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ON CYCLE-SUPERMAGICNESS OF SUBDIVIDED GRAPHS

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Abstract

Lladó and Moragas [‘Cycle-magic graphs’, *Discrete Math.* **307** (2007), 2925–2933] showed the cyclic-magic and cyclic-supermagic behaviour of several classes of connected graphs. They discussed cycle-magic labellings of subdivided wheels and friendship graphs, but there are no further results on cycle-magic labellings of other families of subdivided graphs. In this paper, we find cycle-magic labellings for subdivided graphs. We show that if a graph has a cycle-(super)magic labelling, then its uniform subdivided graph also has a cycle-(super)magic labelling. We also discuss some cycle-supermagic labellings for nonuniform subdivided fans and triangular ladders.

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1. Introduction

Let $G = (V, E)$ be a finite, simple, planar, connected and undirected graph, where V and E are its vertex and edge sets, respectively. A *labelling* (or *valuation*) of a graph is a map that carries graph elements to numbers (usually positive or nonnegative integers). Let H be a graph. An *H-magic labelling* is a total labelling λ from $V(G) \cup E(G)$ onto the integers $\{1, 2, \ldots, |V(G) \cup E(G)|\}$ with the property that, for every subgraph A of G isomorphic to H, there is an integer constant c such that $\sum_{v \in V(A)} \lambda(v) + \sum_{e \in E(A)} \lambda(e) = c$. A graph $G = (V, E)$ is said to be H-magic if every edge of G belongs to at least one subgraph isomorphic to H and it admits an H-magic labelling. Additionally, G is said to be *H-supermagic* if $\lambda(V(G)) = \{1, 2, \ldots, |V(G)|\}$. The notion of H-magic graphs was introduced by Gutierrez and Lladó [6] as an extension of the magic valuation given by Kotzig and Rosa [9], which corresponds to the case $H = K_2$. Classification studies of H-magic labellings have been intensively investigated (see for example [1–4, 7–13]). Ahmad *et al.* [1] studied the super K_2-magicness of an odd union of necessarily nonisomorphic acyclic graphs. Furthermore, they found exponential lower bounds for the number of super K_2-magic labellings of these unions. They also discussed the case when G is not acyclic. Ngurah *et al.* [14] found the H-supermagic labellings

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for chain graphs, fans, ladder graphs, grids and book graphs. In [10], Lladó and Moragas showed the cycle-(super)magic behaviour of several classes of connected graphs including subdivided wheels and subdivided friendship graphs. However, there are no further results on cycle-magic labellings of other families of subdivided graphs. For a detailed study of graph labellings, see the very complete survey by Gallian [5].

In this paper, we discuss cycle-magic labellings of subdivided graphs. The paper is organised as follows. In Section 2, we show that if a graph is cycle-(super)magic, then its uniform subdivided graph is also cycle-(super)magic. In Section 3, we formulate the cycle-supermagic labellings of nonuniform subdivided fans and triangular ladders. At the end, we present an open problem for further study in this area.

2. Cycle-(super)magic labellings of uniform subdivided graphs

Let G be a C_n-supermagic graph and α be the number of cycles C_n in G for $n \geq 3$. An edge $e \in E(G)$ is said to be a *good* edge if e belongs to only one subcycle C_n of the graph G. For $s \geq 1$, B is the collection of good edges obtained by choosing exactly s good edges from each subcycle isomorphic to C_n in G. Let $B = \{x_t^j y_t^j : 1 \leq j \leq \alpha,\ 1 \leq t \leq s\}$ and $|B| = s\alpha$.

Definition 2.1. Let $B \subset E(G)$. A *uniform subdivided graph* $\mathcal{G}$ of the graph G is obtained by subdividing all edges of B with $k \geq 1$ vertices.

Definition 2.2. Let $S = E(G) \backslash B$. A nonuniform subdivided graph is obtained by subdividing the edges of S.

Let $e = x_t^j y_t^j$ be an arbitrary good edge subdivided by k vertices $z_1, z_2, \ldots, z_k$ in a subcycle isomorphic to C_n. After subdivision, we will have a path $P_e \cong x_t^j z_1 z_2 \cdots z_k y_t^j$ corresponding to the edge e.

Next, we define vertex and edge sets of the graph $\mathcal{G}$ as follows:

$$\begin{aligned} V(\mathcal{G}) &= \{x_i : 1 \leq i \leq |V(G)|\} \cup \{z_{i,t}^j : 1 \leq j \leq \alpha, 1 \leq i \leq k, 1 \leq t \leq s\} \\ E(\mathcal{G}) &= \{e_i : 1 \leq i \leq |S|\} \\ &\qquad \cup \{x_t^j z_{1,t}^j,\ z_{i,t}^j z_{i+1,t}^j,\ z_{k,t}^j y_t^j : 1 \leq j \leq \alpha, 1 \leq i \leq k-1, 1 \leq t \leq s\}. \end{aligned}$$

In the following theorems, we present cycle-(super)magic total labellings of uniform subdivided graphs.

Theorem 2.3. *Let G be a C_n-supermagic graph with magic constant c. Then its uniform subdivided graph $\mathcal{G}$ is C_{n+sk}-magic for $n \geq 1$, $s \geq 1$ and $k \geq 1$.*

Proof. Let $p = |V(G)|$, $q = |E(G)|$, $v = |V(\mathcal{G})|$ and $e = |E(\mathcal{G})|$. Since G admits a C_n-supermagic labelling, there exists a labelling $\lambda : V(G) \cup E(G) \to \{1, 2, \ldots, p+q\}$ such that every subcycle isomorphic to C_n has a magic constant, say c, under the labelling λ.

Now we define a labelling $g : V(\mathcal{G}) \cup E(\mathcal{G}) \to \{1, 2, \ldots, v+e\}$ for the uniform subdivided graph $\mathcal{G}$ of graph G as follows.

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- For $1 \le i \le p$: $g(x_i) = \lambda(x_i)$ for all $x_i \in V(G)$.
- For $1 \le i \le q - s\alpha$: $g(e_i) = \lambda(e_i) + 2k\alpha s$ for all $e_i \in S$.
- For $1 \le j \le \alpha, 1 \le i \le k, 1 \le t \le s$: $g(z^j_{i,t}) = p + q + (i-1)s\alpha + \alpha(t-1) + j$.
- For $1 \le j \le \alpha, 1 \le i \le k-1, 1 \le t \le s$:

$$g(z^j_{i,t}z^j_{i+1,t}) = v + e - (i-1)s\alpha - (t-1)\alpha - j + 1,$$
$$g(x^j_t z^j_{1,t}) = \lambda(x^j_t y^j_t),$$
$$g(z^j_{k,t}y^j_t) = v + e - (k-1)s\alpha - (t-1)\alpha - j + 1.$$

It is easy to verify that, under the labelling g, the graph $\mathcal{G}$ is a C_{n+sk}-supermagic graph with magic constant $\acute{c} = c + ks(v + e + p + q + 1)$. □

By Theorem 2.3, we have following corollary.

COROLLARY 2.4. *Let G be a C_n-supermagic graph with magic constant c. Then its uniform subdivided graph $\mathcal{G}$ is C_{n+sk}-magic with magic constant $\acute{c} = c + ks(v + e + p + q + 1)$.*

THEOREM 2.5. *Suppose that G has a C_n-supermagic labelling. Then its uniform subdivided graph $\mathcal{G}$ is C_{n+sk}-supermagic for $n \ge 1$, $s \ge 1$ and $k \ge 1$.*

PROOF. Let $p = |V(G)|$, $q = |E(G)|$, $v = |V(\mathcal{G})|$ and $e = |E(\mathcal{G})|$. Since G is C_n-supermagic, there exists a labelling $\lambda : V(G) \cup E(G) \to \{1, 2, \ldots, p + q\}$ such that every subcycle isomorphic to C_n has a magic constant, say c, under the labelling λ.

Define a labelling $g : V(\mathcal{G}) \cup E(\mathcal{G}) \to \{1, 2, \ldots, v + e\}$ for the uniform subdivided graph $\mathcal{G}$ of graph G as follows.

- For $1 \le i \le p$: $g(x_i) = \lambda(x_i)$ for all $x_i \in V(G)$.
- For $1 \le i \le q - s\alpha$: $g(e_i) = \lambda(e_i) + 2k\alpha s$ for all $e_i \in S$.
- For $1 \le j \le \alpha, 1 \le i \le k, 1 \le t \le s$: $g(z^j_{i,t}) = p + (i-1)\alpha s + \alpha(t-1) + j$.
- For $1 \le j \le \alpha, 1 \le i \le k-1, 1 \le t \le s$:

$$g(z^j_{i,t}z^j_{i+1,t}) = p + 2sk\alpha - (i-1)s\alpha - j + 1,$$
$$g(x^j_t z^j_{1,t}) = \lambda(x^j_t y^j_t) + 2ks\alpha,$$
$$g(z^j_{k,t}y^j_t) = p + 2sk\alpha - (k-1)s\alpha - j + 1.$$

It is easy to verify that, under the labelling g, the graph $\mathcal{G}$ is a C_{n+sk}-supermagic graph with magic constant $\acute{c} = c + 2ks\alpha n + ks(2p + 2ks\alpha + 1)$. □

For $n \ge 3$, $W_n = C_n + \{a\}$ denotes the wheel with centre $\{a\}$ and rim of order n. The subdivided wheel $W_n(r, k)$ is the graph obtained from the wheel W_n by replacing each radial edge av_i, $1 \le i \le n$, by an av_i-path of size $r \ge 2$ and each external edge v_iv_{i+1} by a v_iv_{i+1}-path of size $k \ge 2$. In [10], Lladó and Moragas showed that subdivided wheels $W_n(r, 1)$ admit a C_{2r+1}-supermagic labelling for odd $n \ge 3$. In the following theorem, we discuss cycle-supermagic labelling of wheels W_n and $W_n(1, k)$.

THEOREM 2.6. *For odd $n \ge 3$, the wheel W_n admits a C_3-supermagic labelling.*

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TABLE 1. Ramsey numbers of paths and wheel-like graphs.

Graph	C_n-supermagic	$\alpha,\ s$	Reference
Fan F_n	C_3	$n-1, 1$	[8]
Antiprism A_n	C_3	$2n, 1$	[8]
Tri. ladder TL_n	C_3	$2n-2, 1$	[14]
Wheel W_n	C_3	$n-1, 1$	[10]
Ladder L_n	C_4	$n-1, 2$	[14]
Grid $P_3 \times P_n$	C_4	$2n-2, 1$	[7]

PROOF. For the graph $G \cong W_n$, the vertex set is $V(G) = \{a\} \cup \{v_i : 1 \le i \le n\}$ and the edge set is $E(G) = \{av_i : 1 \le i \le n\} \cup \{v_i v_{i+1} : 1 \le i \le n\}$, where all indices are taken modulo n. Thus, $v = |V(W_n)| = n+1$ and $e = |E(W_n)| = 2n$.

Now we define the labelling $\lambda : V \cup E \to \{1, 2, \ldots, 3n+1\}$ as follows:

$$\lambda(a) = n+1$$
$$\lambda(v_i) = i, \quad 1 \le i \le n$$
$$\lambda(v_i v_{i+1}) = \begin{cases} 3n+1-i & \text{if } 1 \le i \le n-1 \\ 3n+1 & \text{if } i = n \end{cases}$$
$$\lambda(av_i) = \begin{cases} \frac{1}{2}(4n+4-i) & \text{if } 2 \le i \le n-1,\ i \text{ even} \\ \frac{1}{2}(3n+4-i) & \text{if } 1 \le i \le n,\ i \text{ odd.} \end{cases}$$

It is clear that for every subcycle C_3 of the wheel W_n, the sum of all vertex and edge labels is $\frac{1}{2}(15n+13)$. Hence, for $n \ge 3$, the wheel W_n is C_3-supermagic. □

By Theorems 2.5 and 2.6, we have the following corollary.

COROLLARY 2.7. *For $k \ge 2$ and odd $n \ge 3$, subdivided wheels $W_n(1,k)$ admit a C_{3+k}-supermagic labelling with magic constant $\acute{c} = (15n+13)/2 + 2k^2 n + 8kn + 3k$.*

In Table 1 above, we present the cycle-supermagic behaviour of some families of graphs.

We can use Theorems 2.3 and 2.5 to determine cycle-(super)magic labellings for uniform subdivided graphs for the families of graphs shown in Table 1.

COROLLARY 2.8. *The following families of graphs are cycle-(super) magic.*

(1) *For $k \ge 1, n \ge 3$, the uniform subdivided graph $F(n,k)$ of the fan F_n is C_{3+k}-(super)magic.*
(2) *For $k \ge 1, n \ge 3$, the uniform subdivided graph $A(n,k)$ of the antiprism A_n is C_{3+k}-(super)magic.*
(3) *For $k \ge 1, n \ge 3$, the uniform subdivided graph $TL(n,k)$ of the triangular ladder TL_n is C_{3+k}-(super)magic.*
(4) *For $k \ge 1, n \ge 3$, the uniform subdivided graph $L(n,k)$ of the ladder L_n is C_{4+2k}-(super)magic.*

(5) *For $k \geq 1, n \geq 3$, the uniform subdivided graph $G(n,k)$ of the grid graph $G \cong P_3 \times P_n$ is C_{4+k}-(super)magic.*

In the following section, we prove that nonuniform subdivided fans and triangular ladders are cycle-supermagic.

3. Cycle-supermagic labellings of nonuniform subdivided graphs

3.1. Cycle-supermagic labellings of subdivided fans. For $n \geq 3$, a *fan* $F_n \cong P_n + K_1$ is a graph with vertex and edge sets

$$V(F_n) = \{c\} \cup \{x_i : 1 \leq i \leq n\},$$
$$E(F_n) = \{cx_i : 1 \leq i \leq n\} \cup \{x_i x_{i+1} : 1 \leq i \leq n-1\}.$$

For $n \geq 3$, the *spoke-subdivided fan* $F(n,k)$ is the graph obtained by subdividing every edge cx_i of the fan F_n by $k \geq 1$ vertices.

THEOREM 3.1. *For $k \geq 1, n \geq 3$, the spoke-subdivided fan $G \cong F(n,k)$ is C_{3+2k}-supermagic.*

PROOF. Let $v = |V(G)|$ and $e = |E(G)|$, so that $v = n(k+1)+1$ and $e = 2n-1+nk$. We denote the vertex and edge sets of $G \cong F(n,k)$ as follows:

$$V(G) = \{c\} \cup \{x_i : 1 \leq i \leq n\} \cup \{z_i^j : 1 \leq i \leq k, 1 \leq j \leq n\},$$
$$E(G) = \{x_i x_{i+1} : 1 \leq i \leq n-1\} \cup \{x_i z_1^i : 1 \leq i \leq n\}$$
$$\cup \{z_i^j z_{i+1}^j : 1 \leq i \leq k-1,\ 1 \leq j \leq n\} \cup \{z_k^j c : 1 \leq j \leq n\}.$$

We define the labelling $\lambda : V \cup E \to \{1, 2, \ldots, 3n+2kn\}$ as follows:

$$\lambda(c) = n+1,$$
$$\lambda(x_i) = \begin{cases} \frac{1}{2}(i+1) & \text{if } i \equiv 1 \pmod 2, \\ \lfloor \frac{1}{2}(n+i+1) \rfloor & \text{if } i \equiv 0 \pmod 2, \end{cases}$$
$$\lambda(z_i^j) = n+j+(i-1)n+1, \quad 1 \leq j \leq n,\ 1 \leq i \leq k,$$
$$\lambda(x_i x_{i+1}) = n+1+i+2kn, \quad 1 \leq i \leq n-1,$$
$$\lambda(x_i z_1^i) = 3n-i+2kn+1, \quad 1 \leq i \leq n,$$
$$\lambda(z_i^j z_{i+1}^j) = n+2kn-j-n(i-1)+2, \quad 1 \leq j \leq n,\ 1 \leq i \leq k-1,$$
$$\lambda(z_k^j c) = n+2kn-j-n(k-1)+2, \quad 1 \leq j \leq n.$$

It is easy to check that for every subcycle C_{3+2k}^i, $1 \leq i \leq n-1$, of the spoke-subdivided fan $F(n,k)$, the sum of the labels of the vertices and edges is $\lfloor \frac{1}{2}(17n+9) \rfloor + 6kn + 2k(2n+2kn+3)$. Hence, $F(n,k)$ is C_{3+2k}-supermagic. □

For $n \geq 3$, a *subdivided fan* $\mathcal{F}(n,k)$ is a graph obtained by subdividing every edge of the fan F_n by $k \geq 1$ vertices.

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THEOREM 3.2. *For $k \geq 1, n \geq 3$, the subdivided fan $G \cong \mathcal{F}(n,k)$ is C_{3+3k}-supermagic.*

PROOF. Let $v = |V(G)|$ and $e = |E(G)|$, so that $v = n(2k+1) - k + 1$ and $e = (k+1)(2n-1)$. The vertex and edge sets of $G \cong \mathcal{F}(n,k)$ are

$$\begin{aligned} V(G) &= \{c\} \cup \{x_i : 1 \leq i \leq n\} \cup \{y_i^j : 1 \leq i \leq k,\ 1 \leq j \leq n-1\}, \\ &\quad \cup \{z_i^j : 1 \leq i \leq k,\ 1 \leq j \leq n\} \\ E(G) &= \{x_i y_1^i : 1 \leq i \leq n-1\} \cup \{x_i z_1^i : 1 \leq i \leq n\} \\ &\quad \cup \{y_i^j y_{i+1}^j : 1 \leq i \leq k-1,\ 1 \leq j \leq n-1\} \\ &\quad \cup \{z_i^j z_{i+1}^j : 1 \leq i \leq k-1,\ 1 \leq j \leq n\} \\ &\quad \cup \{y_k^j x_{j+1} : 1 \leq j \leq n-1\} \cup \{z_k^j c : 1 \leq j \leq n\}. \end{aligned}$$

Now we define the labelling $\lambda : V \cup E \to \{1, 2, \ldots, n(4k+3) - 2k\}$ as follows:

$$\lambda(c) = n + 1,$$
$$\lambda(x_i) = \begin{cases} \frac{1}{2}(i+1) & \text{if } i \equiv 1 \pmod 2, \\ \lfloor \frac{1}{2}(n+i+1) \rfloor & \text{if } i \equiv 0 \pmod 2, \end{cases}$$
$$\lambda(y_i^j) = n + j + (n-1)(i-1) + 1, \quad 1 \leq j \leq n-1,\ 1 \leq i \leq k,$$
$$\lambda(z_i^j) = n + j + (i-1)n + k(n-1) + 1, \quad 1 \leq j \leq n,\ 1 \leq i \leq k,$$
$$\lambda(x_i y_1^i) = v + e - 2(n-1) + i - 1, \quad 1 \leq i \leq n-1,$$
$$\lambda(x_i z_1^i) = v + e - i + 1, \quad 1 \leq i \leq n,$$
$$\lambda(y_i^j y_{i+1}^j) = v + e - (n-1)(i+1) - j, \quad 1 \leq j \leq n-1,\ 1 \leq i \leq k-1,$$
$$\lambda(y_k^j x_{j+1}) = v + e - (n-1)(k+1) - j, \quad 1 \leq j \leq n-1,$$
$$\lambda(z_i^j z_{i+1}^j) = v + e - (k+2)(n-1) - j - n(i-1), \quad 1 \leq j \leq n,\ 1 \leq i \leq k-1,$$
$$\lambda(z_k^j c) = v + n - j + 1, \quad 1 \leq j \leq n.$$

It follows easily that, for every subcycle C_{3+3k}^i, $1 \leq i \leq n-1$, of the subdivided fan $\mathcal{F}(n,k)$, the sum of the labels of the vertices and edges is $\lfloor \frac{1}{2}(n+3) \rfloor + 3k(v+e-n+3) + 3(v+e+1) - n$. Hence, $\mathcal{F}(n,k)$ is C_{3+3k}-supermagic. □

3.2. Cycle-supermagic labellings of subdivided ladders. Let $G \cong TL_n$ be a *triangular ladder* graph with $V(G) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(G) = \{u_i v_i : 1 \leq i \leq n\} \cup \{u_i u_{i+1}, v_i v_{i+1}, u_{i+1} v_i : 1 \leq i \leq n-1\}$. For $n \geq 3$, a *diagonal-subdivided triangular ladder* $\mathcal{TL}(n,k)$ is a graph obtained by subdividing each edge $u_{i+1} v_i$ of TL_n by $k \geq 1$ vertices.

THEOREM 3.3. *For $k \geq 1$ and $n \geq 3$, the diagonal-subdivided triangular ladder $G \cong \mathcal{TL}(n,k)$ is C_{3+k}-supermagic.*

PROOF. Let $v = |V(G)|$ and $e = |E(G)|$, so that $v = 2n + k(n-1)$ and $e = 4n - 3 + k(n-1)$. We denote the vertex and edge sets of G as follows:

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$$V(G) = \{u_i : 1 \le i \le n\} \cup \{v_i : 1 \le i \le n\} \cup \{z_i^j : 1 \le i \le k, 1 \le j \le n-1\},$$
$$E(G) = \{u_iv_i : 1 \le i \le n\} \cup \{u_iu_{i+1} : 1 \le i \le n-1\}$$
$$\cup \{v_iv_{i+1} : 1 \le i \le n-1\} \cup \{u_{j+1}z_1^j : 1 \le j \le n-1\}$$
$$\cup \{z_i^jz_{i+1}^j : 1 \le i \le k-1, 1 \le j \le n-1\} \cup \{z_k^jv_j : 1 \le j \le n-1\}.$$

Now we define the labelling $\lambda : V \cup E \to \{1, 2, \ldots, 6n + 2k(n-1) - 3\}$ as follows:

$$\lambda(u_i) = 2i - 1, \quad 1 \le i \le n,$$
$$\lambda(v_i) = 2i, \quad 1 \le i \le n,$$
$$\lambda(z_i^j) = 2n + (n-1)(i-1) + j, \quad 1 \le i \le k,\ 1 \le j \le n-1,$$
$$\lambda(u_iv_i) = 4n - 2i + 2k(n-1) + 1, \quad 1 \le i \le n,$$
$$\lambda(u_iu_{i+1}) = 6n - 2i + 2k(n-1) - 1, \quad 1 \le i \le n-1,$$
$$\lambda(v_iv_{i+1}) = 6n - 2i + 2k(n-1) - 2, \quad 1 \le i \le n-1,$$
$$\lambda(u_{j+1}z_1^j) = 4n - 2j + 2k(n-1), \quad 1 \le j \le n-1,$$
$$\lambda(z_i^jz_{i+1}^j) = 2n + (n-1)(2k - i + 1) - j + 1, \quad 1 \le i \le k-1,\ 1 \le j \le n-1,$$
$$\lambda(z_k^jv_j) = 2n + (n-1)(k+1) - j + 1, \quad 1 \le j \le n-1.$$

It is easy to check that, for every subcycle C_{3+k}^i, $1 \le i \le 2n-2$, of the diagonal-subdivided triangular ladder $\mathcal{TL}(n,k)$, the sum of the labels of the vertices and edges is $14n + 6k(n-1) + k(4n + 2kn - 2k + 1)$. Hence, $\mathcal{TL}(n,k)$ is C_{3+k}-supermagic. □

4. Conclusion

In this paper, we described cycle-(super)magic labellings of uniform subdivided graphs. Moreover, we studied cycle-supermagic labellings for nonuniform subdivisions of some particular families of graphs, namely fans and triangular ladders. We believe that if a graph has a cycle-(super)magic labelling, then its nonuniform subdivided graph also has a cycle-(super)magic labelling. Therefore, we propose the following open problem.

Open problem. If a graph has a cycle-(super)magic labelling, determine whether or not its nonuniform subdivided graph has a cycle-(super)magic labelling.

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