PAPER 17

SOME CONSIDERATIONS ON THERMAL CONDUCTION AND MAGNETIC FIELDS IN PROMINENCES

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ABSTRACT

Prominences which extend into the million degree temperature region of the corona will, in the absence of magnetic fields, be heated up to temperatures of the same order of magnitude in the course of at most a few hours. A magnetic field of reasonable magnitude inside the prominence, will, however, be sufficient to cut down thermal conduction and turbulence to such an extent that the long life of some prominences seems understandable.

1. INTRODUCTION

The temperature in prominences has been estimated by various authors [1] to lie around 4000° K. This low value indicates that the prominence in question is, practically speaking, in radiative equilibrium with the bulk of solar surface radiation. The majority of prominences extends well into the region of the corona which is credited with a temperature in the million degree range. We are hence confronted with a close juxtaposition of the matter in the temperature range a million degrees on one hand, and a few thousand degrees on the other. The drop in temperature of may be a million degrees takes place over a distance which may be less than a thousand kilometres. Let us further recall that the life period of prominences of the arched bridge type discussed at length by M. and Mme D'Azambuja [2] may run up into many months or even a year, without suffering serious deterioration until near the end of the period. The form of these filaments is that of a thin nearly vertical sheet, the thickness of which has been estimated by various investigators to range from 5000 to 10,000 km.

These facts suggest various ideas and hypotheses concerning the physical conditions maintaining long-lived prominences. Considering the admitted fact of the high thermal conductivity of the corona [3] it is puzzling to find

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lumps of cold matter like the prominences to exist there for weeks and months, as one would expect them to be broken up by coronal heating in the matter of a few minutes or even seconds.

We suggest that the solution of the puzzle lies in the proper recognition of the part played by magnetic fields in the maintenance of prominences. The magnetic field is long recognized as necessary for the understanding of *the form* of prominences, so that matter is restricted to flow along the magnetic-field lines. Such a field also suppresses turbulent convection of heat, which restricts the interchange of heat between a prominence and the corona to consist of ordinary molecular conduction. It will be shown in the following that the conductivity in the corona is so high—in the absence of magnetic fields—that prominences would not be expected to survive for more than a few minutes. For the understanding of the persistence of long-lived prominences their magnetic field is thus essential.

2. THERMAL CONDUCTIVITY IN THE CORONA

Let us first recall some elementary notions of the gas kinetic theory of conduction. The coefficient of the thermal conduction K is defined by the expression

$$\mathbf{F} = -K\nabla T \tag{1}$$

for the flux $\mathbf{F}$ of thermal energy, T being the absolute temperature. In the following we shall be interested in the case when the conduction of heat is mainly carried on by free electrons. This may be a poor approximation for the central part of a prominence, and will have to be amended in more refined calculations. For our present purpose it is, moreover, essential to keep the picture of the physical processes at work clearly in mind, and for this reason we base the considerations on the free path picture of elementary kinetic gas theory. The coefficient K is then defined by the expression

$$K = \tfrac{1}{2}kCNL, \tag{2}$$

where k is Boltzmann's constant while C, N and L represent the mean thermal velocity, mean number in unit volume, and mean free path of free electrons in the gas. For C the expression $(3kT/m)^{\frac{1}{2}}$ gives a sufficient approximation, m being the electronic mass. Assuming the gas to consist essentially of ionized hydrogen, the motion of the free electrons is mainly interfered with by the free protons, the number of which is approximately equal to N. The mean free path L of an electron is then defined by $1/N\pi a^2$ where a is the effective 'radius of collision' of a proton-electron encounter. Simple considerations, based on the Rutherford scattering formula leads

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to the expression $2e^2/3kT$ for a, $-e$ being the electronic charge. Combining these various terms we find for K the following expression

$$K=\frac{k(3kT)^{5/2}}{8\pi m^{1/2}e^4}. \qquad (3)$$

The general statistical theory of conduction given by Chapman and Cowling [4] leads to nearly the same dependence of K on T as above, but to a slightly different numerical factor. For the following applications this difference is not essential.

To clear the picture as far as the corona is concerned we note that for the temperature $T=10^6$ °K and a coronal electronic density $N=5\times 10^8$ cm^{-3} we find $C=6800$ km/sec, and $L=5200$ km. The average time spent on a free path in the corona becomes $L/C\approx 1$ sec.

In the corona proper the temperature is so high that the above expression for the conductivity should be a fair approximation. As the temperature goes down and approaches that of a prominence, the approximation becomes less good, partly because conduction by heavy particles will gradually have some effect, and partly because the free path will be cut down by collisions with neutral particles in addition to the influence of protons. We do not think it necessary, however, to amplify the theory in this direction for the purpose we have in mind. It seems probable that the speed of the heat wave is likely to be determined by values of the conductivity in the high temperature region rather than in the region of low temperatures, so that the exact expression of K for low values of T matters little.

When a magnetic field of strength H is present [5] the transverse thermal conductivity of completely ionized hydrogen will be cut down by a factor $(1+\omega^2\tau^2)^{-1}$, where $\omega=eH/mc$ is the magnetic gyro-frequency and τ the time spent by an electron on a free path $\tau=L/C$.

The realm of validity of the magnetic factor $(1+\omega^2\tau^2)^{-1}$ appears to be obscure. But it seems to be the best guide we have at the present time.

In many conduction problems it is possible to regard K either as a constant, or as a slowly varying function of space and time. In the present case conditions are different, in that the value of K as given by (3) varies by a factor a million when T drops from the coronal temperature 10^6 °K to a prominence temperature of say 4×10^3 °K. Such a variation must produce a tendency to build up steep temperature gradients in the surface region of the prominence. The consequent expansion of the heated region will, however, tend to reduce the rate of advance of the heat wave. Taken in full generality the problem is thus a very complicated one. But the fact that long-lived prominences seem to persist for a long time with only small changes of form makes it reasonable to use a static working model for our theoretical considerations.

The equation of heat transfer is then given by the simple equation

$$C_v \frac{\partial T}{\partial t} = \text{div}\,(K \nabla T), \tag{4}$$

where C_v is the specific heat per unit volume.

3. SOLUTION OF (4) WHEN K/C_v = CONSTANT

Eq. (4) has a simple solution for the case when K/C_v is constant. The fact that K varies by a factor about a million from the centre of the prominence to the corona then demands that C_v also varies by the same large factor. We imagine this to mean that the density increases outward with this factor, which of course is very far from the truth. However, the influence of this rapid increase of C_v with increasing temperature means a considerable slowing down of the progress of the heat wave into the prominence. As we are mainly interested in deriving upper limits for the life time of a prominence, the solution of this somewhat artificial problem will serve our end.

First of all we change the dependent variable from T to

$$U = \int K\,dT, \tag{5}$$

which, when introduced into (4) gives

$$\frac{\partial U}{\partial t} = \kappa \nabla^2 U; \quad \kappa = K/C_v; \quad C_v = 3Nk. \tag{6}$$

By our assumption κ is constant, Eq. (6) becomes a simple linear equation in U which may be solved in the conventional way by writing

$$U = \sum_s U_s(x, y, z)\, e^{-\lambda_s t}, \tag{7}$$

the functions U_s being functions of the space co-ordinates x, y, z only, and the λ_s being an infinite set of constants, determining the time scale of the problem. When (7) is introduced into (6) it follows that each U_s must satisfy the equation

$$\nabla^2 U_s + (\lambda_s/\kappa)\, U_s = 0. \tag{8}$$

It matters very little what kind of a model is adopted for our prominence, as it will be easily realized that models as different as an infinite plane

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sheet of thickness D or an infinite cylinder of diameter D will give comparable results for the time scale. In the first case the solution of (8) is

$$U_s = A_s \cos\,(\lambda_s/\kappa)^{1/2}\,x, \tag{9}$$

where A_s is a constant and x is the linear coordinate normal to the sheet. In the second case

$$U_s = B_s J_0\,[(\lambda_s/\kappa)^{1/2} r], \tag{10}$$

where B_s is a constant and r the distance from the axis of the cylinder, while J_0 is a Bessel function of zero order. These functions are closely related to the circular ones, and their zeros tend asymptotically to those of a cosine for increasing values of the argument.

As suitable boundary values we may demand that at the time $t=0$ the temperature, and hence also U, shall be constant through the prominence, and rise abruptly to the coronal value at the boundary ($r=x=D/2$). Adhering strictly to these conditions would be inconvenient, as it would make it necessary to handle an infinite and slowly converging series. It is better to date ones time from an instant after the coronal heating has had time to penetrate the skin of the prominence, so that the initial temperature distribution may be represented by a few terms only of the series (7).

Suppose that we consider the simplest case when the series (7) consists of one constant and one variable term, so that the solution is (for the case of the cylinder):

$$U = U(T_c) - [U(T_c) - U(T_0)]\; J_0((\lambda_1/\kappa)^{1/2} r)\; e^{-\lambda_1 t}, \tag{11}$$

where now

$$\lambda_1 = \kappa\,.\,\xi^2 (2/D)^2, \quad \xi = 2{\cdot}40.$$

Here ξ is the first root of the Bessel function J_0. Further are T_c and T_0 the temperatures in the corona and at the centre of the prominence at time $t=0$ respectively.

4. TIME SCALE OF THE PROMINENCE

The structures recognized as prominences have temperatures low enough for the material to show spectra of hydrogen and various metals in non-ionized states. Also, when seen projected on the disk, pictures in Hα-light show the filaments as dark, though with bright borders as a normal feature. This indicates that the bulk of the prominence is at a temperature comparable to or lower than that of the solar surface. The bright borders are naturally interpreted as the result of coronal heating. At a temperature much larger than that of the solar surface, say at 10,000 °K, the emissivity of the prominence material would be expected to exceed that

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of the solar surface considerably, and make the prominence bright all over. To be on the safe side we may increase the limiting value to 20,000 °K, and state that a prominence heated at the centre to such a temperature is to be considered as invisible in the light of Hα. When the corresponding value of U is introduced on the left-hand side (11) and r is put equal to zero, this expression becomes an equation for the time $t=\theta$ during which a non-magnetic prominence embedded in the corona is likely to be visible.

For the case in hand, when K is proportional to $T^{5/2}$ and U, consequently, proportional to $T^{7/2}$, (11) assumes the form

$$T^{7/2} = T_c^{7/2} - (T_c^{7/2} - T_0^{7/2})\, e^{-\lambda_1 \theta},$$

or solved with respect to $e^{-\lambda_1\theta}$:

$$e^{-\lambda_1 \theta} = \frac{T_c^{7/2} - T^{7/2}}{T_c^{7/2} - T_0^{7/2}} \approx 1 - (T/T_c)^{7/2}.$$

This means that to a sufficient approximation

$$\lambda_1 \theta = (T/T_c)^{7/2} = \left(\frac{20{,}000}{1{,}000{,}000}\right)^{7/2} \approx 10^{-6}. \qquad (12)$$

The above method of deriving an expression for θ is easily generalized to the case when the series for U contains any number of terms, provided the exponential factors may legitimately be linearized. Using the data suggested by Öhman [6] $T_0 = 4000$ °K and $N = 3{\cdot}2 \times 10^{10}$ cm^{-3}, and assuming the diameter of the cylinder to be $D = 10{,}000$ km, we find λ_1^{-1} equal to $4{\cdot}6 \times 10^7$ sec, and by (12): $\theta = 46$ sec.

We do not mean to stress the meaning of this figure beyond the inference that by its method of derivation it is in the nature of an upper limit, and that it would have to be extended by a factor of at least 10,000 to provide a semblance of an explanation for the persistence of prominences.

We prefer to think that this extension is provided by the effect of the magnetic field of the prominence. To increase the time scale by a factor 10,000 it is sufficient to have a magnetic field present that makes $\omega^2\tau^2 \approx 10^4$, or $\omega\tau \approx 100$. In the corona we found τ to be of the order of 1 sec. In the surface region of the prominence where the density may be a hundred times higher, τ may be correspondingly less. But in any case only a gyro-frequency of the order $10^4 - 10^5$ has to be considered and this makes the corresponding magnetic field a small fraction of a gauss.

Suppose thus that the magnetic field prevents heat from leaking into the prominence from the corona. The temperature of the former is then determined on one hand by the balance of the energy absorbed from the light radiation of the sun, and the loss by light emission on the other. That prominences depend on ordinary solar light radiation for their thermal state is at least also suggested by the fact that they seem to be easily disrupted by adjacent flares.

One more remark: We have assumed that the unknown heating mechanism which is responsible for the high coronal temperature does not operate inside a prominence of the long-lived type we are considering here.

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Discussion

Cowling: One might consider the reverse problem. A prominence means condensing of coronal material. If a magnetic field is so successful in keeping material in, may it not be difficult to secure the condensation of coronal material into a prominence?

Jensen: The effect of motions has not been taken into account in this investigation, so I do not think that I can answer your question.

Parker: I have seen calculations which seemed to indicate that the enhanced radiation from a prominence due to its low temperature and high density, was sufficient to maintain the low temperature of the prominence immersed in the hot coronal gas without requiring any inhibiting magnetic fields.

Öhman: In my opinion a condensation of prominences from the hot corona gas is perhaps not so typical as we may expect, because coronal prominences do not show characteristics of high temperature when we first see them. They may start from low temperature objects instead. This would help us in overcoming this difficulty.

Gold: The condensation of material into the prominence requires one to suppose that the associated heat transport is radiated away. The luminosity which is observed is perfectly adequate to account for such a cooling.

Piddington: Dr Jensen has shown that the inhibition of thermal conduction across magnetic lines of force may be an important factor in the maintenance of solar prominences. In the corresponding case of electric fields and currents the Hall current plays a major part in causing current to flow across the electric field. This current may, in certain circumstances cause space-charge built up with a resultant potential electric field at right angles to the original field. Finally, this field causes Hall current in the direction of the original field and so apparently causes a large increase in the 'direct' conductivity (this is the σ_3 discussed earlier).

A similar effect should occur in the case of thermal conduction and even if it is on a much smaller scale may have to be considered in connexion with the energy balance in prominences of certain shapes.

Alfvén: This will mean that the thermal cross conductivity has the same position as the electric cross conductivity; and if one goes a little deeper into it, one should perhaps not speak of it at all!

Jensen: In the presented paper it is assumed that thermal conductivity is cut down by the magnetic field by the same factor as the electric 'cross conductivity'. I think this assumption is reasonable, even if no rigorous theory on thermal conduction in a magnetic field has as yet been established.