ON ASSESSING THE EFFICIENCY OF SINGLE-ROOM PROVISION IN HOSPITAL WARDS

By NORMAN T. J. BAILEY

Department of Human Ecology, University of Cambridge

1. INTRODUCTION

In many hospital wards it is usual to have a certain proportion of the beds separately accommodated in single-bed rooms. Current practice shows a great variety not only in the availability of separate accommodation but also in the use made of such a facility. A clear understanding of both the demand and the way in which it can be met is of great importance in the theory and practice of ward design. This problem has been discussed in a recent paper by J. W. D. Goodall (1951), who, using purely medical criteria for the need for separate accommodation, considered that present practice was on the whole largely inadequate. He found it convenient to divide patients into two groups. First, those who have to be kept under close observation, such as dying cases, infants under 2 years, cases of post-traumatic psychosis, patients requiring special treatment, and so on. These are called type A patients, and their beds must be situated for easy observation and attention. Secondly, there are those who need to be separated but who do not require constant nursing supervision, such as patients who are liable to infect others or who are peculiarly susceptible to infection, burns cases and physically unpleasant patients, etc. These are of type B and need not be placed in such immediately accessible parts of the ward. Goodall obtained data from many different hospital wards showing the day-to-day demand for single rooms over, in most cases, at least a month. Examination of these original records showed, as might be expected, not only marked average differences between specialities but also considerable daily fluctuations in demand. Because of these daily fluctuations it became clear that a simple estimate of the *average* requirement would not necessarily give a proper indication of the number of single rooms that should be provided. The present writer was asked to undertake a statistical treatment of the problem, the results of which were quoted by Goodall in his article. It proved possible to employ a simple mathematical model to evaluate (1) the extent to which the demand for single rooms could be met with varying degrees of separate accommodation—the 'efficiency of provision', and (2) the extent to which the single rooms actually provided would be occupied for the proper purpose—the 'efficiency of utilization'. The former can be made very high only at the expense of the latter, and vice versa; for if enough single rooms are provided to cope with nearly all requirements, even on 'peak' days, many of them will not be used for their proper purpose on other days. If a satisfactory balance is to be struck between these conflicting factors, namely, the purely medical needs and the necessity for economizing space and finance, then hospital planners must try to choose an optimum number of single rooms, giving as high an efficiency of provision as possible without allowing the efficiency of utilization to drop too low. Tables

https://doi.org/10.1017/S0022172400066766 Published online by Cambridge University Press

giving these efficiencies for varying amounts of separate accommodation are thus quite essential to making an adequate decision in the matter. It will in general be necessary to have such tables specially constructed for the particular speciality and ward size under consideration, though if very extensive work of this kind were contemplated, it might be worth while computing a series of tables to cover the main cases likely to arise in practice. The following section describes the mathematical model used and shows how the appropriate tables can be obtained for application in specific instances. The final section gives a worked example, based on part of Goodall's data, and comparison is made between calculated and observed efficiencies.

The simplest possible discussion merely considers the question of dealing with all types of demand for separate accommodation, without distinguishing between type A and type B patients. There is, however, some advantage to be gained by making this distinction, and the easiest procedure is to construct the required efficiency tables for type A only, as well as for the whole demand. This is because it is more important to achieve an optimum for type A than for type B. This point is further discussed by Goodall (1951), where it is concluded, for example, that in a sixteen-bed surgical ward three single rooms should be provided, two suitable for type A and one for type B patients. The efficiency tables show that this provision would deal with 76% of type A and 84% of the total requirement; while the two type A rooms would be used for type A patients for only 73% of the time.

2. MATHEMATICAL THEORY

Let us consider the problem of providing just one type of separate accommodation. The general procedure to be adopted where we wish to take account of both type A and type B demands has already been referred to at the end of the previous section, and is further illustrated in Goodall (1951).

Suppose we have a ward with n beds in all, of which a are separately accommodated in single-bed rooms. Let the chance that a patient, selected at random on any particular day, requires separate accommodation be $p = 1 - q$. Then the number r of such patients requiring to be put into a single-bed room on any given day will be distributed binomially, with probabilities given by the terms of $(q+p)^n$. Thus the probability of r is $P(r)$, where

$$P(r) = \binom{n}{r} q^{n-r} p^r. \tag{1}$$

Now if on any given day $r \leqslant a$, r units, in patient-days, of separate accommodation are provided; but if $r > a$, then only a units are provided, $(r-a)$ units of demand remaining unsatisfied. In practice the demands existing on successive days will be fairly highly correlated, as a patient who is seriously ill one day is very likely to be ill on the next day as well. Provided that there is no trend in the average demand it will be quite permissible to calculate efficiencies in the obvious way. In order to estimate the precision of the estimates, on the other hand, it would be essential to take this day-to-day correlation into account. The discussion of precision is beyond the scope of the present paper, as it necessitates a detailed

https://doi.org/10.1017/S0022172400066766 Published online by Cambridge University Press

examination of the random processes involved. It is hoped to take up this point elsewhere.

The average demand for separate accommodation is clearly np, while the average supplied is

$$S = \sum_{r=0}^{a} r\,P(r) + \sum_{r=a+1}^{n} a\,P(r), \tag{2}$$

where the first term represents the contribution of those days for which the demand can be completely satisfied, and the second term represents the occasions when the demand outstrips the supply. It follows that E_1, the efficiency of provision, is given by

$$E_1(a) = S/np. \tag{3}$$

In particular,

$$E_1(0) = 0 \quad \text{and} \quad E_1(n) = 1. \tag{4}$$

Similarly, E_2, the efficiency of utilization, is given by

$$E_2(a) = S/a = (np/a)E_1(a). \tag{5}$$

For practical purposes we have to tabulate E_1 and E_2 for different values of a, given n and p. These computations can be performed *directly* without much difficulty for n not too large, especially if we use the convenient fact that the second differences with respect to a are proportional to $P(a+1)$. Writing Δ to represent forward differences, we have

$$\Delta_a E_1(a) = E_1(a+1) - E_1(a) = \frac{1}{np}\sum_{a+1}^{n} P(r), \tag{6}$$

and

$$\Delta_a^2 E_1(a) = \Delta_a E_1(a+1) - \Delta_a E_1(a) = -\frac{1}{np}P(a+1). \tag{7}$$

In particular,

$$\Delta_a E_1(0) = \{1 - P(0)\}/np, \tag{8}$$

and

$$\Delta_a^2 E_1(0) = -\frac{1}{np}P(1). \tag{9}$$

Thus the initial values of the function and its first and second differences are given by (4), (8) and (9). If we now calculate the series of second differences given by (7), we can build up the whole function very rapidly in the usual way.

A much better procedure, especially if n is at all large, is to represent the partial sums appearing in the expression for S in terms of incomplete B-functions. If tables of the latter (Pearson, 1934) are available then $E_1(a)$ may be expressed quite simply as follows:

$$\begin{aligned} E_1(a) &= \left\{\sum_{r=0}^{a} r\binom{n}{r}p^r q^{n-r} + a\sum_{r=a+1}^{n}\binom{n}{r}p^r q^{n-r}\right\}\Big/ np \\ &= \left\{np\sum_{r=1}^{a}\binom{n-1}{r-1}p^{r-1}q^{n-r} + a\sum_{r=a+1}^{n}\binom{n}{r}p^r q^{n-r}\right\}\Big/ np \\ &= I_q(n-a, a) + a/np\,\{1 - I_q(n-a, a+1)\}, \end{aligned} \tag{10}$$

where $I_x(u, v)$ is the incomplete B-function.* $E_2(a)$ is then obtained by means of (5).

* Alternatively, the partial binomial sums may be read directly from *Tables of the Binomial Probability Distribution* (National Bureau of Standards: Washington, 1949) if these are accessible.

The value of p will of course depend on the type of patients treated in the ward under consideration, and also on whether we are differentiating between type A and type B requirements. We may expect there to be typical values of p for different specialities, though even for a given speciality there may be local differences owing to variations in medical practice or in the type of medical treatment required. The parameter p must therefore be estimated from data obtained under conditions as close as possible to those which will hold in the ward whose design is in question. Suppose that on k successive days in an appropriate ward of size n the requirements for separate accommodation are

$$r_1, r_2, \ldots, r_i, \ldots, r_k. \tag{11}$$

Then a convenient and obvious estimate to use is

$$\check{p} = \frac{1}{nk} \sum_{i=1}^{k} r_i. \tag{12}$$

The precision of this estimate is complicated by the r_i's all being correlated, but, as mentioned above, discussion of this not unimportant topic is beyond the scope of the present treatment. Suffice it to say that if k is fairly large, say of the order of 100, we may expect p to be sufficiently accurate for practical purposes. A numerical example is given in the following section.

3. WORKED EXAMPLE

In the tables and illustrations given by Goodall (1951) the values of p used were based on pooled data for each speciality taken from several different wards of varying sizes in different parts of the country. For the purpose of the present example it is convenient to restrict our attention to data from three medical wards in one hospital. These wards were all of the same size (thirty-four beds each), making it much easier to compare expected and observed efficiencies, since we can pool the data from all three wards, which gave a total of 138 days experience.

It is not intended that the following account shall be an exhaustive description of the numerical work involved; prominence is given to the salient features of the computations which, together with the full formulae set out in § 2 above, should enable easy application to any other data of the same type.

It is obvious that data of this kind can be condensed into a table showing the number of days (f) on which a given demand (r) occurred. These are set out in Table 1 below. The table is more or less self-explanatory, but it is worth mentioning that the quantities F and G are required in calculating *observed* efficiencies. F is the cumulative sum of the f's, starting from the highest r; while G is the cumulative sum of the products rf, starting with the lowest r. Now, with $n=34$ and $k=138$, we have, from (12), the estimate

$$p = 1405/4692 = 0{\cdot}29945.$$

It is convenient to work with $p=0{\cdot}3$ to save extra arithmetic. We require the terms of the binomial expansion of $(0{\cdot}7+0{\cdot}3)^{34}$, starting with $(0{\cdot}7)^{34}=5{\cdot}411696\times10^{-6}$, etc. Then in accordance with the discussion in the preceding section we can build up Table 2, of which only the first four lines are actually given. The values of E_1 and E_2 for $a=6$ to 12 are of greatest interest and are reproduced in Table 3. The above computations are somewhat laborious for n as large as 34, and it is much quicker to use (10) for any value of n, provided that tables of the incomplete B-function are accessible. Thus, for $a=10$, we have

$$E_1(10)=I_{0\cdot7}(24, 10)+\frac{10}{10\cdot2}\{1-I_{0\cdot7}(24, 11)\}=0\cdot886553.$$

If we were simply calculating efficiencies, on the basis of $p=0\cdot3$, for medical wards with thirty-four beds altogether, then one of the two procedures just outlined would complete the work. However, there is considerable interest in seeing how well these efficiencies agree with those actually observed. The observed efficiencies E_1' and E_2' are obtained from Table 1 using the quantities F and G. The actual formulae to use are easily seen to be

$$\left.\begin{aligned}E_1'(a)&=\{G(a)+aF(a+1)\}/G_0,\\ E_2'(a)&=\{G(a)+aF(a+1)\}/ak,\end{aligned}\right\}\tag{13}$$

Table 1. *Data from three medical wards, each with thirty-four beds, for a total of* 138 *days*

Demand (r)	No. of days (f)	rf	$F(r)$	$G(r)$
<5	0	0	138	0
5	4	20	138	20
6	6	36	134	56
7	1	7	128	63
8	17	136	127	199
9	20	180	110	379
10	29	290	90	669
11	18	198	61	867
12	29	348	43	1215
13	8	104	14	1319
14	4	56	6	1375
15	2	30	2	1405
>15	0	0	0	1405

Table 2.

a	$npE_1(a)$	$np\Delta_a E_1(a)$	$np\Delta_a^2 E_1(a)$	$E_1(a)$	$E_2(a)$
0	0·000 000	0·999 995	−0·000 079	0·000	1·000
1	0·999 995	0·999 916	−0·000 558	0·098	1·000
2	1·999 911	0·999 358	−0·002 549	0·196	1·000
3	2·999 269	0·996 809	−0·008 467	0·294	1·000
...	...	...	...	...	...

Table 3. *Observed and calculated efficiencies with* $p=0\cdot3$ *and* $n=34$

	Observed		Calculated	
a	E_1'	E_2'	E_1	E_2
6	0·586	0·995	0·583	0·992
7	0·678	0·985	0·674	0·982
8	0·768	0·977	0·757	0·965
9	0·846	0·957	0·828	0·939
10	0·910	0·927	0·887	0·904
11	0·954	0·883	0·930	0·863
12	0·984	0·835	0·960	0·816

where G_0 is the *total* actual demand for single rooms, and where a is a particular value of r. With $a = 10$, we have from Table 1: $G(10) = 669$ and $F(11) = 61$. Thus if we had provided 10 single rooms we should have been able to satisfy all the 669 patient-days of demand resulting from occasions when the daily demand was 10 or less; while on 61 days the demand would have been more than 10, but only 10 patients could have been separately accommodated on each of these occasions. Thus

$$E_1'(10) = (669 + 10 \times 61)/1405 = 0 \cdot 910,$$
$$E_2'(10) = (669 + 10 \times 61)/10 \times 138 = 0 \cdot 927.$$

A complete set of observed and calculated efficiencies for $a = 6$ to 12 in medical wards of size 34 is shown in Table 3 to three decimal places. Comparing the observed series of efficiencies with the calculated values we see that the absolute differences are nowhere greater than 2·4 %. Until studies on precision have been completed the true significance of these differences must remain in some doubt, though it appears that the agreement between theory and observation is sufficiently close to warrant confidence in the mathematical model employed.

I am indebted to the Research Team of the Investigation into the Functions and Design of Hospitals, sponsored by the Nuffield Provincial Hospitals Trust, for drawing my attention to this problem and for many helpful discussions on the subject.

REFERENCES

GOODALL, J. W. D. (1951). Single rooms in hospital: estimate of the medical need. *Lancet*, 1, 1063.

PEARSON, KARL (1934). *Tables of the Incomplete B-Function*. London: Biometrika.

(*MS. received for publication* 29. VI. 51.)