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INJECTIVITY AND RELATED CONCEPTS IN MODULAR VARIETIES

I. Two commutator properties

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The first property occurred in the investigation of directly representable varieties, and was named *C2* by R. McKenzie, the second one is new. Our analysis is independent of injectivity. However, in the forthcoming second part of this paper we are going to prove that varieties with enough injectives satisfy both properties, and shall use intensively the results proved here.

1. Notation and quoted results

The paper is intended to be self-contained modulo the results of this section. However, we refer the reader to Freese and McKenzie [*1*] and Gumm [*4*] for the background in commutator theory, and to Grätzer [*3*] for general terminology.

Throughout the paper we assume that all algebras considered are in a fixed modular variety V. The smallest and greatest congruence of an algebra A is denoted by 0_A and 1_A, respectively, indices are often omitted. The join and meet of the congruences θ and ψ are denoted by $\theta + \psi$ and $\theta\psi$. The notation $(a, b) \in \theta$, $a \equiv b(\theta)$ and $a\theta b$ are equivalent. The smallest congruence of an algebra A containing a given

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$H \subseteq A \times A$ is denoted by $\mathrm{Cg}_{\mathbf{A}}(H)$, where A is the underlying set of $\mathbf{A}$; $\mathrm{Cg}(a, b)$ stands for $\mathrm{Cg}_{\mathbf{A}}(\{(a, b)\})$. The restriction of an $\alpha \in \mathrm{Con}\,\mathbf{A}$ to a $\mathbf{B} \leq \mathbf{A}$ is $\alpha/\mathbf{B}$. The subalgebra generated by the set H is $\langle H \rangle$. If $f : \mathbf{A} \to \mathbf{B}$ is a homomorphism, and $\mathbf{C} \leq \mathbf{A}$, $\gamma \in \mathrm{Con}\,\mathbf{C}$, then the image of $\mathbf{C}$ and γ under f is $f^{\gg}\mathbf{C}$ and $f^{\gg}\gamma$, or simply $\mathbf{C}^{\gg}$ and $\gamma^{\gg}$ if it is clear that the homomorphism we consider is f.

The smallest nontrivial congruence of a subdirectly irreducible algebra $\mathbf{S}$ is called the *monolith* of $\mathbf{S}$, and is denoted by $\mu(\mathbf{S})$. An algebra $\mathbf{A}$ is called *finitely subdirectly irreducible*, if $\alpha\beta = 0$ implies $\alpha = 0$ or $\beta = 0$ for any two congruences of $\mathbf{A}$. We use the abbreviations Si and FSi for these concepts.

The join $V_1 \vee V_2$ of two varieties is $\mathsf{V}(V_1 \cup V_2)$, where the operator V stands for HSP. We use also the operators P_s (subdirect products), P_f (products of finitely many components), P_{sf} (subdirect products of finitely many components), P^2 (direct squares), P_u (ultraproducts), D (direct unions), $\mathsf{Si}(K)$ (Si elements of the class K). Logical conjunction is often denoted by $+$, for example, $CD + CP$ stands for arithmetical varieties.

LEMMA 1.1. (1) $\mathsf{SP}(K) \subseteq \mathsf{P}_s\mathsf{S}(K)$.

(2) *The finite elements of* $\mathsf{SP}(K)$ *are in* $\mathsf{SP}_f(K)$.

(3) *The free algebra* $\mathbf{F}_{\mathsf{V}(K)}(X)$ *of* $\mathsf{V}(K)$ *generated by the set* X *is in* $\mathsf{SP}(K)$.

(4) $\mathsf{S}(K) \subseteq \mathsf{HP}_s(K)$ (see Grätzer [3], Theorem 23.3).

Let $\mathbf{A} \leq \mathbf{A}_1 \times \mathbf{A}_2$, $\alpha_i \in \mathrm{Con}\,\mathbf{A}_i$ $(i = 1, 2)$. The *product* of the congruences α_1 and α_2 on $\mathbf{A}$ is defined by

$$(a_1, a_2)(\alpha_1 \times \alpha_2)(b_1, b_2) \text{ if and only if } a_1\alpha_1 b_1 \text{ and } a_2\alpha_2 b_2 .$$

Congruences of this form are called product congruences. If $\mathbf{A}$ is a subdirect subalgebra of $\mathbf{A}_1 \times \mathbf{A}_2$, then $\alpha \in \mathrm{Con}\,\mathbf{A}$ is a product congruence

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if and only if $\alpha = (\alpha+\eta_1)(\alpha+\eta_2)$, where η_i is the kernel of the ith projection.

DEFINITION 1.2. An algebra A is called *affine* if there exists an Abelian group structure $(A, +, -, 0)$ on its underlying set such that

(i) each operation f of A is affine, that is, $f - c_f$ is a group homomorphism with respect to + for a suitable constant $c_f \in A$,

(ii) $x - y + z$ is a term function of A .

Affine algebras of a given type form a variety, and are polynomially equivalent to modules over associative rings.

DEFINITION 1.3 (Gumm [4]). Let A be an algebra, $\alpha, \beta \in \mathrm{Con}\, A$. Define

$$\Delta_{\alpha,\beta} = \mathrm{Cg}_\alpha\big(\{(aa, bb) : a\beta b\}\big) ,$$
$$[\alpha, \beta] = \{(a, b) : aa\Delta_{\alpha,\beta}ab\} .$$

Here aa, bb and ab abbreviate the corresponding ordered pairs, and α is considered as a subalgebra of $A \times A$. $[\alpha, \beta]$ is the *commutator* of α and β .

THEOREM 1.4 (see Gumm [4]). *Let V be a congruence modular variety,* $A, B \in V$, $\alpha, \beta, \beta_i \in \mathrm{Con}\, A$ $(i \in I)$. *Then the following hold:*

(1) $[\alpha, \beta] = [\beta, \alpha] \in \mathrm{Con}\, A$;

(2) $[\alpha, \beta] \le \alpha\beta$;

(3) $[\alpha, \Sigma\beta_i] = \Sigma[\alpha, \beta_i]$;

(4) *for any epimorphism* $f : A \to B$ *we have* $f^{\gg}[\alpha, \beta] = [f^{\gg}\alpha, f^{\gg}\beta]$;

(5) A *is affine if and only if* $[1_A, 1_A] = 0_A$.

DEFINITION 1.5 (Hagemann and Herrmann [5]). An algebra A is called *neutral* if A satisfies the commutator identity $[x, y] = xy$ (for each pair of congruences). An $\alpha \in \mathrm{Con}\, A$ is *perfect* if $[\alpha, \alpha] = \alpha$. The algebra A is *prime* if $[\alpha, \beta] = 0$ implies $\alpha = 0$ or $\beta = 0$, and

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semiprime if $[\alpha, \alpha] = 0$ implies $\alpha = 0$. A congruence α is prime (semiprime) if and only if A/α is.

THEOREM 1.6 (Hagemann and Herrmann [5]). *The following are equivalent for an algebra* $A \in V$:

(1) A *is neutral;*

(2) *each congruence of* A *is perfect;*

(3) *if* $\alpha \in \mathrm{Con}\, A$, *then* $\mathrm{Con}\, \alpha$ *is distributive.*

The class of all neutral algebras of V *is closed under* D, H *and* P_{sf} .

THEOREM 1.7 (Hagemann and Herrmann [5]). *A congruence* α *of an algebra is semiprime if and only if it is the meet of prime congruences.*

THEOREM 1.8 (Generalized Jónsson's Theorem, see Hagemann and Herrmann [5]). *Let* A_i $(i \in I)$ *be algebras of* V , B *a subalgebra of their direct product and* α *a prime congruence of* B . *Then there is an ultrafilter* U *on the index set* I *such that the corresponding congruence* $\mathrm{Cg}_B(U) \le \alpha$. *Consequently, the prime algebras of* $V(K)$ *are contained in* $HSP_u(K)$ *for any* $K \subseteq V$.

2. The two properties

The first property, named *C2* , seems to be the "join" of affineness and neutrality, the second one, called *S* , is a sort of commutator extension property saying that « forming the commutator » commutes with « restricting congruences to subalgebras ».

PROPOSITION 2.1. *The following are equivalent for an algebra* A :

(1) $A \models [x, y] = xy[1, 1]$;

(2) $A \models [x, x] = x[1, 1]$;

(3) *if* $a[1, 1]b$ *then* $\mathrm{Cg}(a, b)$ *is perfect;*

(4) $A \models [x, yz] = [x, y]z$;

(5) $A \models [x, y] = [x, 1]y$.

Proof. (1) ⇒ (4), (1) ⇔ (5) and (1) ⇔ (6) are evident. (4) ⇒ (3) holds, since x is a join of principal congruences. For (3) ⇒ (2) apply

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(3) to $x[1, 1]$, and for (2) ⇒ (1) apply (2) to xy . □

DEFINITION 2.2. An algebra A has property *C2* if any of the conditions in Proposition 2.1 holds in A , and has property *S* if for arbitrary $B \leq A$ and $\alpha, \beta \in \text{Con } A$ we have $[\alpha/B, \beta/B] = [\alpha, \beta]/B$.

The property *S* will always be investigated together with *C2* . In this case it is enough to require only the special case $\alpha = \beta = 1$.

PROPOSITION 2.3. *If all the subalgebras of* A *are* C2 *, and for every* $B \leq A$ *we have* $[1_B, 1_B] = [1_A, 1_A]/B$ *, then* A *satisfies* S . □

A class of algebras is said to satisfy *C2* (*S*) if so do all of its members. Neutral algebras are clearly *C2* , and affine ones are *C2* + *S* . Further examples are found in the second part of the present paper.

3. Preservation

We investigate the behaviour of our properties with respect to standard algebraic constructions.

PROPOSITION 3.1. *The class of* C2 *algebras is closed under the operators* H, D *and* P_{sf} .

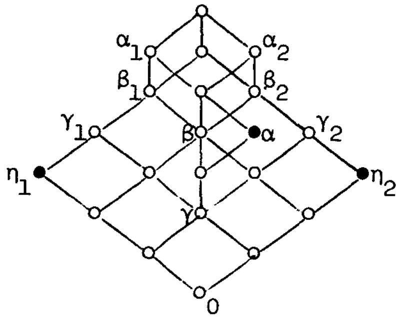
Proof. Since the assertion is straightforward for D and H , it suffices to verify the following lemma.

LEMMA 3.2. *Suppose that* A_1 *and* A_2 *are* C2 *algebras, and* B *is a subdirect subalgebra of their direct product. Then*

(1) B *satisfies* C2 ,

(2) *if* $\alpha \leq [1_B, 1_B]$ *, then* α *is a product congruence, in particular* $[1_B, 1_B] = ([1_{A_1}, 1_{A_1}] \times [1_{A_2}, 1_{A_2}])/B$.

Proof. To show (2), let η_1 and η_2 be the projection kernels on B and consider the sublattice of Con B generated by α, η_1 and η_2 . It is a homomorphic image of the following lattice which is the free modular lattice generated by α, η_1 and η_2 subject to the relation $\eta_1\eta_2 \leq \alpha$ (see Grätzer [2]):



Let $\alpha_i = \alpha + \eta_i$ $(i = 1, 2)$, we have to prove that $\alpha = \alpha_1\alpha_2$. Consider the diamond between β and γ . An easy computation shows $[\beta, \beta] \le \gamma$. Now B/η_1 is isomorphic to A_1 , so this factor is *C2* . Hence $\alpha \le [1_B, 1_B]$ implies that there is no affine interval between α_1 and η_1 . Thus $\beta_1 = \gamma_1$, consequently $\beta = \gamma$, which yields $\alpha = \alpha_1\alpha_2$. So (2) holds.

To get (1) observe that if $\alpha \le [1_B, 1_B]$, then α and $[\alpha, \alpha]$ are both product congruences by (2). As A_1 and A_2 have *C2* , the image of α under the projections is perfect. Hence α and $[\alpha, \alpha]$ are the product of the same congruences, and therefore they are equal. □

Unfortunately, in the previous proof we had to use the perfectness of $\beta_1^{\gg}$ and $\beta_2^{\gg}$. But if B is the direct product of A_1 and A_2 , then $\eta_1 + \eta_2 = 1$, hence $\beta_i = \gamma_i$, so our reasoning yields

LEMMA 3.3. *Let* $\alpha \in \mathrm{Con}(A_1 \times A_2)$, A_1, A_2 *arbitrary. If the image of* α *under the projections is perfect, then* α *is perfect as well.*

PROPOSITION 3.4. *The property* $C2 + S$ *is closed under* H, D, P_{sf} . *If all the subalgebras of* A *have* *C2* , *and* A *has* *S* , *then every* $B \le A$ *has* *S* .

Proof. The statement is straightforward for D, H , and the rest is an easy consequence of Proposition 2.3 and Lemma 3.2. □

4. Subdirectly irreducible algebras

We give characterisations of *C2* and $C2 + S$ varieties by means of

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conditions imposed on their subdirectly irreducible members. Sometimes these conditions involve $P_u(K)$ for some class K. Though such a condition is not very easy to handle and check in general, it becomes clear when K is axiomatic, or, in particular, if K is a finite set of finite algebras.

PROPOSITION 4.1. *A finitely subdirectly irreducible C2 algebra is either prime or affine.*

Proof. If $[1, 1] \neq 0$, then $0 = [\alpha, \beta] = \alpha\beta[1, 1]$ yields $\alpha = 0$ or $\beta = 0$ by the FSi property. □

PROPOSITION 4.2. *A variety V has C2 if and only if each* Si *member of V is affine or prime.*

Proof. Suppose that $[1, 1] \geq \alpha \in \text{Con } A$, $A \in V$, and the Si members of V are either affine or prime. If $[\alpha, \alpha] < \alpha$, then there is a congruence θ of A such that A/θ is Si, $\theta \geq [\alpha, \alpha]$, but $\theta \not\geq \alpha$. Then $\theta + \alpha \not\geq \alpha$, so $[\theta+\alpha, \theta+\alpha] \leq \theta + [\alpha, \alpha] = \theta$ shows that A/α is not prime. Hence it is affine, yielding $\theta \geq [1, 1] \geq \alpha$, which is a contradiction. The converse is Proposition 4.1. □

THEOREM 4.3. *A variety V satisfies C2 + S if and only if*

(i) $\text{Si}(V)$ *satisfies C2 and*

(ii) $P_u\text{Si}(V)$ *satisfies S.*

Proof. Suppose V satisfies *(i)* and *(ii)*. Then V is C2 by Propositions 4.1 and 4.2. The FSi algebras of V are either affine, or contained in $HSP_u\text{Si}(V)$ by the Generalized Jónsson's Theorem, so they have S by Proposition 3.4. We are going to find a method of factorising algebras into FSi ones. Call a congruence δ *subperfect* if every $\delta' \leq \delta$ is perfect.

LEMMA 4.4. *If* $\alpha, \beta, \delta \in \text{Con } A$ *and* δ *is subperfect, then* $\delta + \alpha\beta = (\delta+\alpha)(\delta+\beta)$.

Proof. By setting $\delta' = \delta(\alpha+\beta)$ we obtain

$$\delta' = [\delta', \delta'] \leq [\delta, \alpha+\beta] \leq \delta\alpha + \delta\beta .$$

Hence $\delta(\alpha+\beta) = \delta\alpha + \delta\beta$. It is an elementary result of modular lattice theory that this implies our assertion (see Grätzer [2]). □

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LEMMA 4.5. *Let* δ *be a subperfect congruence of a* $B \leq A$. *If* $\delta \not\supseteq \gamma/B$ *for some* $\gamma \in \mathrm{Con}\, A$, *then* $\delta^{\gg} \not\supseteq (\gamma/B)^{\gg}$ *holds in an appropriate* FSi *factor of* A .

Proof. Choose $(a, b) \in (\gamma/B) - \delta$, and let θ_0 be maximal among the congruences θ of A satisfying $(a, b) \notin \theta/B + \delta$. Then A/θ_0 is a FSi factor of A by Lemma 4.4, and the image of (a, b) yields $\delta^{\gg} \not\supseteq (\gamma/B)^{\gg}$. □

Now the proof of Theorem 4.3 is finished by the following corollary, which comes from Lemma 4.5 with $\delta = [1_B, 1_B]$ and $\gamma = [1_A, 1_A]$.

COROLLARY 4.6. *Let* $B \leq A$ *be* $C2$ *algebras and assume that every prime factor of* A *satisfies* S . *Then* $[\alpha, \beta]/B = [\alpha/B, \beta/B]$ *holds for all congruences* α, β *of* A . *In particular, a* $C2$ *variety has* S *if and only if its prime algebras have* S .

5. Generator classes

Given a class K of algebras we would like to know whether $V(K)$ satisfies $C2$ or $C2 + S$. For the first question - even in the case of neutrality - we have only a partial answer, so we start with a problem.

PROBLEM 5.1. Let K be a class of algebras such that $SP_u(K)$ is neutral $(C2)$. Does it follow that $V(K)$ is neutral $(C2)$?

As partial solutions we can state

PROPOSITION 5.2. *Let* K *be a class of algebras. If* $P_u(K)$ *is neutral* $(C2)$, *then so is* $P(K)$.

PROPOSITION 5.3. *Let* K *be a class of algebras,* $V = V(K)$. *If* $SP_u(K)$ *satisfies* $C2 + S$, *and* $F_V(2)$ *is either finite or* $C2$, *then* V *has* $C2 + S$.

PROPOSITION 5.4. *Let* K *be a finite set of finite algebras. Then* $V(K)$ *has* $C2$ $(C2{+}S)$ *if and only if* $S(K)$ *does.*

Proposition 5.4 is clear: $V = DHP_{sf}S(K)$, hence Propositions 3.1 and 3.4 apply. For Proposition 5.2 we need a definition.

DEFINITION 5.5. Let A_i $(i \in I)$ be algebras, $B \leq \Pi\{A_i : i \in I\}$, and $a, b \in B$. A subset X of I is called a *set of perfectness* for a, b and B, if projecting B to $\Pi\{A_i : i \in X\}$, the image of $Cg_B(a, b)$ is perfect.

In what follows, the kernel of this projection is denoted by $Cg_B(X)$. If U is an ultrafilter on I, the corresponding congruence on B is named $Cg_B(U)$. Algebras of the form $B/Cg_B(U)$ are called *sub-ultraproducts*.

LEMMA 5.6. *With the notation above,* I *is the join of finitely many sets of perfectness for* a, b *and* B *if and only if the image of* $Cg_B(a, b)$ *is perfect in all the sub-ultraproducts formed from* B.

Proof. If I is the join of a finite number of sets of perfectness, and U is an ultrafilter on I, then one of these sets, say $X \in U$. Hence $Cg(X) \leq Cg(U)$, thus the image of $Cg(a, b)$ in the sub-ultraproduct is an image of a perfect image of $Cg(a, b)$. Conversely, let F be the ideal of finite joins of sets of perfectness, and U an ultrafilter disjoint from F. The perfectness of a principal congruence can be described by finitely many equations, as it turns out from the definition of the commutator. So if the image of $Cg(a, b)$ is perfect in the sub-ultraproduct corresponding to U, then - since U is closed under finite intersections - the image of $Cg(a, b)$ is perfect in $B/Cg(X)$ for an appropriate $X \in U$. But then X is a set of perfectness in U, and this is a contradiction. □

Now Lemma 3.3 asserts that if B is the whole direct product, then the join of finitely many sets of perfectness is a set of perfectness again. So the neutral case of Proposition 5.2 is clear by applying Lemma 5.6 to all pairs (a, b) of B, while for the case of $C2$ one has to consider the pairs in $[1_B, 1_B]$.

Finally, we show Proposition 5.3. Note first that if $F_V(2)$ is finite, then it is in $P_{sf}S(K)$, hence satisfies $C2$ by Proposition 3.4. We copy the proof of Theorem 4.3. Let $A_i \in K$, and

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$C \le B \le A = \Pi\{A_i : i \in I\}$. Suppose $(a, b) \in [1_B, 1_B]/C - [1_C, 1_C]$. Then for $D = \langle a, b\rangle$ we have $(a, b) \in [1_A, 1_A]/D - [1_D, 1_D]$. But A has $C2$ by Proposition 5.2, D has $C2$, as it is a homomorphic image of $F_V(2)$, and the prime algebras have S by the assumption. So Corollary 4.6 gives a contradiction. Hence,

$$(*) \qquad [1_B, 1_B]/C = [1_C, 1_C] .$$

Now apply (*) to $C = \langle a, b\rangle$ for some $a[1_B, 1_B]b$. As C has $C2$, we get the perfectness of $Cg_B(a, b)$. Thus B has $C2$. Hence (*) and Proposition 2.3 show that B has $C2 + S$. So $V = HSP(K)$ has $C2 + S$ by Proposition 3.4. □

6. Joins of varieties

We examine the behaviour of the join of two subvarieties of the fixed large modular variety with respect to our properties.

PROPOSITION 6.1. *Let* V_1 *and* V_2 *be varieties. Then* $V_1 \vee V_2$ *satisfies* $C2$ $(C2+S)$ *if and only if both* V_1 *and* V_2 *do also. If* $V_1 \vee V_2$ *has* $C2$ *, then each of its elements has a subdirect decomposition into three factors: one from* V_1 *, one from* V_2 *, and the third factor is affine.*

Proof. Since $V_1 \vee V_2 = HP_{sf}(V_1 \cup V_2)$, the first statement follows from Propositions 3.1 and 3.4. The Generalized Jónsson's Theorem (applied with a 2-element index set) shows that the prime algebras of $V_1 \vee V_2$ are in $V_1 \cup V_2$. So the second assertion follows from Proposition 4.1 and the Birkhoff theorem. □

7. Disconnected varieties

We look for conditions under which a variety is the join of an affine and a congruence distributive variety. These varieties have been investigated by Herrmann [*6*], their structure is described in Freese and McKenzie [*1*].

DEFINITION 7.1. An algebra is called *disconnected*, if it is the direct product of a neutral and an affine algebra. A variety is disconnected if and only if it is the join of an affine and a congruence distributive variety.

THEOREM 7.2. *Let* K *be the class of all neutral* Si *algebras of a variety* V *. The following are equivalent for* V *:*

(1) V *is disconnected;*

(2) *each algebra in* V *is disconnected;*

(3) *the* Si *elements of* V *are either affine or neutral, and* $SP_u(K)$ *is neutral.*

Disconnected varieties satisfy $C2 + S$ *. If* $F_V(3)$ *is finite, then* (3) *can be replaced by*

(3') *the* Si *elements of* V *are either affine, or each of their subalgebras is neutral.*

REMARK. In the second part of the present paper we give an example of a locally finite $C2 + S$ variety, which is not disconnected.

Proof. (1) ⇒ (2). This is Hermann's result (see [6]).

(2) ⇒ (3). A disconnected Si algebra is either affine or neutral. Subdirect products of neutral algebras are semiprime by Theorem 1.7, so they are neutral by (2). Now Lemma 1.1 (4) gives $SP_u(K) \subseteq HP_s(K)$, so $SP_u(K)$ is neutral.

(3) ⇒ (V satisfies $C2+S$) . V has $C2$ by Proposition 4.2. If an element of $P_u Si(V)$ is not affine, then it is in $P_u(K)$. Hence the non-affine elements of $SP_u Si(V)$ are in $SP_u(K)$, so they satisfy S by (3). Thus Theorem 4.3 applies.

(3) ⇒ (1). Proposition 5.3 shows that $V_1 = V(K)$ is neutral. Let V_2 be the subvariety of all affine algebras. Then $Si(V) \subseteq V_1 \cup V_2$ by (3), so $V = V_1 \vee V_2$.

(3') ⇒ (3) provided $F_V(3)$ is finite. Let $V_1 = V(K)$. The

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3-generated free algebra over V_1 is finite, so it is in $P_{sf}S(K)$ by Lemma 1.1, which is congruence distributive by (3') and Theorem 1.6. Thus V_1 is CD by Jónsson [7]. □

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