

SOME SUMMARY REMARKS

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I. THE HISTORICAL BACKGROUND: DEFINITION OF THE PROBLEM

Of the classical problems of physics there is one which might be picked out as simplest to state but slowest to approach solution. "What is the fate of a self-gravitating system of point masses interacting according to Newton's laws?" The frustration felt by theoreticians, who, despite the accumulation of more than 300 years of mathematical knowledge and numerical technology, have not solved the problem, even to understanding the qualitative late evolution of such systems, is compounded by the existence of observations known since Messier's time of over 100 globular star clusters. These are real and ancient systems, well approximating the idealized problem; astronomical study of their properties should have led us to an understanding of the physical situation. Galactic clusters containing smaller numbers of stars also exist in abundance. However, most of these are much younger systems and they are also subject to a variety of additional complex processes, not important in the large N , globular cluster, systems. Because of this complexity they are less well understood than globulars and will not be covered in this theoretically oriented review.

In the last decade and especially in the last few years progress in understanding large N systems on both observational and theoretical fronts has been rapid. New findings concerning this old question have accumulated from studies by diverse groups working in various parts of the world. Thus, this IAU Symposium occurs at an extremely propitious moment. Personally, this meeting has been more valuable to me scientifically than any other I can recall. It is one of the happy instances when the whole is greater than the sum of the parts. Pieces of the puzzle put together by many of the lecturers have combined to generate a clear, if not definitive, conception of the evolution of stellar clusters.

As early as 1930 Heckman and Seidentopf had argued that clusters should tend to establish an isothermal equilibrium with a Maxwellian distribution of velocities, and that the appropriate time to approach this state was, for many clusters, comparable with cosmic timescales. They further realized that the clusters would be paradoxically unable

to reach a true equilibrium since the only possible stationary state was that of an isothermal (Emden) sphere infinite in mass and radius. They exposed the basic problem but did not attempt to solve it.

Ambartsumian (1938) and Spitzer (1940) soon showed the way that evolution must proceed. Both pointed out that the average escape velocity for a cluster star was only twice the rms random velocity, so that each relaxation time, as gravitational interactions tended to set up a Maxwellian velocity distribution, of order 1% of the cluster would be given energy sufficient to escape. Thus the cluster would evaporate. But now a new paradox presented itself. Since the escaping stars have zero or positive energy and the cluster as a whole is gravitationally bound (negative total energy), it is impossible for the cluster to evaporate totally. Furthermore, Spitzer noted, the evaporation once started "proceeds at a continually accelerating rate as the cluster contracts", and would formally lead to total evaporation in a finite time. He then proposed that evaporation "would presumably proceed until one of two alternatives occurred. Collisions between stars can become important. Or as the cluster continued to lose stars, the remaining ones might conceivably find themselves in periodic orbits".

We will return to these two possibilities. But it was clear from astronomical observations that neither of them, nor in fact the whole evaporation scenario, was a complete description of the problem. Many astronomers have pointed out that a significant fraction of all globular clusters had central relaxation times less than or comparable to 10^8 years (e.g., Ostriker, Spitzer and Chevalier, 1975; Lightman, Press and Odenwald, 1978). Does this mean that these clusters will soon totally evaporate? Two objections to this hypothesis immediately come to mind. First, it would give to our epoch a preferred status since it would imply that many of the globular clusters waited 1.5×10^{10} years until just now, to self-destruct. It could be shown that this is not statistically probable (cf. Lightman, 1982). Also, the relaxation times at the half-mass point are much longer in these same clusters and it is difficult to see why anything occurring in the inner 1% of mass fraction will very much alter the slowly changing equilibrium state of the bulk of the cluster.

But the problem is more complex. Antonov (1962) and Lynden-Bell and Wood (1968) provided a theoretical explanation for numerical findings of Henon and Spitzer in the 1960s. To wit: the central regions of concentrated clusters will tend to collapse due to conduction of heat outwards even in the absence of evaporation of stars to infinity. Both this phenomenon and evaporation are driven by the "negative specific heat" of self-gravitating systems and both proceed on the two-body time scale, but they are distinct. The "gravitational collapse", unlike evaporation, proceeds at fixed total mass and energy. Recent investigations by (cf. Spitzer, 1975, for a review of earlier work) Cohn (1979, 1980), Heggie (1979), Lynden-Bell and Eggleton (1980), Marchant and Shapiro (1980), and others established the nature of the asymptotic approach to the singular state but gave no insight into how a system could pass through this trial or what, if any, recovery was possible for its post-collapse existence. From the observations previously mentioned it seems that many real globular clusters must have in

fact passed through this phase so the question was of more than theoretical interest.

Thus the development of theory and numerical modelling provided increasingly precise detail concerning the question raised by Spitzer and Ambartsumian in their early work. What happens to these systems accelerating towards infinite central density? An important clue was provided in Henon's thesis (cf. Henon, 1961, 1965, 1975) where he showed that, if clusters had central energy sources, they could survive for very long times and would in fact gradually and slowly expand with the nature of the expansion quite insensitive to the energy source. Recent work by Inagaki and Lynden-Bell (1983), Goodman (1984) and others (cf. review by Heggie in this volume) has provided much more insight into this hypothetical post-collapse state. The problem may be seen as analogous to the one solved decades earlier in the field of stellar evolution. As the hydrogen fuel is exhausted in a stellar core the central regions become degenerate and ever more massive as more and more fuel is burned. Are they due to collapse? The answer found was, no, they have a reprieve so long as other fuel sources exist. The central cores will contract until, at last, helium-burning begins abruptly and then a new equilibrium phase is possible as an expanding red giant. In the case of a stellar cluster, what are the possible energy sources? Three mechanisms have been proposed. They are associated with

- a) A central black hole,
- b) Central binary systems,
- c) Central mass loss;

we will take each up in turn. Even if we can envision a post-collapse asymptotic expansion phase fueled by one of these energy input processes another question remains. Can a collapsing core "bounce", that is, arrest the contraction and re-expand? It was not a priori obvious that it can find a way to jump from a point on the sequence approaching collapse to one on the sequence of slowly expanding states (but see Heggie, 1984).

The possibility that stellar systems can harbor massive black holes has been attractive for some time (cf. Zeldovich and Podurets, 1965; Bahcall and Ostriker, 1975) with the current consensus of opinion that no known clusters contain black holes, but many galactic nuclei are good candidates. In this context it has been shown, first with relatively crude calculations (Shapiro, 1977) and then with more sophisticated two-dimensional Fokker-Planck numerical codes (Duncan and Shapiro, 1982) that the presence of a massive central black hole will invariably halt and reverse core collapse. Furthermore, the expansion phase satisfies the $r_{\text{core}} \propto t^{2/3}$ law pre-figured in Henon's work. The mechanism is simple. Stars are consumed by the central hole either due to tidal disruption or (for massive systems) they are "eaten whole". As we follow a prospective victim we find it diffusing downward into the cluster center and heating other stars (releasing kinetic energy) as it sinks deeper and deeper into the potential well. When, at last, it is consumed by the central hole, its kinetic energy and gravitational binding energy both disappear from the accounting, but, since its mass is added to that of the central hole, the other stars do not notice the event. Thus, for all stars except the victim, the result is a

For the sake of completeness, a number of recently-discovered objects are included here as possible to certain globular clusters. These include: AM 1 = E 1 = ESO 201-SC 10 (Holmberg *et al.* 1975; Lauberts 1976; Cannon, Hawarden, and Tritton 1978; Madore and Arp 1979), Eridanus = ESO 551-SC 01 (Cesarsky *et al.* 1977; Lauberts *et al.* 1981b), Reticulum = Sé 40/3 = ESO 118-G 31 (Sérsic 1974; Holmberg *et al.* 1975), AM 2 = ESO 368-SC 07 (Holmberg *et al.* 1978b; Madore and Arp 1979), E 3 = ESO 037-SC 01 (Lauberts 1976; Holmberg *et al.* 1978a; Cannon, Hawarden and Tritton 1978), ESO 093-SC?08 (Holmberg *et al.* 1977), AM 4 (Madore and Arp 1982), BH 176 = ESO 224-SC 08 (van den Bergh and Hagen 1975; Holmberg *et al.* 1977; Cannon, Hawarden and Tritton 1978), ESO 452-SC 11 (Lauberts *et al.* 1981a), TJ 5 (Terzan and Ju 1980), TJ 16 (Terzan, Bernard, and Ju 1978b; Terzan and Bernard 1978; Terzan and Ju 1980), TJ 15 (Terzan and Ju 1980), TJ 17 (Terzan, Bernard, and Ju 1978a; Terzan and Bernard 1978; Terzan and Ju 1980), Grindlay 1 (Grindlay and Hertz 1981), Liller 1 (Liller 1976a,b), TJ 23 (Terzan and Ju 1980), UKS 1 (Malkan, Kleinmann and Apt 1980), and Kodaira 1 (Kodaira 1983). UKS 2 = ESO 166-SC 12 = BH 66? (van den Bergh and Hagen 1975; Holmberg *et al.* 1977; Malkan 1981) was discussed by Malkan (1981) as a globular cluster, and is included here, although in the opinion of Holmberg *et al.* (1977) and this author, it appears to be an open cluster. In addition to these objects, several previously known objects have been added to the list: Ruprecht 106 = ESO 218-SC 10 (Ruprecht 1959; Holmberg *et al.* 1977), Terzan 3 = ESO 390-SC 06 (Terzan 1968; Holmberg *et al.* 1978b; Cannon, Hawarden, and Tritton 1978), Terzan 8 = ESO 398-SC 21 (Terzan 1968; Cannon, Hawarden, and Tritton 1978; Lauberts *et al.* 1981a), and Terzan 10 = ESO 521-SC 16 (Terzan 1968; Lauberts *et al.* 1981a). In this author's opinion, only the last of these four objects remains in doubt as a true globular cluster.

The Reticulum cluster noted above, NGC 1466, and NGC 1841 are sometimes discussed as members of the Large Magellanic Cloud (LMC) complex. Although they share a very similar apparent distance modulus with the LMC, they have been included here among the galactic globular clusters, NGC 1466 because of the large velocity difference from the LMC found by Cowley and Hartwick (1981) (although Freeman, Illingworth and Oemler [1983] find a much smaller difference), and Reticulum and NGC 1841 because, at angular distances of more than 10° from the center of the LMC, they can scarcely now be bound to the LMC, even though they may well share a common origin with it.

In the following section, the methods are discussed by which the observational data were obtained which form the basis of this study. These data are catalogued in Table I. The basic assumptions employed in deriving the individual cluster structure parameters are discussed in the subsequent section, and these parameters are listed in Table II.

is every reason to believe that our growing understanding of the collapse, rebound and asymptotic expansion phases is not an artifact of one particular, and perhaps flawed manner of approximating the physics of this old and for so long untractable N-body problem.

II. CONFERENCE PAPERS

My review is not intended to be complete nor even to identify the most significant contributions. Rather, I will focus on those papers which, it seemed to me, contributed most to the central question defined in my opening paragraph. [References given without dates are to papers presented at the conference and included in this volume.]

Papers by Aarseth, Lightman and Jernigan showed that significant progress is being made on the computational front. My own perhaps partial view is that these techniques (which accurately treat small N systems) will be needed to understand galactic clusters, but that globular clusters and galactic nuclei can be efficiently modeled with one- or two-dimensional Fokker-Planck codes (cf. papers by Cohn and Shapiro) which can be enhanced to include large energy changes in close encounters (cf. Goodman) and even binary formation (cf. Ostriker; Cohn, Goodman and Hut). Spitzer and Heggie's reviews showed the understanding we have reached on pre- and post-core collapse behavior although the details of the post-collapse evolution are not yet agreed upon by all participants (cf. Sugimoto and Bettwieser).

To me the most exciting discussions were those which focused attention on close interactions amongst stars showing in detail how processes involving tidal capture binaries (cf. Ozernoy, Ostriker), central black holes (Shapiro), binaries made by three-body processes (Cohn, Lightman) would enable clusters to make the transition between contraction and re-expansion. Mass loss from stellar evolution can also be important. Most impressive were the calculations of Stodolkiewicz which, using rather minimal computing power, combined all of these processes and others of importance such as the galactic tidal field to produce some fascinating results. To summarize, it is now clear that evolving clusters can, by various processes, pass through the collapse phase into a slowly expanding quasi-equilibrium state.

My own view (cf. Ostriker, this conference) is that, of the various possible processes, those which follow from the two-body tidal capture (Fabian, Pringle and Rees, 1975; Press and Teukolsky, 1977) are dominant. These lead either to hard binary formation or to stellar fusion but in either case result in mass loss from the cluster center at a rate determined by the two-body capture cross section. Such processes lead to predictions that in the condensed clusters (cf. Djorgovski, Penner and King) the central regions will contain hundreds of cataclysmic variables, WUrsae Majoris systems or rapidly rotating fused stars of $\sim 1.4 M_{\odot}$. It is comforting for theoretical interpretations which make heavy use of the tidal capture process that Grindlay finds the distribution of X-ray luminosities consistent with that expected from neutron star-main sequence and degenerate dwarf-main sequence binaries formed by tidal capture.

- Fabian, A. C., Pringle, J. E. and Rees, M. 1975, M.N.R.A.S., 172, 15P.
- Goodman, J. 1984, Ap. J., 280, 298.
- Hachisu, I., Nakada, Y., Nomoto, K. and Sugimoto, D. 1978, Prog. Theor. Phys. (Japan), 60, 393.
- Heckman, O. and Seidentopf, H. 1930, Zs. f. Ap., 1, 67.
- Heggie, D. C. 1975, M.N.R.A.S., 173, 729.
 1979, M.N.R.A.S., 188, 525.
 1984, M.N.R.A.S., 206, 179.
- Henon, M. 1961, Ann. d. Ap., 24, 369.
 1965, Ann. d. Ap., 28, 62.
- Henon, M. 1975, IAU Symposium #69, Dynamics of Stellar Systems, ed. A. Hayli (Dordrecht: Reidel) p. 133.
- Inagaki, S. and Lynden-Bell, C. 1983, M.N.R.A.S., 205, 913.
- Lightman, A. P. 1982, Ap. J. (Lett.), 263, L19.
- Lightman, A. P., Press, W. H. and Odenwald, S. F. 1978, Ap. J., 219, 629.
- Lynden-Bell, D. 1967, M.N.R.A.S., 136, 101.
- Lynden-Bell, D. and Wood, R. 1968, M.N.R.A.S., 138, 495.
- Marchant, A. B. and Shapiro, S. L. 1980, Ap. J., 239, 685.
- Lynden-Bell, D. and Eggleton, P. 1980, M.N.R.A.S., 191, 483.
- McMillan, S. L. W. 1983, Ph.D. Harvard University.
- Ostriker, J. P., Spitzer, L. and Chevalier, R. A. 1975, Ap. J. (Lett.), 176, L51.
- Ozernoy, L. M. and Dokuchaev, V. I. 1982, Astron. Astrophys., 111, .1.
- Press, W. H. and Teukolsky, S. A. 1977, Ap. J., 213, 183.
- Shapiro, S. L. 1977, Ap. J., 217, 281.
- Spitzer, L. 1940, M.N.R.A.S., 100, 397.
 1975, IAU Symposium #69, Dynamics of Stellar Systems, ed. A. Hayli (Dordrecht: Reidel), p. 3.
- Spitzer, L. and Mathieu, R. D. 1980, Ap. J., 241, 618.
- Spitzer, L. and Saslaw, W. 1966, Ap. J., 143, 400.
- Stodolkiewicz, J. S. 1982, Acta Astronomica, 32, 63.
- Zeldovich, Y. A. and Podurets, M. A. 1965, Astron. Zh., 42, 963
 (English translation in Sov. Ast. A.J., 9, 742).

DISCUSSION

SPITZER: I would like to add a historical note, about the first evidence of the gravothermal instability. I think that one should pay tribute to Dick Larson's work with his gas-dynamical code, which showed the first evidence that something else was happening beyond evaporation.

OSTRIKER: My apologies (some laughter).

GOODMAN: I'd like to take issue with that (lots of laughter). Not that Dr. Larson's work was not extremely important to get this subject going, but in fact Michel Hénon in his thesis, which was published in *Annals d'Astrophysique* 24, 369 (1961), first considered the time-dependent evolution of a stellar system with a finite central density, using the Fokker-Planck equation. He could not follow the evolution for very long, on his ancient IBM machine, but his numerical calculations showed that the central density rapidly increased. From there, he jumped to the conclusion that one could only hope to find a self-similar solution for a finite-mass system in a post-collapse, singular state. Thus, I believe that his was really the first numerical, time-dependent evidence that core collapse actually occurs.

SPITZER: Thank you for pointing out this important early work by Hénon.

OSTRIKER: A comment on a topic I did not touch on: I want to point out that Haldan Cohn, and perhaps others, are unwillingly supporting gravothermal oscillations if they say that a post-core-collapse cluster can look the same as a pre-core-collapse cluster, because the equations are all reversible, and there is nothing to prevent it from collapsing again.

VAN LEEUWEN: I am here as an observer of open clusters, and you have not heard much about my field during this conference. Therefore I would like to make a few comments. First, despite all modern instrumentation, we heavily depend, and will depend in the future, on what people have done fifty or eighty years ago. We simply need the old plates in order to measure proper motions in order to identify cluster members, since without knowing which stars are members we cannot do anything like what observers of globular clusters do. That leads me to one question and one request. The question is: if anyone knows of old deep plates in the field of the Pleiades, within five degrees around the centre, I would be happy to add them to my catalogue of proper motions. The request is: if you are erecting a new telescope which is good in the photometric sense and has a reasonable field, be careful that you take first-epoch plates of open clusters. These will be the plates that people, later on, will depend heavily on in the study of the dynamics of these clusters. The old plates we have available at the moment are generally not deeper than 14th or 15th magnitude. We need old plates, and the old plates can be the present plates, for the future, exposed to 20th or 21st magnitude.

TREMAINE: One comment about the origin of globular clusters. If one ever does arrive at a theory of the origin of globular clusters, one obvious thing that has to be explained, which I don't think has been mentioned, is the following. Both in our galaxy and in other galaxies, there is a very sharp cut-off in the mass and the luminosities of the globulars, corresponding to masses of slightly over one million solar

masses. As Mike Fall and other people have mentioned, there is no known reason why a cluster of, say, three or five million solar masses could not survive for a Hubble time. Somehow one has to explain why the formation process produces this very sharp upper limit.

TOOMRE (seeing the many hands raised): Aha, theories are coming (laughter).

SPITZER: Mike, what is the explanation (more laughter)?

FALL: Martin Rees and I have recently proposed a theory for the origin of globular clusters in which roughly the observed masses and sizes arise naturally in the collapsing gas of a proto-galaxy.

SPITZER: Are there any other problems which can be solved quickly (laughter)?

COHN: I would like to amplify on the final point which Jerry Ostriker made in his very nice summation. As has been pointed out by members of the panel, globular clusters may well serve as analogs for galactic nuclei, which are necessarily more poorly observed than globulars because of their greater distance. If we can develop and verify a sound dynamical theory for clusters including all of the relevant physical effects, this should be of great aid in developing models for active galactic nuclei in which the fireworks of which Dr. Toomre is so fond are observed to occur.

WEBBINK: I think it is important over the next few years to pursue problems of the *hydrodynamics* of globular clusters: Observationally, just how little gas do clusters contain? And theoretically, how is gas so efficiently removed from clusters? In this connection, I think it is important to use the Space Telescope to attempt identifications of weak cluster X-ray sources. Some years ago (at the NATO workshop in Cambridge, England) Andy Fabian floated the idea that globular cluster X-ray sources could be single neutron stars passing through the winds of nearby giants. A very rough calculation indicates that, even appealing only to ambient gas (at the present upper limits to its density), isolated neutron stars should begin to appear near $L_x \sim 10^{30} \text{ erg s}^{-1}$. One might expect that post-core-collapse clusters would be very efficient at collecting gas and neutron stars in their cores, and so particular attention to this phenomenon would appear warranted in the case of clusters showing central cusps.

OSTRIKER: Are you referring to globular clusters collecting neutron stars from the background, from outside the clusters?

WEBBINK: No, I am referring to neutron stars formed in the clusters, from the initial mass function.

GRINDLAY: We looked at that possibility, but it was not consistent with our observations of low-luminosity X-ray sources in globular clusters.

WEBBINK: I agree.

OSTRIKER: Another point is that given the high velocities of neutron stars seen in the field around the sun, it is a bit hard to understand why there are as many in globular clusters as there are.

WEBBINK: That is true, but notwithstanding that, there seems to be an embarrassing number of neutron stars in globular clusters. The question is: are they all tied up in binaries?

SEMENZATO: I would like to mention a problem which nobody seems to have touched upon: the possibility of existence of massive halos

around globular clusters. There seems to have been some controversial evidence, recently, about their existence.

OSTRIKER: A comment on that: If I saw correctly the velocity dispersion vs. radius curves, presented on the first day of this symposium, I think there is strong evidence against massive halos. They would produce flat velocity dispersion curves, in analogy to flat rotation curves. If the velocity dispersion really significantly decreases in the outer parts, that would be a dramatic and definite test. But one would have to make a detailed comparison with Peebles' models.

BAHCALL: Perhaps Josh Grindlay could tell us about observations expected to be made by AXAF.

GRINDLAY: A few brief comments: As probably everybody here knows, the AXAF is the X-ray analog of Space Telescope, and is of course not nearly as far along but will appear one day, we all hope. If it does, and if it has the properties we hope it will have, it should allow these low-luminosity X-ray sources to be observed and located to sub-arcsecond precision right down to luminosities of order 10^{30} erg/s, well below where we think the peak is in the luminosity function. In a typical cluster one might have ten or twenty of these sources, and one can play the game of using the radial offsets once again to map out the potential within a single cluster, rather than applying a statistical analysis over a variety of different clusters, as we have been forced to do so far. Therefore, the possibilities for studying cluster dynamics that this will provide are really exciting.

AXAF will also allow to study the whole area which we did not touch on here at all, except for the comment we just heard, the question of diffuse gas in globular clusters. We did have significant evidence in a couple of clusters for hot diffuse X-ray emitting gas, in ω Cen, in 47 Tuc, and to a lesser extent in M22. This hot gas, detected by Einstein, was interpreted by us as being due to mass loss from individual cluster giants, the gas on its way out of the cluster being shock-heated running into the interstellar medium, given the large proper motion of the cluster. That process can be studied by AXAF in enormous detail, and has a whole range of exciting implications for densities of the interstellar medium and the galactic halo, and for mass loss rates and stellar evolution of cluster giants.

APPLEGATE: Will AXAF be able to see X-rays from M dwarfs in globular clusters? If so we have another method of studying the population of low mass stars in clusters.

GRINDLAY: I am glad you mentioned this third possibility of generation of X-ray sources in globular clusters. The answer is very definitely yes, one expects to detect pop. II M dwarfs.