cambridge University Press

Table 2. Low mitigation subsidy ($v = 0$)

Steady state ($\times 10^{-2}$)	*Laissez-faire*	*Laissez-faire*	Taxation	Taxation	Optimum
h	2.9	15	2.9	15	4
k	44.012	18.281	64.249	40.979	18
y	78.176	60.062	87.571	76.519	59.784
m	7.8109	8.7624	1.3155	3.8448	9
c	23.453	18.019	19.106	16.6 95	30
E	29.341	12.187	42.833	27.319	41
H	4.875	20	4.875	20	6
u	26.3998	28.1884	20.3523	22.3937	31.3968

Table 3. High mitigation subsidy ($v = 0.9$)

Steady state ($\times 10^{-2}$)	*Laissez-faire*	*Laissez-faire*	Taxation	Taxation	Optimum
h	2.9	15	2.9	15	4
k	0.007502	1.4047	0.003703	0.71348	18
y	5.7883	27.815	4.553	22.699	59.784
m	1.1443	3.0656	0.65627	2.0333	9
c	1.7365	8.3444	0.99339	4.9526	30
E	0.050013	9.3647	0.022469	4.7565	41
H	4.875	20	4.875	20	6
u	9.0742	14.8610	6.4418	10.6410	31.3968

A first important result is that the level of subsidy v significantly determines capital intensity at steady state. When the subsidy is low (Table 2), the economy is in a situation of capital over-accumulation, whereas a high subsidy (Table 3) causes capital to fall drastically, and the economy is then characterized by under-accumulation. We confirm the results in Proposition 2. The increase in expenditure on adaptation ($\bar{h}$ increases from 0.029 to 0.15) reduces the capital stock per capita in a situation of over-accumulation. Conversely, in a situation of under-accumulation, the rise in $\bar{h}$ increases k. In both cases, adaptation is beneficial in terms of well-being because k moves closer to its optimum value. More generally, any increase in the subsidy v worsens the capital stock per capita, whatever the adaptation effort.

A second significant result relates to the impact on mitigation m chosen by households. Private spending on mitigation has a twofold impact on pollution. A direct effect is to reduce the stock of pollution through the accumulation of pollutants, and an indirect effect is to reduce savings and hence consumption, the sole source of pollution. When the subsidy to mitigation is low (Table 2), the increase in m is accompanied by a reduction in economic activity, consumption and therefore pollution. This situation is pareto-improving because the economy is over-accumulating capital. On the other hand, when the subsidy is high (Table 3), the opposite mechanisms are at work, since the increase in m goes hand in hand with an increase in k, c and therefore E. In this case too, these changes are beneficial in terms of welfare despite the rise in pollution. Indeed, the increase in m, c and k follows an increase in $\bar{h}$: households consume more and are better protected against pollution. In this example, we show that any increase in $\bar{h}$ leads to an increase in m, whatever the tax scheme (v, τ^y, τ^c). We are therefore in a situation where adaptation and mitigation are complements (see Proposition 3).

We find the results of corollary 1 on the impact of instruments (*i.e., laissez-faire vs* taxation) on the capital stock per capita. When the latter is high (i.e., v low, Table 2), higher taxes increase the capital stock, whereas the effect is negative when the capital stock is low (Table 3).

https://doi.org/10.1017/S1365100525000252 Published online by Cambridge University Press

When the mitigation subsidy is small (Table 2), the level of mitigation is low (relative to the optimum), and counter-intuitively, the level of pollution is also small. This result can be explained by the high level of the capital stock per capita, which absorbs the greater part of the production of goods, and which does not pollute, unlike consumption. This effect is even stronger when expenditure on adaptation is high ($\bar{h} = 0.15$), reflecting a substitution between consumption and adaptation to the environment. As a result, welfare increases. Note that when the mitigation subsidy is high, private mitigation choices and capital per capita are highly sensitive to public decisions on adaptation ($\bar{h}$).

5. Extensions

5.1. Proportional adaptation

We suppose a case in which the adaptation level is determined by the flow of pollutants. This implies that, in a sense, mitigation influences adaptation. Hence, if mitigation increases, it implies a lower consumption level (budget constraint) and adaptation level. We extend the previous results to the case where the investment in adaptation is set proportionally to the flow of pollution at a constant rate $z \geq 0$:

$$h_t = z \cdot \epsilon c_t. \tag{38}$$

This policy implies a relationship between the current adaptation h_t and previous mitigation m_{t-1}. Indeed, h_t must increase if consumption c_t increases. This is possible if savings s_{t-1} increases and mitigation m_{t-1} decreases, *ceteris paribus*.

By substituting (38) into (11) and using (8), (9), and (12), the dynamics of capital stock per young are given by

$$\begin{aligned} k_{t+1} &= \Gamma_z(k_t) \\ &= \frac{(1-\nu)\{1-\alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma+\epsilon z)\}\beta A k_t^{\alpha} - \phi(1-\beta)(1-\delta_E)k_t}{(1-\nu)\beta-(1-\beta)\phi}. \end{aligned} \tag{39}$$

In this case, we have two steady states with k_{ss}^1 and k_{ss}^2 such that

$$k_{ss}^1 = 0; \quad k_{ss}^2 = \left[\frac{(1-\nu)\left\{1-\alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma+\epsilon z)\right\}\beta A}{(1-\nu)\beta-(1-\beta)\delta_E\phi}\right]^{\frac{1}{1-\alpha}}. \tag{40}$$

Let us consider the conditions for positive capital stock in a non-trivial steady state, $k_{ss}^2 > 0$. This is easily attained by assuming that $\frac{1-\alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma+\epsilon z)}{(1-\nu)\beta-(1-\beta)\delta_E\phi} > 0$. On the one hand, the denominator becomes negative (resp. positive) if the mitigation subsidy is greater than $\bar{\nu}$ (resp. lower than $\bar{\nu}$). On the other hand, the numerator can also be negative (resp. positive) if the adaptation provision rate z is greater than $\bar{z}$ (resp. lower than $\bar{z}$), as defined below.

$$\bar{z} \equiv \frac{1-\alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma)}{\alpha\frac{1-\tau^y}{1+\tau^c}\epsilon} > 0. \tag{41}$$

From Assumption 1(b), $\bar{z}$ is positive. For the nontrivial steady state with k_{ss}^2, we suppose that the environmental policy tools satisfy the following:

Assumption 5. *We assume that either (i) $z < \bar{z}$ and $\nu < \bar{\nu}$ or (ii) $z > \bar{z}$ and $\nu > \bar{\nu}$.* □

This assumption requires either sufficiently large or sufficiently small mitigation and adaptation. We therefore exclude intermediate cross-cases in order to restrict the analysis to situations where there exist two stationary states with at least one non-trivial stable steady state.

To examine the stability of the steady state, we define $\overline{k_z}$, $\underline{k_z}$, and $\hat{k}_z$ as follows.

$$\Gamma_z'(\overline{k_z}) = 0 \quad \Leftrightarrow \quad \overline{k_z} \equiv \left[\frac{(1-\nu)\left\{1-\alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma+\epsilon z)\right\}\alpha\beta A}{(1-\beta)(1-\delta_E)\phi} \right]^{1/1-\alpha}, \tag{42}$$

$$\Gamma_z'(\underline{k_z}) = -1 \quad \Leftrightarrow \quad \underline{k_z} \equiv \left[\frac{(1-\nu)\left\{1-\alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma+\epsilon z)\right\}\alpha\beta A}{(1-\beta)(2-\delta_E)\phi-(1-\nu)\beta} \right]^{1/1-\alpha}, \tag{43}$$

$$\Gamma_z'(\hat{k}_z) = 1 \quad \Leftrightarrow \quad \hat{k}_z \equiv \left[\frac{(1-\nu)\left\{1-\alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma+\epsilon z)\right\}\alpha\beta A}{(1-\nu)\beta-(1-\beta)\phi\delta_E} \right]^{1/1-\alpha}. \tag{44}$$

Consequently, from (39), Proposition 5 characterizes the steady-state equilibrium with proportional adaptation.

PROPOSITION 5. *Suppose a proportional adaptation to the flow of pollutants, and Assumptions 1, 2, 3, and 5 are satisfied. Then, there are two steady-state equilibria with capital stocks* $k_{ss}^1 = 0$ *and* $k_{ss}^2 > 0$*. The stability properties of these steady states are presented in the following four cases.*

- (i) *Suppose* $\nu > \bar{\nu}$ *and* $z > \bar{z}$*. Then, the steady state with* k_{ss}^1 *is unstable, whereas that with* k_{ss}^2 *is stable.*
- (ii) *Suppose* $\nu < \bar{\nu}$ *and* $z < \bar{z}$*. Especially, if* $\nu < \underline{\nu}$ *and* $k_{ss}^2 < \underline{k_z}$*, the steady state with* k_{ss}^1 *is unstable, whereas that with* k_{ss}^2 *is stable.*
- (iii) *Suppose* $\nu < \bar{\nu}$ *and* $z < \bar{z}$*. Especially, if* $\nu < \underline{\nu}$ *and* $k_{ss}^2 \geq \underline{k_z}$*, both the steady states are unstable.*
- (iv) *Suppose* $\nu < \bar{\nu}$ *and* $z < \bar{z}$*. Especially, if* $\underline{\nu} \leq \nu < \bar{\nu}$*, the steady state with* k_{ss}^1 *is stable, and that with* k_{ss}^2 *is unstable.*

Proof. *See Appendix C.* □

As well as Proposition 1, the mitigation subsidy is a key to the analysis of the dynamics of the capital stock. Depending on whether the subsidy is low or high enough, three cases for each exist. However, as the adaptation level is not fixed in this case, the analysis of the capital stock dynamics is simplified.

From this proposition, we focus on the steady state with k_{ss}^2 in cases (i) and (ii) of Proposition 5. The nontrivial steady-state equilibrium is described as follows:

$$c_{ss} = \frac{\tau^c}{1+\tau^c}\alpha(1-\tau^y)A{k_{ss}^2}^{\alpha}, \tag{45}$$

$$h_{ss} = \epsilon z\frac{\tau^c}{1+\tau^c}\alpha(1-\tau^y)A{k_{ss}^2}^{\alpha}, \tag{46}$$

$$H_{ss} = \frac{1}{\delta_H}\left\{\bar{H}+h_{ss}\right\}, \tag{47}$$

$$m_{ss} = \left\{1-\alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon z)\right\}A{k_{ss}^2}^{\alpha} - k_{ss}^2. \tag{48}$$

$$E_{ss} = \frac{\gamma}{\delta_E}\left[k_{ss}^2 - \left\{1-\alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon z+\tau^c\epsilon/\gamma)\right\}A{k_{ss}^2}^{\alpha}\right]. \tag{49}$$

https://doi.org/10.1017/S1365100525000252 Published online by Cambridge University Press

Next, we derive the derivatives of k_{ss}^2 with respect to the policy variables as a comparative statics study.

$$\frac{dk_{ss}^2}{dz} = -\frac{k_{ss}^2}{1-\alpha} \cdot \frac{\epsilon\alpha\frac{1-\tau^y}{1+\tau^c}}{1-\alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma+\epsilon z)}. \tag{50}$$

$$\frac{dk_{ss}^2}{dv} = \frac{k_{ss}^2}{(1-\alpha)(1-v)} \cdot \frac{(1-\beta)\delta_E\phi}{(1-v)\beta-(1-\beta)\delta_E\phi}. \tag{51}$$

$$\frac{dk_{ss}^2}{d\tau^y} = \frac{k_{ss}^2}{1-\alpha} \cdot \frac{\alpha\frac{1}{1+\tau^c}(1+\epsilon/\gamma+\epsilon z)}{1-\alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma+\epsilon z)}. \tag{52}$$

$$\frac{dk_{ss}^2}{d\tau^c} = \frac{k_{ss}^2}{1-\alpha} \cdot \frac{\alpha\frac{1-\tau^y}{(1+\tau^c)^2}(1+\epsilon/\gamma+\epsilon z)}{1-\alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma+\epsilon z)}. \tag{53}$$

From (50)–(53), we immediately obtain the following corollary.

COROLLARY 2. *Suppose a proportional adaptation to the flow of pollutants, and Assumptions 1, 2, 3, and 5 are satisfied. If $v > \bar{v}$ and $z > \bar{z}$, the adaptation rate (z) increases the steady-state capital stock, whereas the mitigation subsidy rate (v), output tax (τ^y), and consumption tax (τ^c) decrease it. By contrast, if $v < \bar{v}$ and $z < \bar{z}$, their effects on steady-state capital stock are reversed.*

This corollary shows that capital stock does not always increase with the adaptation rate z. This also implies, together with (45) and (47), that the adaptation rate, z, does not always increase the adaptation flow, h_{ss}, or the adaptation stock, H_{ss}, in the steady state. To observe this in detail, we have from (46):

$$\frac{dh_{ss}}{dz} = \frac{h_{ss}}{z} + \alpha\frac{h_{ss}}{k_{ss}^2} \cdot \frac{dk_{ss}^2}{dz}.$$

The first term is a direct increase in z whereas the second is an indirect increase via k_{ss}^2. We define the fiscal scheme in (38). If z decreases the capital stock, the consumption shown in (45) decreases, and thus the indirect effect decreases the adaptation flow and stock. We can rearrange this expression using (46) and (50) as:

$$\frac{dh_{ss}}{dz} = \frac{\epsilon\frac{\alpha}{1-\alpha}\frac{1-\tau^y}{1+\tau^c}}{1-\alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma+\epsilon z)} \cdot (1-\alpha)\left\{1-\alpha\frac{1-\tau^y}{1+\tau^c}\left(1+\frac{\epsilon}{\gamma}+\frac{\epsilon z}{1-\alpha}\right)\right\}\tau^c A{k_{ss}^2}^{\alpha}. \tag{54}$$

We define $\underline{z}$, which satisfies $\frac{dh_{ss}}{dz} = 0$:

$$\underline{z} \equiv (1-\alpha)\bar{z}. \tag{55}$$

Therefore, the derivative in (54) is strictly positive for $z < \underline{z}$ and strictly negative for $z > \underline{z}$. Likewise, the effect on mitigation from the adaptation rate is ambiguous, as shown in (48):

$$\frac{dm_{ss}}{dz} = \frac{\epsilon\frac{\alpha}{1-\alpha}\frac{1-\tau^y}{1+\tau^c}}{1-\alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma+\epsilon z)} \cdot \left[\left\{1-\alpha\frac{1-\tau^y}{1+\tau^c}\left(1+(1-\alpha)\frac{\epsilon}{\gamma}+\epsilon z\right)\right\}A{k_{ss}^2}^{\alpha} - k_{ss}^2\right]. \tag{56}$$

https://doi.org/10.1017/S1365100525000252 Published online by Cambridge University Press

From (54) and (56), we obtain the following expression.

$$\frac{dm_{ss}/dz}{dh_{ss}/dz} = \frac{dm_{ss}}{dh_{ss}} = \frac{\left[1 - \alpha\frac{1-\tau^y}{1+\tau^c}\left\{1 + (1-\alpha)(\epsilon/\gamma) + \epsilon z\right\}\right] A{k_{ss}^2}^{\alpha} - k_{ss}^2}{(1-\alpha)\left\{1 - \alpha\frac{1-\tau^y}{1+\tau^c}\left(1 + \epsilon/\gamma + \frac{\epsilon z}{1-\alpha}\right)\right\}\tau^c A{k_{ss}^2}^{\alpha}}. \tag{57}$$

The denominator becomes positive if $z < \underline{z}$, whereas the square brackets of the numerator become positive if $z < \hat{z}$ defined as

$$\hat{z} \equiv \bar{z} + \frac{\alpha}{\gamma}. \tag{58}$$

Then, we have $\underline{z} < \bar{z} < \hat{z}$. In addition to the adaptation rate, the numerator also has a threshold k^{ℓ} that determines its sign.

$$k^{\ell} \equiv \left[\left\{1 - \alpha\frac{1-\tau^y}{1+\tau^c}\left(1 + (1-\alpha)\frac{\epsilon}{\gamma} + \epsilon z\right)\right\} A\right]^{1/(1-\alpha)}. \tag{59}$$

From the discussion thus far, the substitutability between mitigation and adaptation can be summarized as follows:

PROPOSITION 6. *Suppose a proportional adaptation to the flow of pollutants, and Assumptions 1, 2, 3, and 5 are satisfied. The mitigation and adaptation are complements if (i)* $z \leq \underline{z}$ *and* $k_{ss}^2 < k^{\ell}$*, (ii)* $\underline{z} < z < \hat{z}$ *and* $k_{ss}^2 > k^{\ell}$*, or (iii)* $z \geq \hat{z}$*.*

Proof. *See Appendix D* □

In the case of constant adaptation investment, mitigation and adaptation can be both complements and substitutes, but this depends on the adaptation rate and capital stock level at the steady state.

5.2. Public debt financing of adaptation

Results thus far assume that public investment in adaptation and mitigation subsidies is financed by taxes on current generations. However, as adaptation is a stock or durable good, it benefits not only the present generations but also future generations. Debt is an instrument that affects intertemporal transfers and intergenerational welfare. We assume that investment in adaptation is partly financed by public debt. This financing policy allows future generations to benefit from the accumulated adaptation stock and partly bear its present cost.

In particular, we suppose that adaptation investment is set at an optimal level at steady state and is totally financed by debt issuing. We focus on equilibria with constant debt per capita, that is, $B_t = B > 0$ for all $t > 0$. Debt is constant so that steady-states can be explicitly determined. We consider public debt as an instrument of environmental policy, it finances investments in the adaptation stock. This instrument replaces the production tax. Optimal debt helps to achieve the optimal level of adaptation, as defined by (30) and (31), by substituting h^* into $\bar{h}$. Thus, the government budget is given as

$$B = R_t B + \bar{h} - T_t - \tau^c c_t + v m_t. \tag{60}$$

We have the following capital market equilibrium:

$$k_{t+1} = s_t - B. \tag{61}$$

https://doi.org/10.1017/S1365100525000252 Published online by Cambridge University Press

Eq. (61) is rewritten as the dynamics of capital stock:

$$k_{t+1} = \Gamma_B(k_t) = \frac{1}{(1-\nu)\beta - (1-\beta)\phi}[T_1 + T_2 + T_3 + T_4], \tag{62}$$

$$\begin{aligned} \textit{where} \quad & T_1 = B(1-\beta)\delta_E\phi - \beta(1-\nu)\bar{h}, \\ & T_2 = -(1-\delta_E)(1-\beta)\phi k_t, \\ & T_3 = -\frac{B(1-\nu)}{1+\tau^c}(1+\epsilon/\gamma)\alpha A k_t^{\alpha-1}, \\ & T_4 = (1-\nu)\left\{1 - \frac{\alpha}{1+\tau^c}(1+\epsilon/\gamma)\right\}\beta A k_t^{\alpha}. \end{aligned}$$

With this expression, we obtain the following proposition for the steady-state equilibrium. For stability, we derive the derivatives of (62) as

$$\Gamma_B'(k_t) = \frac{1}{(1-\nu)\beta - (1-\beta)\phi}\left[-(1-\delta_E)(1-\beta)\phi + B\frac{1-\nu}{1+\tau^c}(1-\alpha)(1+\epsilon/\gamma)\alpha A k_t^{\alpha-2} + (1-\nu)\left\{1 - \frac{\alpha}{1+\tau^c}(1+\epsilon/\gamma)\right\}\alpha\beta A k_t^{\alpha-1}\right]. \tag{63}$$

$$\Gamma_B''(k_t) = \frac{-1}{(1-\nu)\beta - (1-\beta)\phi}\left[B\frac{1-\nu}{1+\tau^c}(1-\alpha)(2-\alpha)(1+\epsilon/\gamma)\alpha A k_t^{\alpha-3} + (1-\nu)\left\{1 - \frac{\alpha}{1+\tau^c}(1+\epsilon/\gamma)\right\}\alpha(1-\alpha)\beta A k_t^{\alpha-2}\right]. \tag{64}$$

Furthermore, to summarize the stability conditions, we define $\underline{k_B}$, $\overline{k_B}$, and $\hat{k_B}$ as $\Gamma_B'(\underline{k_B}) = -1$, $\Gamma_B'(\overline{k_B}) = 0$, and $\Gamma_B'(\hat{k_B}) = 1$. These three levels are unique because (64) exhibits monotonicity of $\Gamma_B'(k_t)$.

PROPOSITION 7. *Suppose investment in adaptation and public debt are constant, and Assumptions 1, 2, and 3 are satisfied. Then, the steady-state equilibria and the stability conditions are characterized in the following six cases.*

- *(i) Suppose $\nu < \underline{\nu}$ and $\Gamma_B(\hat{k_B}) < \hat{k_B}$. Then, there is a unique steady state at $k_{ss} = 0$, and this is stable.*
- *(ii) Suppose $\nu < \underline{\nu}$ and $\Gamma_B(\hat{k_B}) = \hat{k_B}$. Then, there are two steady states with $k_{ss}^1 = 0$ and $k_{ss}^2 > 0$. The steady state with $k_{ss}^1 = 0$ is locally stable and that with k_{ss}^2 is unstable.*
- *(iii) Suppose $\nu < \underline{\nu}$ and $\Gamma_B(\hat{k_B}) > \hat{k_B}$. Then, there are three steady states with $k_{ss}^1 = 0$ and $k_{ss}^2, k_{ss}^3 > 0$, where $k_{ss}^2 < k_{ss}^3$. The steady state with $k_{ss}^1 = 0$ is stable, and that with k_{ss}^2 is unstable. Furthermore, the steady state with k_{ss}^3 is locally stable if $k_{ss}^3 < \underline{k_B}$, whereas it is unstable if $k_{ss}^3 \geq \underline{k_B}$.*
- *(iv) Suppose $\nu > \underline{\nu}$ and $\Gamma_B(\hat{k}) > \hat{k_B}$. Then, there is no steady state in a finite range.*
- *(v) Suppose $\nu > \underline{\nu}$ and $\Gamma_B(\hat{k}) = \hat{k_B}$. Then, there is a unique steady state with $k_{ss} > 0$, and this is unstable.*
- *(vi) Suppose $\nu > \underline{\nu}$ and $\Gamma_B(\hat{k}) < \hat{k_B}$. Then, there are two steady states with $k_{ss}^1 > 0$ and $k_{ss}^2 > 0$, where $k_{ss}^1 < k_{ss}^2$. The steady state with k_{ss}^2 is unstable. By contrast, the steady state with k_{ss}^1 is locally stable if $k_{ss}^1 > \underline{k_B}$, and unstable if $k_{ss}^1 \leq \underline{k_B}$.*

Proof. *See Appendix E.* □

As well as the previous models and as shown in Proposition 1 and 5, the capital stock dynamics has two cases, depending on the subsidy rates. For each case, the dynamics have further three cases, depending on the capital stock.

From Proposition 7, in the presence of a constant adaptation financed partly by debt, we focus on the locally stable nontrivial steady state, such as k_{ss}^3 in Case (iii) with $k_{ss}^3 < \underline{k_B}$ and k_{ss}^1 in Case (vi) with $k_{ss}^1 > \underline{k_B}$.

We then develop comparative statics with respect to debt. By differentiating $\Gamma_B(k_t)$ with respect to the constant debt B, we have[5]

$$\frac{\partial \Gamma_B(k_t; B)}{\partial B} = \frac{(1-\beta)\delta_E\phi - \frac{B(1-\nu)}{1+\tau^c}(1+\epsilon/\gamma)\alpha A k_t^{\alpha-1}}{(1-\nu)\beta - (1-\beta)\phi}. \tag{65}$$

This implies that if $\nu < \underline{\nu}$, the dynamics of the capital stock shift upward for $k_t > k^o$ and downwards for $k_t < k^o$, where

$$k^o \equiv \left\{ \frac{B(1-\nu)(1+\epsilon/\gamma)\alpha A}{(1+\tau^c)(1-\beta)\delta_E\phi} \right\}^{1/(1-\alpha)}. \tag{66}$$

Because B can take zero or larger values, k^o can also take a wide range of values. By contrast, if $\nu > \underline{\nu}$, the dynamics shift upward if $k_t < k^o$ and downward if $k_t > k^o$.

Given this result, we consider the steady-state effect of debt. For this purpose, we suppose locally stable steady states. First, suppose $\nu < \underline{\nu}$, $\Gamma_B(\hat{k}_B) > \hat{k}_B$, and $k_{ss}^3 < \underline{k_B}$, as in case (iii) of Proposition 7. Then, steady-state capital stock k_{ss}^3 is locally stable. Suppose further that the economy lies on $k_{ss} = k_{ss}^3$; if $k_{ss}^3 > k^o$, a marginal increase in the constant debt to finance the constant adaptation increases the capital stock. Conversely, if $k_{ss}^3 < k^o$, debt decreases it. On the contrary, suppose $\nu > \underline{\nu}$, $\Gamma_B(\hat{k}_B) < \hat{k}_B$, and $k_{ss}^1 > \underline{k_B}$ as in case (vi) of Proposition 7. Then, steady-state capital stock k_{ss}^1 is locally stable. Suppose further that the economy lies on $k_{ss} = k_{ss}^1$; if $k_{ss}^1 < k^o$, the debt increases the capital stock. If $k_{ss}^1 > k^o$, the debt decreases it.

We can summarize the results thus far in the following corollary.

COROLLARY 3. *Suppose investment in adaptation and public debt are constant, and Assumptions 1, 2, and 3 are satisfied. Then, the per capita capital stock at locally stable steady states, k_{ss}, increases with debt if (i) $\nu < \underline{\nu}$ and $k_{ss}^3 > k^o$ or (ii) $\nu > \underline{\nu}$ and $k_{ss}^1 < k^o$.*

Corollary 3 says that when B is sufficiently low, the initial capital stocks tend to be characterized by under-accumulation in both cases (i) and (ii). Then, an increase in public debt B is welfare improving if it leads to an increase in the capital stock. When the government increases B and thereby the adaptation, there are two opposing effects on k. Any improvement in adaptation should reduce the need for mitigation (substitution effect), and thus increase savings. At the same time, public debt is increasing, which could lead to a crowding-out effect and reduce capital accumulation. The main effects come through, on the one hand, the interest rate level which impacts the cost of debt repayment, and, on the other hand, the level of savings to satisfy consumption needs when retired. For low ν and high k as Case (i), the interest rate and mitigation subsidies are low, which means that the increase in debt can finance a high amount of adaptation investments. Pollution has a lower welfare effect and private spending on mitigation is reduced. Conversely, for high ν and low k as Case (ii), pollution stock is low, and adaptation is high enough to fight its consequences on welfare.

Case (ii) of Corollary 3 requires the mitigation subsidy to be sufficiently high in low capital stock countries. By contrast, case (i) shows that sufficiently low subsidies is required in countries with high per capita capital stock. Therefore, when the government is indebted and the mitigation subsidy is low, financing public investment for mitigation or adaptation with public debt can be beneficial when the capital stock is high enough but is detrimental otherwise.

https://doi.org/10.1017/S1365100525000252 Published online by Cambridge University Press

The debt policy is of great interest because it allows an optimal situation to be reached in terms of adaptation while sharing the cost of the policy between generations. The latter makes environmental policy more acceptable in the short term and under certain conditions, without compromising economic growth.

5.3. Public mitigation and private adaptation

Whether adaptation and mitigation are private or public is not obvious and may depend on the technologies, the scale, pollution characteristics for instance. To limit the number of cases, we have considered the case where adaptation is a public good, focusing on investments such as buildings, dikes, health and education spending. The converse is an alternative case where adaptation is a local and private decision, (i.e., an "individual choice"), such as cooling systems or crop diversification for a farmer in developing countries. Meanwhile, the government provides the mitigation services, for example, by building and operating carbon capture and storage (CCS) facilities. In this section, we consider the case where adaptation is endogenous and chosen by households, while the government decides on the level of mitigation spending.

An individual born at period t invests part of her disposable income when young in adaptation services h_t. The rest of income at t is saved and consumed with an interest rate, R_{t+1}, when old. The private adaptation h_t is subsidized at a rate of v whereas the consumption is taxed at τ^c. Thus, the individual maximizes her lifetime utility (3) subject to the following intertemporal budget constraint:

$$(1-v)h_t + \frac{1+\tau^c}{R_t}c_{t+1} = w_t - T_t. \tag{67}$$

With (67), the individual chooses the consumption at $t+1$ as

$$c_{t+1} = \frac{\beta}{\mu(1-\beta)} \cdot \frac{1-v}{1+\tau^c} R_{t+1} H_{t+1}. \tag{68}$$

We assume that the mitigation services are fixed over time: $m_t = \bar{m}\ \forall t$. The government's budget is balanced at each period:

$$T_t = vh_t + \bar{m} - \tau^c c_t + \tau^y y_t. \tag{69}$$

The capital market equilibrium writes: $k_{t+1} = s_t$. By using (68), we obtain

$$k_{t+1} = \frac{\beta(1-v)}{\mu(1-\beta)} H_{t+1}. \tag{70}$$

From (67), we can derive the adaptation investment, h_t:

$$h_t = \frac{\mu(1-\beta)\left[\left\{-\frac{\beta(1-v)}{\mu(1-\beta)}\bar{H} - \bar{m}\right\} + \left\{(1-\alpha)(1-\tau^y) + \tau^y + \frac{\tau^c}{1+\tau^c}\alpha(1-\tau^y)\right\} Ak_t^\alpha + (1-\delta_H)k_t\right]}{\mu(1-\beta) + \beta(1-v)}. \tag{71}$$

By using (71), we finally derive the dynamics of the capital accumulation (70) as

$$\begin{aligned} k_{t+1} &\equiv \Gamma_H(k_t) \\ &= \frac{\beta(1-v)}{\mu(1-\beta)+\beta(1-v)}(\bar{H}-\bar{m}) + \frac{\mu(1-\beta)(1-\delta_H)}{\mu(1-\beta)+\beta(1-v)}k_t \\ &\quad + \frac{\beta(1-v)}{\mu(1-\beta)+\beta(1-v)}\left\{1 - \frac{\alpha(1-\tau^y)}{1+\tau^C}\right\} Ak_t^\alpha. \end{aligned} \tag{72}$$

https://doi.org/10.1017/S1365100525000252 Published online by Cambridge University Press

As previously shown, we define $\hat{k}_H$ such that $\Gamma'_H(\hat{k}_H) = 1$, given as

$$\hat{k}_H = \left(\frac{\beta(1-\nu)\left\{1 - \frac{\alpha}{1+\tau^c}(1-\tau^y)\alpha A\right\}}{\delta_H \mu(1-\beta) + \beta(1-\nu)} \right)^{\frac{1}{1-\alpha}}. \tag{73}$$

We then have the following cases.

PROPOSITION 8. *Suppose that the government finances the adaptation while households provide the adaptation. The steady-state equilibria are characterized by the following cases.*

(i) If $\bar{H} > \bar{m}$, there exists a positive, unique, and stable steady state.
(ii) If $\bar{H} = \bar{m}$, there exist two steady states, $k^1_{ss} = 0$ and $k^2_{ss} > 0$. Then, k^1_{ss} is unstable, whereas k^2_{ss} is stable.
(iii) If $\bar{H} < \bar{m}$ and $\Gamma'_H(\hat{k}_H) > \hat{k}_H$, there exist three steady states, $k^1_{ss} = 0$, $0 < k^2_{ss} < k^3_{ss}$. Then, k^1_{ss} is locally stable, k^2_{ss} unstable, and k^3_{SS} is positive and locally stable.
(iv) $\bar{H} < \bar{m}$ and $\Gamma'_H(\hat{k}_H) = \hat{k}_H$, there exist two steady states, $k^1_{ss} = 0$ and $k^2_{ss} > 0$. Then, k^1_{ss} is locally stable, whereas k^2_{ss} is unstable (one-side stable).
(v) $\bar{H} < \bar{m}$ and $\Gamma'_H(\hat{k}_H) < \hat{k}_H$ there exists a unique and stable steady state, $k^1_{ss} = 0$.

Proof. *See Appendix F.* □

When mitigation is provided by the government, its size relative to natural adaptation is key to determining the characteristics of the steady state. As public investment in abatement crowds out capital accumulation, the dynamics of capital shown in (72) shifts downward and modifies the steady states.

Then, are the public mitigation and private adaptation complementary? To answer this question, we first derive how the capital stock is affected by an increase in the mitigation from (72) at the stable steady state.

$$\frac{dk_{ss}}{d\bar{m}} = -\frac{\beta(1-\nu)}{\beta(1-\nu)\left[1 - \left\{1 - \frac{\alpha}{1+\tau^c}(1-\tau^y)\right\}\alpha A k_{ss}^{\alpha-1}\right] + \mu(1-\beta)\delta_H}. \tag{74}$$

The mitigation increases k_{ss} when the capital stock is sufficiently small but decreases k_{ss} when it is large enough. For more detail, we substitute $k_{ss} = \hat{k}_H$ of (73) into (74). Then, the denominator of (74) turns to 0. As the capital stock approaches to $\hat{k}_H$ from 0, $\frac{dk_{ss}}{d\bar{m}}$ diverges to $+\infty$. From Proposition 8 and Appendix F, the capital stock at the stable steady states is greater than $\hat{k}_H$. Therefore, at the stable steady states, we always have $\frac{dk_{ss}}{d\bar{m}} < 0$.

Second, we derive a total derivative of h_{ss} and $\bar{m}$ in (71) and obtain the following:

$$\frac{dh_{ss}}{d\bar{m}} = \frac{\mu(1-\beta)\left[-1 + \left\{(1-\tau^y)(1-\alpha) + \tau^y + \frac{\alpha}{1+\tau^c}(1-\tau^y)\alpha\right\}\alpha A k_{ss}^{\alpha-1}\frac{dk_{ss}}{d\bar{m}} - (1-\delta_H)\frac{dk_{ss}}{d\bar{m}}\right]}{\mu(1-\beta) + \beta(1-\nu)}. \tag{75}$$

From this expression, since the third term in square brackets on the numerator is positive $(-(1-\delta_H)\frac{dk_{ss}}{d\bar{m}} > 0)$, the adaptation may increase when k_{ss} is large enough. However, we have $\lim_{k_{ss}\to+\infty} \frac{dk_{ss}}{d\bar{m}} = -\frac{\beta(1-\nu)}{\beta(1-\nu)+\mu(1-\beta)\delta_H}$ from (74), and thereby $\lim_{k_{ss}\to+\infty} \frac{dh_{ss}}{d\bar{m}} = -\frac{\mu(1-\beta)\delta_H}{\beta(1-\nu)+\mu(1-\beta)\delta_H} < 0$ from (75). Discussion so far provides the following proposition.

https://doi.org/10.1017/S1365100525000252 Published online by Cambridge University Press

PROPOSITION 9. *Suppose that adaptation is provided by households and mitigation is provided by the government. Then, mitigation and adaptation are substitutes for all stable steady states.*

Whether adaptation and mitigation are complements or substitutes depends not only on the level of the capital stock and the policy instruments (propositions 3 and 6), but also on who provides what (Proposition 9). Public mitigation directly crowds out private adaptation as a substitution effect. Although savings and thus the capital stock could be increased instead, the steady-state capital stock does not increase from (74). This implies that the government intervention is so large that the income effect from the increased mitigation is negative for private adaptation.

6. Conclusion

This study examines the interactions between adaptation, mitigation, and fiscal policies. Adaptation comprises public infrastructure financed by tax revenue and public debt, whereas mitigation is a private decision supported by subsidies. Pollution results from household consumption, but its welfare consequences are limited by adaptation investment. To consider these features, we study the dynamics of capital in an overlapping generation model. Fiscal policy has a twofold effect on households' budget constraints through taxation for financing public adaptation and through subsidies for private mitigation. First, we find that it is possible to explicitly determine a policy to decentralize the optimal solution. This policy requires setting tax rates on consumption and production, subsidies for mitigation, and public spending on adaptation at specific levels, which depend on both environmental parameters (pollution rate) and economic development (capital stock). We also show that adaptation and mitigation may be substitutes or complements depending on the level of economic development, but also on the level of mitigation subsidy. Finally, when the government is indebted, we show that financing public adaptation and/or subsidizing private mitigation through an increase in public debt could be beneficial for countries with high capital stock, but is detrimental when the capital stock is too low.

Our results can easily be extended by considering the government's exogenous mitigation investment at the optimal level, for example, in addition to public investment in adaptation infrastructure. In this case, the government has a direct instrument (mitigation) and no longer needs to provide incentives to households for mitigation. Moreover, our results remain valid when the decision to invest in an adaptation stock becomes private, instead of mitigation. However, in our framework, the consequences of adaptation concern households' welfare; it neglects the direct consequences on biodiversity and animal welfare, and to a lesser extent, the indirect effects on the environment (as only consumption is a source of pollution). Therefore, the consequences of adaptation are likely to be underestimated. Finally, we assume homogeneity of the agents within each generation. It would be interesting to extend this model to heterogeneous agents whose contributions to mitigation would be different. Depending on the form of the utility function, the adaptation needs would also be different, which would have a significant effect on equilibrium and the definition of efficient policy tools in the absence of coordination.

In this paper, we focus on stable steady-states in order to analyze long-term consequences of environmental policies on pollution and public finance. Note that our comparative statics refers to locally stable steady states, but our figures allow for a global stability analysis. Our complete analysis of paths and equilibria allows us to determine the conditions for unicity and stability. Nevertheless, our results show that cycles and bifurcations can emerge for some specific policy parameters. A detailed analysis of these complex dynamics need to be carried out in future works.

Further issues to be considered are uncertainty and/or a tipping point of climate change.[6] Indeed, the environmental damage caused by climate change could be subject to uncertainty. We expect that not only optimal policies but also the substitutability between mitigation and adaptation will be affected by the probabilistic features of climate change. Finally, further extensions could include considering other sources of pollution (such as fossil energy or production) instead of consumption, as well as changes in production technologies. These issues are thus significant and therefore to be analyzed in future works.

Supplementary material. To view supplementary material for this article, please visit 10.1017/S1365100525000252

Acknowledgements. This paper was presented at seminars of Univ. Paris Ouest Nanterre, ETH Zurich, Montpellier, Nantes, and Seikei, and conferences of EENR at Univ. Orléans (2022) and PET (2022) at Aix-Marseille School of Econ. The authors are grateful to participants of seminars and conferences, along with two anonymous referees from this journal, for their valuable feedback that improved this paper. Hiroaki Yamagami is grateful for the research grant from Seikei University and JSPS Grant-in-Aid for Scientific Research (C) (No. 21K01489).

Notes

1 Another approach would be to assume that the pollution emission rate decreases with mitigation. In this case, mitigation only tackles the instantaneous flow of pollution. The advantage of our assumption is that it allows the stock of pollutants to be reduced and, potentially, carbon dioxide removal technologies to be taken into account. In addition, it allows the model to be solved analytically. Much of the theoretical literature on these issues considers similar assumptions.

2 We provide some examples. First, the atmosphere protects us from the sun's X-rays and allows us to maintain a certain temperature on the planet's surface, even if mankind has an effect on its atmosphere through activities, such as emitting CFCs that reduce the ozone layer. However, at first, the atmosphere exists by itself, and when economic activity stops, nature regains its natural level and the ozone layer is restored. This is different from natural regeneration, which in some ways consists of absorbing or dissolving some pollutants. With adaptation, pollutants do not disappear. Second, the water cycle uses rainfall to supply water to the groundwater, lakes, and reservoirs. Water regenerates naturally, but artificial adaptation can be added, such as dams, artificial lakes, and canals. These infrastructures do not eliminate pollutants or cancel the effects of climate change but make it possible to live better despite pollution. Finally, we may also consider genetic capital, resistance to external threats, and resilience to changes in our ecosystems. In our model, adaptation is a human activity aimed solely at protecting households and requires corresponding investments. These investments complement natural adaptation capacity without replacing or influencing it. An alternative would have been to assume that adaptation expenditure h affects natural capacity in the long run. This is an interesting assumption, but it would significantly complicates the analysis.

3 The derivative of $\Gamma_0(k_t)$ is given as $\Gamma_0'(k_t) = \frac{\beta\{1-\alpha(1+\epsilon/\gamma)\}Ak_t^{\alpha} - \phi(1-\beta)(1-\delta_E)k_t}{\beta-(1-\beta)\phi}$.

4 See for instance Bréchet et al. (2013) and Ingham et al. (2013).

5 One illustration is to assume that the debt is set at $B = \eta\bar{h}$ with $\eta \in [0, 1]$, and the debt marginally increases as η increases in order to maintain the adaptation at its optimal level: $\bar{h} = h^*$.

6 See Pindyck (2013, 2021) for general discussions of uncertainty and tipping points. Suzuki and Yamagami (2024) are examples that incorporate the climate uncertainties related to the tipping point as a Markov process.

References

Ayong le Kama, A. and A. Pommeret (2017) Supplementing domestic mitigation and adaptation with emissions reduction abroad to face climate change. *Environmental and Resource Economics* 68, 875–891.

Bachner, G., B. Bednar-Friedl and K. Knittel (2019) How does climate change adaptation affect public budgets? Development of an assessment framework and a demonstration for Austria. *Mitigation and Adaptation Strategies for Global Change* 24(7), 1325–1341.

Barrage, L. (2020) The fiscal costs of climate change. *AEA Papers and Proceedings* 110, 107–112.

Bonen, A., P. Loungani, W. Semmler and K. Koch (2016) *Investing to mitigate and adapt to climate change: A framework model*. International Monetary Fund. IMF Working Papers 2016/164.

Bovenberg, A. L. and B. Heijdra (1998) Environmental tax reform and intergenerational distribution. *Journal of Public Economics* 67, 1–24.

Bovenberg, A. L. and B. Heijdra (2002) Environmental abatement and intergenerational distribution. *Environmental and Resource Economics* 23, 45–84.

Bréchet, T., N. Hritonenko and Y. Yatsenko (2013) Adaptation and mitigation in long-term climate policy. *Environmental and Resource Economics* 55, 217–243.

Carleton, T., A. Jina, M. Delgado, M. Greenstone, T. Houser, S. Hsiang, A. Hultgren, R. E. Kopp, K. E. McCusker, I. Nath, J. Rising, A. Rode, H. K. Seo, A. Viaene, J. Yuan and A. T. Zhang (2022) Valuing the global mortality consequences of climate change accounting for adaptation costs and benefits. *The Quarterly Journal of Economics* 137, 2037–2105.

Catalano, M., L. Forni and E. Pezzolla (2020a) Climate-change adaptation: the role of fiscal policy. *Resource and Energy Economics* 59, 101111.

Catalano, M., L. Forni and E. Pezzolla (2020b) Fiscal tools to reduce transition costs of climate change mitigation. Marco Fanno Working Papers 0265, Dipartimento di Scienze Economiche, Marco Fanno.

Chiroleu-Assouline, M. and M. Fodha (2006) Double dividend hypothesis, golden rule and welfare distribution. *Journal of Environmental Economics and Management* 51, 323–335.

Enríquez-de-Salamanca, A., R. Díaz-Sierra, R. M. Martín-Aranda and M. J. Santos (2017) Environmental impacts of climate change adaptation. *Environmental Impact Assessment Review* 64, 87–96.

Fodha, M. and T. Seegmuller (2012) A note on environmental policy and public debt stabilization. *Macroeconomic Dynamics* 16, 477–492.

Fodha, M. and T. Seegmuller (2014) Environmental quality, public debt and economic development. *Environmental and Resource Economics* 57, 487–504.

Fodha, M., T. Seegmuller and H. Yamagami (2018) Environmental tax reform under debt constraint. *Annals of Economics and Statistics* 129, 33–52.

Fried, S., K. Novan and W. Peterman (2018) The distributional effects of a carbon tax on current and future generations. *Review of Economic Dynamics* 30, 30–46.

Habla, W. and K. Roeder (2017) The political economy of mitigation and adaptation. *European Economic Review* 92, 239–257.

Hultgren, A., T. Carleton, M. Delgado, D. R. Gergel, M. Greenstone, T. Houser, S. Hsiang, A. Jina, R. E. Kopp, S. B. Malevich, K. McCusker, T. Mayer, I. Nath, J. Rising, A. Rode and J. Yuan (2022). Climate Change Impacts on Global Agriculture Accounting for Adaptation. Availabale at SSRN: http://dx.doi.org/10.2139/ssrn.4222020

Ingham, A., J. Ma and A. M. Ulph (2013) Can mitigation and adaptation be complements? *Climatic Change* 120, 39–53.

John, A. and R. Pecchenino (1994) An overlapping generations model of growth and the environment. *Economic Journal* 104, 1393–1410.

Jondeau, E., G. Levieuge, J. G. Sahuc and G. Vermandel (2022) Environmental Subsidies to Mitigate Transition Risk. Swiss Finance Institute Research Paper, 22-45.

Karp, L. and A. Rezai (2014) The political economy of environmental policy with overlapping generations. *International Economic Review* 55, 711–733.

Mariani, F., A. Pérez-Barahona and N. Raffin (2010) Life expectancy and the environment. *Journal of Economic Dynamics and Control* 34-4, 798–815.

Ono, T. (1996) Optimal tax schemes and the environmental externalities. *Economics Letters* 53, 283–289.

Pindyck, R. S. (2013) Climate change policy: what do the models tell us? *Journal of Economic Literature* 51, 860–872.

Pindyck, R. S. (2021) What we know and don't know about climate change, and the implications for policy. *Environmental and Energy Policy and the Economy* 2, 4–43.

Schumacher, I. (2019) Climate policy must favour mitigation over adaptation. *Environmental and Resource Economics* 74, 1519–1531.

Suzuki, S. and H. Yamagami (2024) On the effects of pessimism toward pollution-driven disasters on equity premiums. *Economic Theory Bulletin* 12, 167–181.

Cite this article: Fodha M and Yamagami H (2025). "Mitigation or adaptation to climate change? The role of fiscal policy." *Macroeconomic Dynamics* **29**(e105), 1–24. https://doi.org/10.1017/S1365100525000252