

Index

- 1-form, 36
- Abelian group, 170
- absorption in dielectrics, 25
- A.C. conductivity, 319
- action
- classical particle mechanics, 52
 - complex scalar fields, 425
 - Dirac field, 430
 - freedom to change, 64
 - non-Abelian fields, 471
 - physical meaning, 71
 - real scalar field, 416
 - Yang–Mills theory, 471
- action principle, 50
- fields, 61
 - preferred directions, 157
- adjoint field
- Dirac, 430
- adjoint group, 203
- adjoint representation, 182, 185, 472
- centre of group, 203
- advanced boundary conditions, 75
- advanced Green function, 82, 96
- Aharonov–Bohm effect, 28, 164, 264
- algebra
- group, 172
 - N -ality, 205
- Anderson–Higgs mechanism, 278
- angular momentum, 195, 303
- anisotropy, refractive index, 28
- anti-commutator Green function, 89
- anti-particles, non-relativistic limit, 340
- anti-symmetric products, reduction formulae, 505
- anti-symmetric tensors, 15, 40
- properties, 504
- anyons, 164, 304
- Baker–Campbell–Hausdorff formula, 512
- BCH formula, 512
- Bianchi identity, Maxwell field, 15, 455
- Bloch’s theorem, 212, 213
- Bode’s law, 108
- Boltzmann factor, 124
- boosts, 220
- Bose–Einstein distribution, 126
- bound states, 257
- boundary conditions, 74, 219
- advanced, 75
 - and complexity, 419, 426
 - electromagnetism, 461
 - Feynman, 75
 - on field, 57
 - Green functions, 82
 - magnetic field, 213
 - retarded, 75
 - symmetry breaking, 254
- bra-ket notation, 378
- Brownian motion, 160
- bulk viscosity, 317
- canonical hyper-surface, 64
- canonical momentum, 55
- canonical position, 283

- canonical transformations, 363
 Cartan–Weyl basis, 191
 Casimir invariant, quadratic, 184, 468
 causality, 74, 90
 Green functions, 82
 centre of group, 170, 175
 Clifford algebra, 433
 charge, 257
 generalized, 466
 Cherenkov radiation, 147
 Chern–Simons field
 action, 486
 continuity, 489
 field equations, 488, 489
 tensor density, 487
 Chern–Simons Green function, 165
 Chern–Simons theory, 164
 energy–momentum tensor, 490
 Chevalley normalization, 193
 chiral invariance, 441
 Christoffel symbol, 37
 classical particle mechanics, 50
 Clebsch–Gordon series, 232
 Clifford algebra, 431
 commutation relations, 89, 377
 angular motion, 303
 commutator Green function, 89
 commuting derivatives, 260
 compactness of group, 220
 complementary function, 138
 completeness and Poisson bracket, 368
 complex scalar, Green function solution,
 426
 complex scalar field
 action, 425
 as two-component real field, 428
 components of a vector, 38
 conductivity, 335
 configuration space, 34
 conformal energy–momentum tensor,
 301
 conformal group, 208
 gauge fields, 235
 conformal invariance, 208, 247
 congruency class, 180
 conjugacy of group elements, 180
 conjugate elements of a group, 174
 conjugate field
 Dirac, 442
 Maxwell, in ambient vacuum, 454
 Proca field, 464
 conjugate momentum, 55
 fields, 64
 scalar field, 417
 Schrödinger field, 411
 conjugate pairs, 285
 conjugate variable, 328
 connection, 187
 conservation
 electromagnetic field, 21
 law, 20, 206, 325
 of energy–momentum tensor, 295,
 322
 conserved current, 20, 325
 conserved inner product, 333
 conserved probability, 332
 constancy of c , 207
 constitutive relation, 158, 317, 318
 constrained form of field solution, 74
 constraint
 equation of motion, 74
 on field, 19
 of motion, 54
 contact potential, 65
 continuity
 complex scalar, 425
 Dirac field, 441
 of the field, 55
 Maxwell field, 453
 in dielectric, 461
 real scalar field, 417
 continuity condition, 20
 continuum hypothesis, 5
 contour integration, 92
 contravariant components of a vector, 39
 coordinate invariance, 207
 coordinate transformations, 207
 coordinate translations, generator, 366
 correlation length, 117
 correlations, 4
 cosets, 173
 group of, 175

- cosmological constant, 65
Coulomb gauge, 457
covariant components of a vector, 39
covariant derivative, 261
 physical assumption, 262
coverings of groups, 180
CPT theorem, 263
curl, 40
currents, 20, 77
 classical particle mechanics, 329
 conserved, 325
 electric, 330
curvature, 492, 495
cyclotron radiation, 148
- damping in dielectrics, 25
D.C. conductivity, 319
de Broglie relation, 376
de Broglie waves, 42, 375
degeneracy, 253
delta function, 501
derivatives
 commuting, 260
 coordinate symmetry, 258
 covariant, 262
 form invariance, 261
 Hermitian property, 258
 substantive, 246
 symmetry in action, 66
 time-reversal invariance, 66
de Moivre's theorem, 507
differential Hamiltonian operator, 284
diffusion, 162
dimension of a group, 170, 184, 199
dimension of algebra representation, 184, 468
dimension of group, 468
dimension of representations, 199
dimensions, engineering, 400
dipole moment, 24
Dirac delta function, 501
Dirac field
 action, 430
 adjoint, 430
 conjugate, 442
 continuity, 441
- energy–momentum tensor, 448
Dirac Green function, 106, 447
 non-relativistic limit, 348
Dirac notation, 378
disconnected sub-groups, 224
dispersion relation, 19, 73
dissipation, 59
Doppler effect, 25
dual vector, 36
duality, 336
dynamical symmetry, 262
dynamical variables and groups, 172
Dynkin index, 184, 468
- effective action
 as a function of source, 115
 $W[J]$, 115
effective field theory, 4, 5, 77
 dielectrics, 452
effective mechanical force, 6
Einstein and gravity, 491
Einstein's A, B coefficients, 124
Einstein's equations, 496
Einstein's relation for mobility, 163
Einstein's summation convention, 404
electric components of a tensor, 229
electric current, 330
electric susceptibility, 25
electromagnetic waves, 18
electromagnetism and Lorentz group, 230
energy constraint, relativistic, 416
energy density of E.M. field, 21
energy density of Maxwell field, 460
energy–momentum tensor, 22, 496
 Chern–Simons theory, 490
 complex scalar field, 427
 conformally improved, 301
 conservation of, 295, 322
 Dirac, 448
 gauge theories, 296
 isotropic fluid, 314
 Maxwell field, 458
 in particle mechanics, 293
 scalar field, 293, 422
 symmetry, 296

- energy-momentum tensor (*cont.*)
 $T_{\mu\nu}$, 298
 $\theta_{\mu\nu}$, 289
 trace, 300
 energy-momentum waves, 42
 ensemble, 372
 equation of motion, 54
 equations
 Einstein, 496
 Langevin, 160
 Lippmann–Schwinger, 413
 London, 335
 Navier–Stokes, 314
 equilibrium, statistical, 114
 ergodic hypothesis, 372
 Euclidean Green function, 113
 Euclidean spacetime, 47
 Euler–Lagrange equations, derived, 54
 expectation values
 statistical, 373
 exponentiation, 179
 factor group, 175
 Faraday effect, 139
 Feynman boundary conditions, 75
 Feynman Green function, 82, 96
 Feynman proof of Maxwell equations, 66
 field, 5
 continuity condition, 57
 as dynamical potential, 72
 free, 73
 as a potential/source, 7
 as wavefunction, 72
 field strength tensor $F_{\mu\nu}$, 14
 field theory, as quantum mechanics, 87
 finite group transformations, 178
 first quantization, 3
 flavour, 467
 fluctuation force, 161
 fluctuations, 121
 and Green functions, 114
 fluids as fields, 314
 flux tubes, 164
 force, 6, 310
 mechanical, 6
 form-invariant derivative, 261
 Fourier transform of Green function, 83
 free fields, 73
 friction, 59
 functional differentiation, 60
 functional integral, 394
 fundamental representation, 182
 Galilean invariance, 243
 gamma-matrices (γ -matrices) in $n + 1$ dimensions, 431
 gauge coupling, 11
 gauge fixing, 108, 456
 gauge invariance, 17, 264
 $F_{\mu\nu}$ in non-Abelian fields, 470
 gauge parameter α , 108
 gauge symmetry, 11
 gauge transformation
 global, 332
 non-Abelian, 470
 rigid, 332
 gauge-invariant variations, 70
 Gauss' theorem, 511
 Gell-Mann matrices, 476
 generalized forces, 310
 generating functional, 121
 generator, 177
 algebra, 183
 conjugate variables, 388
 coordinate translations, 366
 derivation from action principle, 366
 time translations, 365
 geodesic equation, 58, 493
 global gauge transformation, 332
 global symmetries, 256
 Goldstone bosons, 274, 281
 Grassman variables, 263
 gravitational field, action, 496
 Green function
 advanced, 96
 anti-commutator, 89
 boundary conditions, 82
 causality, 90
 Chern–Simons, 165
 commutator, 89
 Dirac field, 106, 447

- Green function (*cont.*)
Euclidean, 113
 $n = 2$, 102
Feynman, 96
 $n = 3$, 101
as matrix, 98
non-relativistic limit, 345
as operator inverse, 81
and Poisson bracket, 370
retarded, 91
 $n = 3$, 100
scalar field, 420
Schrödinger field, 104
symmetry problems, 107
Wightman functions, 91
Green functions, 79
relationships between, 421
group
Lie, 183
 $U(N)$, 199
 $Z(N)$, 201
group algebra, 172
group axioms, 170
group centre, 175
and cover group, 203
group generators, normalization, 193
group of cosets, 175
group representation, 177
group theory, 169
group vectors, 171
groups, 206
- Hamilton–Jacobi equation, 367
Hamilton's equations, 359
Hamilton's operator equations of motion, 381
Hamiltonian, 22
formulation, 358
Hamiltonian density
Maxwell field, 454, 460
scalar field, 417
heat bath, 123
Heaviside step function, 503
Heisenberg picture, 377
helicity, 240
massless fields, 242
- Helmholtz relations, 66
Hermitian derivatives, 258
hidden momentum, 285
higher derivative theories, 68
Hilbert space, 185
holonomic, 7
- ignorable coordinates, 181, 360
index notation, 35
infinitesimal group transformations, 178
inner product defined from conserved
current, 333
interactions, 79, 310
invariant length, relativity, 40
invariant probability, 419
invariant sub-group, 174
invariant sub-space, 181
invariants
boundary conditions, 75
E.M. field, 16
inverse of ∇^2 in two dimensions, 102
inverse problem, 79
irreversibility, 67
isospin, 467
- Jacobi identity, 183
Maxwell field, 15, 455
Jacobi identity and Poisson bracket, 368
- Kaluza–Klein theory, 496
kernel, 81
kinetic energy, 50, 51
KMS relation, non-relativistic limit, 354
Kramers–Kronig relation, 25, 108
- Lagrange multiplier, 108
Lagrangian, 51
and $T - V$, 62
Landau damping, 150
Langevin's equation, 59, 160
large transformations, 179, 209
large-scale approximation, 5
laser cooling, 153
length of a vector, 38
Levi-Cevita tensor, 15, 40
Lie algebra, 368, 466
Lie groups, 183, 466

- limits
 - of action principle, 66
 - defined, 409
- line element, 38
 - Minkowski spacetime, 207
- linear derivatives, 71
- linear response theory, 79, 335
- Liouville's theorem, 374
- Lippmann–Schwinger equation, 413
- little group, 241
- Liénard–Wiechert potential, 142
- local coupling to E.M. field, 11
- local symmetry, 278
- localization of massless particles, 309
- London equation, 335
- long-range order, 122, 130, 274
- long-wavelength approximation, 5
- longitudinal components, 43
- longitudinal field, 236
- Lorentz gauge, 108, 456
- Lorentz group, 12
 - electromagnetism, 230
 - irreducible representations, 231
- Lorentz invariance, 219
- magnetic components of a tensor, 229
- magnetic field, boundary conditions, 213
- magnetic susceptibility, 460
- magnetization, 460
- Markov property, 67, 172, 248
- mass shell, 73, 87, 88, 386
- matrix factorization formula, 512
- Matsubara frequencies, 123
- Maxwell field
 - action, in ambient vacuum, 452
 - ambient dielectric, 459
 - conjugate field, 454
 - continuity, 453
 - in a dielectric
 - continuity, 461
 - energy–momentum tensor, 458
- $F_{\mu\nu}$ defined, 14
- Green function solution, 455
- Hamiltonian density, 454, 460
 - in a vacuum, 452
- Maxwell Green function
 - gauge invariance, 456
- Maxwell's equations, 9
 - covariant form, 13
- measurements as interactions, 79
- mechanical force, 6
- metric tensor, Minkowski spacetime, 41
- Minkowski spacetime, 33
- mobility, 163
- momentum operator, 284, 377
- momentum space, 33
- momentum, definitions, 283
- multi-component fields, 257
- multi-valuedness, 201
- multiple coverings, 180
- Nambu, 274
- Nambu–Goldstone modes, 281
- Navier–Stokes equations, 314
- negative energy solutions, 75
 - scalar field, 418
- Newton's law, 51
- Noether's theorem, 325
- non-Abelian field, 257, 466
 - action, 471
 - gauge transformation, 470
 - strength, 470
- non-Abelian group, 170
- non-holonomic, 7
- non-integrable phase, 212
- non-relativistic limit, 88, 340
 - curvature, 357
 - Green function, 345
- observables, 376
- ODLRO, 122
- off-diagonal long-range order, 122
- Ohm's law, 127, 335
- ohmic dissipation, 318
- on shell, 73, 88
- operator action principle, 383
- operator equations of motion, 394
- operator Hamilton equations, 381
- operator variations, quantum field theory, 393
- optical activity, 139
- optical molasses, 155
- optically active electrons, 24

- order, 274
 - of a group, 170
- parity transformations, 209
 - helicity, 234
- particle energy, 283
- particle field, 24
- particle mechanics
 - classical, 50
 - Hamiltonian formulation, 358
 - Lagrangian formulation, 369
 - relativistic, 57
- particle trajectories, 283
- particular integral, 138
- path integral, 394
- Pauli–Lubanski vector, 239, 306
- PCT theorem, 263
- periodic potential, 213
- permeability of free space, 460
- permeability, non-Abelian field theory, 471
- phase invariance, 264
- phase transitions, 117, 277
- phonons, 277
- Planck scales, 400
- plane waves, 42
- Poincaré invariance, 219
- Poisson brackets, 362
 - fields, 369
 - Green functions, 370
- polarizability, 24
- polarization, 23
- position coordinate in field theory, 283
- position operator, 377
- positive definite metric, 41
- positive energy solutions, 75
- potential as a field/source, 7
- potential energy, 51
- potentials, 10, 12
- Poynting vector, 21
- pressure
 - radiation, 321
- principal value, 108
- probability
 - Dirac field, 442
 - scalar field, 419
- probability interpretation
 - quantum mechanics, 332
 - real fields, 334
- Proca field, conjugate field, 464
- propagator, 81
- proper group, 179
- proper Lorentz group, 224
- proper time, 40
- QED, 348, 450
- quantum, defined, 3
- quantum action principle, 383, 391
- quantum field theory, 277
- quantum mechanics
 - classical correspondence, 382
 - relativistic fields, 385
 - statistical interpretation, 381
- quasi-non-Abelian fields, 467
- quasi-particles, 467
- radiation
 - Cherenkov, 147
 - cyclotron, 148
 - pressure, 321
 - synchrotron, 148
- radiation from moving charge, 142
- radiation gauge, 457
- rank of a group, 170
- rate of work, E.M. field, 21
- real fields
 - become complex, 419
 - probability interpretation, 334
- real scalar field
 - action, 416
 - continuity condition, 417
- reciprocal lattice, 34
- reciprocal lattice space, 33
- reflection symmetry Z_2 , 201
- reflection, Z_2 invariance, 256
- refractive index, 26
- relativistic energy constraint, 416
- relativistic quantum mechanics, 77, 385
- renormalization group, 5
- representation
 - non-Abelian gauge field, 472
 - dimension of, 184, 468

- representation (*cont.*)
 of group, 177
 of Lie algebra, 184, 468
 representation space, 172, 184
 reservoir, thermal, 123
 restricted Lorentz group, 224
 retardation, Galilean, 245
 retarded boundary conditions, 75
 retarded Green function, 82, 91
 independent of state, 126
 reversibility, 67
 rigid gauge transformation, 332
 role of position coordinate in field theory, 283
 root vector, 192
 roots of unity, 201
 rotation eigenvalues, 195
 rotational invariance, 214
- scalar electrodynamics
 action, 278
 scalar field
 action, 416
 conjugate momentum, 417
 conserved currents, 419
 energy–momentum tensor, 422
 gauged, 278
 Green function, 420
 Hamiltonian density, 417
 scalar potential, 12
 scalar product, 40
 scale invariance, 252
 scaling, 117
 Schrödinger action, 410
 Schrödinger field
 conjugate momentum, 411
 conserved currents, 413
 differential Hamiltonian operator, 410
 energy–momentum tensor, 414
 Green function, 104
 Schrödinger picture, 377
 Schur's lemma, 170
 Schwinger action principle, 383, 391
 second quantization, 3
 semi-simple group, 206
- shear viscosity, 317
 simple group, 206
 Slater determinant, 391
 source as a field/potential, 7
 source theory, 5
 sources, 77, 310
 spacelike hyper-surface, 64
 speed of light, constancy, 207
 spin, 195, 197, 303, 305, 306
 double-valued nature, 201
 Lorentz theory background, 231
 of a tensor field, 198
 spinor electrodynamics, 348, 450
 spring model of dielectric, 24
 standing waves, 74
 state independent Green function, 126
 stationary waves, 74
 statistical expectation values, 373
 statistical mechanics, 372
 step function, 503
 Stokes' theorem, 511
 structure constants, 183, 468
 sub-groups, 170
 $SU(2)$, 476
 substantive derivative, 156, 246
 summation convention, 404
 susceptibility, 352
 Green function, 82
 Magnetic, 460
 thermal, 124
 symmetry breaking
 by boundary condition, 253
 dynamical, 281
 global, 274
 local, 278
 spontaneous, 130, 336
 symmetry, Hamiltonian view, 360
 symplectic coordinates, 362
 symplectic transformations, 360
 synchrotron radiation, 148
- tangent space, 36
 TCP theorem, 263
 thermal conductivity, 317
 thermal susceptibility, 124
 time, special role of, 358

- time-ordered products, 114
time-reversal transformation, 209
time-translation generator, 365
trace of energy–momentum tensor, 300
transformation
 of coordinates, 207
 of group vectors, 171
transformation function, 380
translation in periodic lattice, 213
transversality, 236
transverse components, 43
travelling waves, 74
triality, 205

unitarity, 88
 and macrostate, 126
unitary gauge, 280
unitary matrices, 199
units, defined, 399
universal cover group, 203

vacuum, 4
variation
 classical dynamical variables, 365
 dynamical, 68
 gauge-invariant, 70
 non-dynamical, 68
variation of an operator, 380

variational principle, 52
vector
 length of, 38
 potential, 12
 product, 40
Verdet's constant, 142, 510
vielbein, 300
virtual processes, 121
viscosity, 317
vortices, 164

wavefunction, 376
 gauge transformation, 264
wavenumber k_μ , 42
waves, electromagnetic, 18
Weyl spinors, 441
Wick rotation, 47, 48, 113
Wightman functions, 83, 88
 Green functions, 91
 $n = 3$, 101
Wilson loop, 213
world-lines, 57

Yang–Mills theory, 467
 action, 471
Zeeman effect, 139
Zitterbewegung, 351

