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ARTICLE

Economic resilience and the dynamics of capital stock

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Abstract
The role of capital in measuring resilience is investigated. Focusing on the current short-run and potential long-run growth paths of the economic system, we propose new indexes to separately measure adaptability and resistance to shocks, which are the essence of a system's resilience. Capital dynamics during the transition and along the balanced growth path are used here instead of employment to represent the evolution of the size and composition of the economy. Our indexes measure adaptability and resistance by comparing the two capital growth rates. They are built by mimicking the average and variance of the difference in growth rates. In this new setting, investment and depreciation flows play an important role in explaining what the partial index of adaptability reveals. The available data on the USA and Spanish capital allow us to empirically compute the indexes and draw conclusions about their ability to resist shocks and absorb their effects. We conclude that the US economy is more adaptable and has a greater capacity to absorb impacts than the Spanish economy, but it is less resistant to disturbances.

Keywords: adaptability; capital; resilience; resistance

1. Introduction

This paper discusses the concept of resilience, which was first proposed in the 1970s in connection with the study of natural systems. From the ecology literature, Holling (1973) refers to the preservation of ecosystem organizational structure and functioning in the presence of exogenous stresses. In this conservation context, a system is said to be resilient if it is able to absorb external shocks without suffering catastrophic changes. Or changing perspective, when the system is characterized by multiple stable equilibria what matters for resilience is the magnitude of the perturbation that can be absorbed before the trajectory of the system moves from one domain of attraction to another. To sum up, in natural sciences resilience accounts for the capability of a system to withstand absorbing disturbances, but also to recover from them, while retaining the same basic structure and functioning (Liu et al. 2007; Ehrlich et al. 2012).

On the other hand, ecosystems and economies are related in a way that shocks in one of them quickly affect the other. They interact basically through inputs and outputs. Moreover, both ecological and socioeconomic systems are nonlinear and sometimes even complex systems. Hence, the concept of resilience is important not only for natural systems but also for economic systems as well. From a technical point of view, resilient systems are characterized by their structural stability, even though they can be dynamically unstable. In this context, economic resilience is defined as the capacity of an economy to keep the qualitative basic organization face to disturbances caused by natural events or human activity (Levin et al. 1998), but also as the ability of such an economy to recover from perturbations and surprises (Dasgupta, 1995). After all, let us not forget that as the magnitude of the disturbance increases, there is an increasing probability that the system will cross the threshold of structural stability and the dynamic characterization of the system will be substantially modified.

Of course, we are interested in economic systems but we dissent on how the concept of resilience has been addressed in the economic literature. Our purpose here is to provide an alternative that is consistent with the predictions of macroeconomic models, in particular models of long-run sustained growth, and to suggest a new operational way of measuring economic resilience. Resilience should be viewed as the combination of two related but distinct attributes: resistance (i.e., the ability to avoid or minimize the effects of shocks) and adaptability (i.e., the aptitude to absorb shocks quickly). Economic systems are dynamic by nature and sudden, discontinuous, and irreversible changes in their dynamic behavior raise serious problems for business and policy management. This is why the robustness and flexibility of a given economy is so highly appreciated. In other words, this is why researchers focus on and strive to measure the property of resilience, which encompasses both the ability to resist the inevitable shocks and to bounce back after them (Pimm, 1984; Pendall et al. 2010). Then, a straightforward index of resilience should be based on the measurement of resistance and adaptability to shocks. In addition, the study of economic resilience broadens the focus on economic growth and efficiency in the neighborhood of equilibrium and also suggests a useful way of thinking about the sustainability of the system (Arrow et al. 1995; Levin et al. 1998; Perrings, 1998, 2006).

Economies are complex networks of interacting components linked by nonlinear dynamic processes, and both are open to exchanges across their boundaries (Limburg et al. 2002). Consequently, just what constitutes the system is somewhat arbitrary and depends on how the boundaries are drawn by the observer. In fact, economic systems, like ecological systems, have boundaries that expand and contract, and a critical first step is to delineate their boundaries. Setting the system boundaries is sometimes easy (i.e., the borders of a country) but others are not because of shifts over space and time (i.e., the extent of a wetland). In the case of economic systems, the decision on where to fix the limits depends on the analytical context of the research. A second necessary step is to identify the components of the system. Once the boundaries and components have been specified, we can undertake the study of economic resilience, not earlier.

At the aggregate level, the economic system can be characterized by two features: size and composition, equivalent to mass, or spatial scale, and diversity in biological systems. From an economic point of view, the best alternative for the area occupied by an ecosystem and the population of living organisms (species) that represents the biomass and biodiversity of ecosystems is not physical space (territory), human population, or employment but capital, or the sum of all the cumulative productive factors on which economic growth is based. Capital should not be considered as just another factor of production since it is the material support of economic activity, and, according to the vintage capital theory, it is also an important vehicle of modernization (Boucekkine et al. 1998, 2005). The magnitude of the aggregate reflects the size of the economic system, and its composition is given by the amounts of the different categories of capital: physical (private and public), human, technological, natural, and social.

In economics, equilibrium is fundamentally dynamic and is represented by time trajectories. The literature on economic growth and development, in particular, emphasizes the balanced growth path (BGP), which is shaped by the evolution of the state variables that integrate the system, that is, the variables that are the engine of growth in economic models. This is the framework in which we consider that economic resilience should be studied, given that we can benefit from advances in engineering and physics where the definition of resilience is also grounded in the concept of equilibrium. But equilibrium trajectories in economics are differentiated between short-run and long-run trajectories. It is important to realize that, although the transition path may fluctuate, the system may still be very resilient. In fact, we should not limit ourselves to studying the short-run trajectory because resilience is conditional in the sense that it is necessary to compare the continuously transforming evolution of the system during the transition with the corresponding continuously transforming evolution along the BGP. Overall, an economic system is resilient as long as the state variables of the system resist disturbances or adapt to them, while following a trajectory that remains in the vicinity of the BGP (Batabyal, 1998a, 1998b).

Consider the equilibrium trajectory arising from a system in which the effects of external shocks have been avoided or absorbed so that there are no perturbations operating, and take this long-run or BGP as a benchmark trajectory. Resilience then refers to the ability to resist and adapt in order to continue to develop along this trajectory (Hill et al. 2008; Becker, 2014). Since resistance and adaptability are defined with respect to a BGP, for a system to persist and be resilient over time it is required that the state variables during the transition remain as close as possible to the equilibrium values of the state variables prevailing throughout the BGP. Nevertheless, it is well known that economic systems are constantly affected by shocks, and if we observe them as they are it is because they have resisted and adapted, and are therefore resilient. But how resilient are they? Our purpose here is to provide new statistical tools for measuring the degree of economic resilience.

The paper is organized as follows. Section 2 provides our conceptual framework to interpret economic resilience. There, we establish a connection with the evolution of capital, a state variable, instead of the variable employment, which is the most commonly used in empirical studies. We address the question of why it is important to use the capital stock as a proxy for characteristics such as the size and composition of an economic system. We also make explicit the dynamic trajectories of capital to introduce the benchmark measure of perfect resilience. In Section 3, we propose the indexes that better account for the cases of imperfect resilience. They are designed to measure the two key dimensions of resilience: adaptability and resistance, by comparing the capital growth rates of the trajectories corresponding to the short-run transition and the long-run BGP. In Section 4, we adapt the indexes to a discrete-time context and put them in relation to the available series of private and productive capital stock. In Section 5, we provide the results of an exercise where the concepts and instruments proposed in previous sections are applied to the USA and Spanish data. Our exercise covers and summarizes the entire sample period, 1960–2016 for USA and 1964–2016 for Spain. Consequently, what we offer is a comparative structural and static characterization of the two economies in terms of adaptability and resistance. Section 6 concludes.

2. Economic resilience and capital dynamics

There is no generally accepted definition of economic resilience, nor is there a theory of economic resilience as such (CARRI, 2013; Palekiene et al. 2015). Moreover, there is also no single, universally accepted empirical measure of economic resilience. In physics and engineering, resilience is usually defined as the capacity of a system to absorb the impact of a major disturbance and reorganize in order to preserve the same function, structure, and identity. It can also refer to the time required for a system to return to an equilibrium path or state following a perturbation. In economics, it has been assumed that resilience is the ability of the system to adapt and recover quickly its original size and structural shape, after a distortion caused by exogenous shocks. But it is also the ability to resist the shocks themselves and avoid being expelled from its previous equilibrium trajectory. Two main attributes of economic systems emerge from the aforementioned that take us to the core of the concept of economic resilience: the adaptation-absorption capacity and the resistance ability to stay close to a benchmark equilibrium path.

According to Martin (2012), there are three main perspectives to address the concept of resilience: engineering, based on the existence of a unique dynamic equilibrium path; ecological, which admits the existence of multiple equilibria; and adaptive, founded in complex adaptive systems theory. The definition of economic resilience outlined in the previous paragraph, which is the definition that we will use in this paper, does correspond to the first of the three above perspectives. Consequently, we use a concept of resilience grounded in physics and mathematics that shows a strong connection with the elements of modern dynamic macroeconomics and the theory of economic growth. One feature of these new developments in economic theory that we will retain in our study of economic resilience is the emphasis in the relationship between short-run and long-run dynamics, that is, between transitional dynamics and long-run growth.[1]

To address the measurement issue, we must take into account that resilience is a property of systems that can be analyzed in terms of their performance. In the case of an economic system, we have to choose the variable that better represents it, and, in our opinion, the most suitable candidate to serve as a performance indicator is social welfare. Moreover, given that resilience is a property of dynamic systems, the ideal indicator should be the social welfare index evaluated along the economy's dynamic equilibrium trajectory.

From a macroeconomic perspective, in the standard formulation of optimal growth models the objective functional is an expression that determines the total welfare. This functional is represented as an intertemporal welfare function, which is time additively separable given the constancy of the discount rate. Social welfare also includes the intratemporal sum, for the entire population, of all individual instantaneous utility functions that are defined mainly on the grounds of per capita consumption. In general, the optimal control problem can be transformed and the relevant variables can be written in per capita terms.[2] In addition, given the subjectivity of preferences, and although it may not be an accurate measure of well-being, scholars always focus on per capita income as the key variable to measure the performance of the economy. More specifically, what is representative of economic performance is the evolution of per capita income over time and, therefore, its growth rate.

The relationship between the trajectories corresponding to welfare achievements, per capita consumption, and per capita income is established through the resource constraint and the dynamics of the state variables that shape the intertemporal optimization problems. In other words, what ultimately concerns us is the dynamics of the state variables, commonly representing the stock or stocks of capital. In the context of endogenous growth models, according to which the economy grows during the transition but also in the long run, the basis for economic growth lies in the accumulation of one or more types of capital. It is the dynamics of capital that determines the progress of output, per capita income, and welfare.

Let us return to the economic system as a whole and try to characterize it in terms of resilience. By analogy with natural systems, its resilience depends on size and composition. The best proxies for the biological mass and biodiversity of an economy are the size and variety of capital. That is, the magnitude of the economy is reflected in the size of its capital and the composition represented by the amounts of different categories of capital: physical private and public capital (equipment, structures, buildings), human capital (labor supply with skills, formation, training, and health status), technological capital (knowledge and ideas), social capital (institutions and norms), and natural capital (natural resources, environmental quality, emission reservoirs, and landscape). In addition, some, if not all, of them are genuine engines of growth. Thus, regardless of the influence they may have in the short term, the determinants of economic growth are not the increase in human population, labor supply, or employment, but the dynamics of capital, or the combined dynamics of several cumulative productive factors.

Beyond the exhaustive enumeration of the different components of capital just provided, since we want to study at an aggregate level the resilience of an economy moving relatively close to its BGP, but also for the sake of simplicity, we can summarize and consider a single capital index. In our opinion, the dimension, the composition, or any other quasi-permanent feature of an economic system is better represented by a structural variable like capital than by a flow like labor input or the unemployment rate.[3] This is because the latter tends to overreact more than capital face to marginal changes in the economic environment. Employment usually obeys conjunctural movements.[4] Therefore, we assume that the path of capital growth is representative of the dynamic behavior of the economy as a whole and that it is a good proxy for the path of economic welfare and per capita income in the medium and long term.

Once the variable to be used to measure the state of the economy has been chosen, and the two relevant characteristics associated with resilience have been identified, it remains to decide whether to take the capital levels or its growth rates as the reference for computations. The usual specification of growth models gives rise to exponential solution trajectories. Consequently, we will assume that trajectories are ideally represented in continuous time and that they adopt an exponential form. This is consistent with the representation of an economy evolving close to a long-run trajectory, which is commonly characterized as a BGP where the levels of the relevant variables grow at a positive constant rate of growth. Then, we can study the characteristics of the economic system, and implement comparative analyses, just observing the current and long-run rates of growth instead of the levels.

In this context, the variable $KL(t)$ will represent the level of capital along the BGP in the long run. This variable is assumed that follows a long-run trajectory characterized by a constant rate of growth, $\bar{\gamma}_{KL}$, which applies at every point of the path

$$KL(t) = KL(t_0) \cdot \exp\left\{ \int_{t_0}^{t} \bar{\gamma}_{KL} d\tau \right\}. \tag{1}$$

Conceptually, the long-run values associated with this path represent the equilibrium values that prevail either in the absence of any shock or after the effects caused by different shocks have been completely interiorized. This is the reference trajectory to which we will compare the short-run current values of capital.

The variable $KS(t)$ will represent the level of capital along the transition in the short run. This variable, in equilibrium, evolves according to the trajectory

$$KS(t) = KS(t_0) \cdot \exp\left\{ \int_{t_0}^{t} \gamma_{KS}(\tau) d\tau \right\}, \tag{2}$$

where $\gamma_{KS}(t)$ is the current rate of growth.

Resilience is a property of the economic system that has to do with its capacity to keep the economy's growth path as close as possible to the potential one. Resilience is in fact a characteristic of economies that may be analyzed by studying the relationship between the two variables $KS(t)$ and $KL(t)$. The $KS(t)$ variable is assumed that moves in the short-run subject to any shock experienced by the economy. Shocks that either throw the economy off its growth path or have the potential to throw it off its growth path but do not. Here, different patterns may be found for $KS(t)$: it could monotonically explode away; it could fluctuate around the long-run levels $KL(t)$, drawing either explosive, dampened, or regular oscillations; but it also could remain stuck to the long-run levels with no transitional dynamics.

For the sake of simplicity, we will consider that the two series of capital start from the same initial value, $KS(t_0) = KL(t_0)$. Then, the trivial case of **perfect resilience**[5] may be associated with the equality $KS(t) = KL(t)\ \forall t$. In this extreme case, none of the multiple and repeated shocks experienced by the economy diverts the short-run values from the corresponding long-run values. Consequently, from (1) and (2) we get the result of perfect resilience in terms of the growth rates,

$$\int_{t_0}^{t} \left(\gamma_{KS}(\tau) - \bar{\gamma}_{KL} \right) d\tau = 0 \qquad \forall\, t, \tag{3}$$

together with

$$\gamma_{KS}(t) = \bar{\gamma}_{KL} \qquad \forall\, t. \tag{4}$$

Finally, since we have made a clear distinction between adaptability and resistance as key dimensions of resilience, it is important that both be independently measurable. We will address this issue in the next section.

3. Imperfect resilience: indexes of adaptability and resistance

Let us start by inspecting the alternative cases of less-than-perfect resilience in which, as a consequence of some shock(s), the state variable capital moves away from the long-run trajectory, $KS(t) \neq KL(t)$ from t_0 onwards. In particular, the series $KS(t)$ can explode, in which case $KS(t) \neq KL(t)$ forever and $\gamma_{KS}(t) \neq \bar{\gamma}_{KL}$ $\forall t > t_0$. But we can also observe the more interesting case where after some finite interval of time, at t_1 for example, the series $KS(t)$ reaches again the long-run value of the variable $KL(t)$, that is $KS(t_1) = KL(t_1)$.[6]

It is the latter case that deserves more attention because we could differentiate between the capability of adaptation and the success of absorption on the one hand and the resistance to shocks on the other. Leaving aside for the moment the property of resistance, we can say that the time elapsed from t_0 to t_1 is the amount of time required by the economy to completely absorb the effects caused by shocks or to become fully adapted to them. This clearly involves the speed with which the economic system returns to its long-run BGP after the shock.

Given that $KS(t)$ and $KL(t)$ match each other at t_0 and again at t_1, we can establish the result $\ln\left(\frac{KS(t_1)}{KS(t_0)}\right) = \ln\left(\frac{KL(t_1)}{KL(t_0)}\right)$ from which, because of the exponential forms introduced in (1) and (2), we get

$$\int_{t_0}^{t_1} \left(\gamma_{KS}(\tau) - \bar{\gamma}_{KL}\right) d\tau = 0. \tag{5}$$

Now, based on this result, we will characterize the extreme case of **complete adaptation-absorption**. It is said that the economy is completely adapted to the effects of a shock after $(t_1 - t_0)$ periods if

$$Average\left\{\gamma_{KS}(r) - \bar{\gamma}_{KL}\right\}_{t_0 \le r \le t_1} = \frac{1}{(t_1 - t_0)} \int_{t_0}^{t_1} \left(\gamma_{KS}(\tau) - \bar{\gamma}_{KL}\right) d\tau = 0. \tag{6}$$

However, since the economy is continuously being perturbed by shocks that overlap, we rather expect to find partial adaptation while the absorption is on the way to being completed. In such a case, the sample mean of the difference between the short- and long-run rates of growth of capital stock, calculated for any interval of $(t - t_0)$ periods, would be different from zero. We consider that it is possible to use this non-null value as the basis for the measure of the incomplete degree of adaptability. However, we have to take into account that, as shown in (5), every time that $KS(t)$ touches $KL(t)$ the accumulated value of the difference between the rates of growth becomes zero. This could imply that a unique non-null value of the sample mean might be associated with two very different adaptive processes: one that adapts slowly and rarely crosses and other that adapts quickly and crosses repeatedly.

This possibility leads us to introduce a correction for avoiding indeterminacy and better conclude about the degree of adaptability. We propose as the *index of adaptability* (AI)

$$AI(t) = \frac{1}{(1+n)} \frac{1}{(t - t_0)} \int_{t_0}^{t} \left(\gamma_{KS}(\tau) - \bar{\gamma}_{KL}\right) d\tau. \tag{7}$$

Here, frequency $n \in [0, \infty[$ represents the number of times that the $KS(t)$ series comes into contact with the $KL(t)$ series, crossing or bouncing, immediately or after a lapse of time, but excluding the initial period in which they are equal by assumption. According to (7), the closer the absolute value of $AI(t)$ is to zero, the greater the degree of adaptability of the economic system.

Finally, we come back to the property of resistance. We find the trivial case of **perfect resistance** associated with the equality $KS(t) = KL(t)$ $\forall t$. This implies that we can also identify this extreme case with the result

$$\ln(KS(t)/KL(t)) = 0 \qquad \forall\, t. \tag{8}$$

Consequently, imperfect resistance will be associated with non-null values of the difference of logarithms of the two series of capital stock. One way of defining the different degrees of imperfect resistance is by calculating the variance of the difference of logarithms, which we propose as the *index of resistance* (RI). Of course, the closer the variance is to zero, the greater the degree of resistance to shocks displayed by economic systems. Under the ideal representation of trajectories in the exponential form, and given the initial equality $KS(t_0) = KL(t_0)$, we have

$$RI(t) = Variance\{\ln KS(r) - \ln KL(r)\}_{t_0 \leq r \leq t}$$

$$= Variance\left\{\int_{t_0}^{r} \left(\gamma_{KS}(\tau) - \bar{\gamma}_{KL}\right) d\tau\right\}_{t_0 \leq r \leq t}. \tag{9}$$

With the two previous indexes, we suggest a way to separately measure adaptability and resistance, the two dimensions of resilience. Although they are complementary to each other, it is evident that they cannot be ranked hierarchically. In consequence, even if they offer a quantitative representation of the components of resilience, we cannot unify them to provide a unique and one-dimensional quantitative statistic for resilience. Any study on resilience implemented according to the approaches of this work must necessarily compute independently the two indexes *AI* and *RI*, and try to manage them in the best way to correctly conclude about the resilience in the aggregate.

4. The indexes at work

In this section, we revisit the previous indexes of adaptability (AI) and resistance (RI) and reformulate the mathematical expressions by adapting them to a format in discrete time. In this way, we make operational the concepts of resilience, adaptability, and resistance, establishing a direct and clear connection between theory and practice. But, for the sake of equanimity, it must be recognized that, although we have previously argued that the best option for the study of resilience is to use a single index representing all the different categories of capital, the uneven nature of physical, human, technological, natural and social capitals, together with the shortcomings detected in their measurement, make it impossible to have such an index. Accordingly, we accommodate this challenging scenario regarding the availability of data on capital, and confine ourselves to using a narrow database to undertake a first prospective exercise of estimating the resistance and adaptability indexes according to our own methodology. In fact, we abstract from any other capital and focus on the stock of private and productive physical capital.[7]

The database with the series used in this paper is available in Escribá-Pérez et al. (2018, 2022, 2023). Our empirical analysis is based on the USA and Spanish data for the series of gross investment, depreciation, and capital stock. Given the series for gross investment, I_t^G, the annual series for the short-run capital stock, $KS(t)$, does correspond to the *economic measure* of productive capital, called K_t^* in the above-mentioned papers. There too, δ_t^* is the associated *economic* depreciation rate. Then, the short-run capital stock evolves according to

$$K_t^* = I_t^G + \left(1 - \delta_t^*\right) K_{t-1}^*. \tag{10}$$

On the other hand, the annual series for the long-run capital stock, $KL(t)$, refers to the *statistical measure* of productive capital called K_t. In this case, δ_t represents its associated *statistical* depreciation rate. Then, the long-run capital stock evolves according to

$$K_t = I_t^G + (1 - \delta_t) K_{t-1}. \tag{11}$$

This last measure corresponds to the capital values generated with the Perpetual Inventory Method, according to which it is assumed a fixed service life for each different type of capital goods. Hence, the variability of the depreciation rate reflects mechanically the changes in capital composition. Instead, the former measure is obtained according to a more sophisticated algorithmic procedure that allows the depreciation rate and the capital stock to be measured endogenously. This method is based on the agents' optimal decisions once they have been transformed into market valuations.

According to what has been said in the previous sections, to study economic resilience we adopt a standard framework commonly associated with economic growth theory. That is, in discrete terms the levels of the relevant variables evolve geometrically following the paths

$$K_t = K_{t_0} \cdot \prod_{t_0}^{t} \left(1 + \bar{\gamma}_K\right), \tag{12}$$

$$K_t^* = K_{t_0}^* \cdot \prod_{t_0}^{t} \left(1 + \gamma_{K_\tau^*}\right), \tag{13}$$

where $\bar{\gamma}_K$ is the constant long-run rate of growth of capital stock along the BGP, and $\gamma_{K_\tau^*}$ is the variable or current rate of growth of capital stock in the short run along the transition. As in the continuous time specification, we assume that $K_{t_0}^* = K_{t_0}$.

We shall now conform the *index of adaptability* (7) to the discrete context,[8] which for sufficiently small γ-values becomes the following expression as a first-order approximation

$$AID_t = \frac{1}{(1+n)} \frac{1}{(t-t_0)} \sum_{t_0}^{t} \left(\gamma_{K_\tau^*} - \bar{\gamma}_K\right). \tag{14}$$

Recall that n stands for the number of times the K_t^* series encounters the K_t series and crosses it or bounces off it, immediately or after a time interval, but not counting the initial period. This expression may be rewritten as the product of two terms where the first one is always positive and contributes to modify the second one by reducing its absolute value. The second term determines the sign of the index,

$$AID_t = \left(\frac{1}{1+n}\right) \left(\left(\frac{1}{(t-t_0)} \sum_{t_0}^{t} \gamma_{K_\tau^*}\right) - \bar{\gamma}_K\right). \tag{15}$$

The first term does not affect the sign of the adaptability index, which is negative or positive depending on whether the average value of the current rates of growth experienced by the capital stock K^* during the sample period is lower or higher than the constant rate of growth associated with the capital stock K. In any case, the important thing to interpret the result of this index is not the sign, but how far it is from zero, that is, its absolute value. As in the continuous case, the closer the absolute value of AID_t is to zero, the greater the degree of adaptability of the economic system.

From (10) and (11), we get $\frac{K_t^* - K_{t-1}^*}{K_{t-1}^*} = \frac{I_t^G}{K_{t-1}^*} - \delta_t^* = i_t^* - \delta_t^*$ and $\frac{K_t - K_{t-1}}{K_{t-1}} = \frac{I_t^G}{K_{t-1}} - \delta_t = i_t - \delta_t$. These expressions suggest the direct substitution of $\gamma_{K_\tau^*} - \bar{\gamma}_K = i_\tau^* - \delta_\tau^* - i_\tau + \delta_\tau$ in the index of adaptability (14). In such a case, we would have a disaggregation of the index into two components: the *depreciation component* that is related to the difference between statistical and economic depreciation rates, and the *investment component* that is related to the difference between economic and statistical investment rates,

$$AID_t^d = \frac{1}{(1+n)} \frac{1}{(t-t_0)} \sum_{t_0}^{t} \left(\delta_\tau - \delta_\tau^*\right) + \frac{1}{(1+n)} \frac{1}{(t-t_0)} \sum_{t_0}^{t} \left(i_\tau^* - i_\tau\right). \tag{16}$$

The computational procedure involved in equation (15) introduces a discrepancy with the theoretical approach outlined above. Conceptually, $\bar{\gamma}_K$ is the constant growth rate associated with the geometric specification of the long-run path for capital stock, rather than the current rate of growth of the statistical measure of capital stock. Ideally, they should be the same but in practice they are not. In consequence, before we implement empirically the discrete index of adaptability, there is an important remark to be done. That is, since it is not possible to know the exact value of such a theoretical parameter, and we have to numerically compute the constant rate of growth as the sample mean $\bar{\gamma}_K = \frac{1}{T}\sum_{t=1}^{T}\frac{K_t - K_{t-1}}{K_{t-1}} = \frac{1}{T}\sum_{t=1}^{T}(i_t - \delta_t)$, then the specification that appears in (16) requires a correction. Given that $\gamma_{K^*_\tau} - \bar{\gamma}_K = i^*_\tau - \delta^*_\tau - \frac{1}{T}\sum_{s=1}^{T}(i_s - \delta_s) = \delta_\tau - \delta^*_\tau + i^*_\tau - \frac{1}{T}\sum_{s=1}^{T} i_s - \delta_\tau + \frac{1}{T}\sum_{s=1}^{T}\delta_s$, we show next how the adaptability index (14) can be correctly decomposed in the components that substitute the investment and depreciation rates for the rates of growth of the two involved capital stocks,

$$\tilde{AID}^d_t = \frac{1}{(1+n)}\frac{1}{(t-t_0)}\sum_{t_0}^{t}\left(\delta_\tau - \delta^*_\tau\right) + \frac{1}{(1+n)}\frac{1}{(t-t_0)}\sum_{t_0}^{t}\left(i^*_\tau - \tilde{i}_\tau\right) \equiv AID_t, \tag{17}$$

where $\tilde{i}_\tau = \frac{1}{T}\sum_{s=1}^{T} i_s + \left(\delta_\tau - \frac{1}{T}\sum_{s=1}^{T}\delta_s\right)$.[9]

The above expression may be rewritten as the product of two terms where the first one makes apparent the role of the frequency n, while the second one plays the important role of bringing to the foreground the sign of the *depreciation* and *investment components*,

$$AID_t = \left(\frac{1}{1+n}\right)\left(\left(\frac{1}{T}\sum_{s=1}^{T}\delta_s - \frac{1}{(t-t_0)}\sum_{t_0}^{t}\delta^*_\tau\right) + \left(\frac{1}{(t-t_0)}\sum_{t_0}^{t} i^*_\tau - \frac{1}{T}\sum_{s=1}^{T} i_s\right)\right) \equiv \tilde{AID}^d_t. \tag{18}$$

The sign of the depreciation component is negative (positive) if the average value up to period t of the economic depreciation rate δ^* is higher (lower) than the average value for the whole sample period of the statistical depreciation rate δ. In other words, a negative (positive) depreciation effect may be interpreted as the result of an over-depreciation (infra-depreciation) during the sample period.[10] Likewise, the sign of the investment component is positive (negative) if the average value up to period t of the economic investment rate i^* is higher (lower) than the average value for the entire sample period of the statistical investment rate i. In other words, a positive (negative) investment effect would be the result of an over-investment (infra-investment) during the period under study. In any case, a given absolute value of the adaptability index can be associated with any of the four feasible combinations of signs, even though a negative (positive) depreciation component is most likely associated with a positive (negative) investment component.

Finally, we can also adapt the *index of resistance* (9) to the discrete context in the following way

$$RID_t = Variance\left\{\ln K^*_r - \ln K_r\right\}_{t_0 \le r \le t}$$

$$\approx Variance\left\{\sum_{t_0}^{r}\left(\gamma_{K^*_\tau} - \bar{\gamma}_K\right)\right\}_{t_0 \le r \le t}$$

$$= \frac{1}{(t-t_0)}\sum_{t_0}^{t}\left(\sum_{t_0}^{s}\left(\gamma_{K^*_\tau} - \bar{\gamma}_K\right) - \frac{1}{(r-t_0)}\sum_{t_0}^{r}\left(\sum_{t_0}^{s}\left(\gamma_{K^*_\tau} - \bar{\gamma}_K\right)\right)\right)^2. \tag{19}$$

The second row is an approximation for sufficiently small γ-values. Once again, as in the continuous case, the closer the value of RID_t is to zero, the greater the degree of resistance to shocks displayed by economic systems. In other words, lower variance implies more resistance.

5. Resilience results for USA and Spain

Next, we provide in the following table the main results for the USA and Spanish economies. The results we show are the values of the different indexes for the entire samples: 1960–2016 for USA and 1964–2016 for Spain.

Resilience indexes: comparative results.

Country	**Adaptability** Index	Depreciation effect	Investment effect	Country	**Resistance** Index
USA	− 0.00089	− 0.00192	+ 0.00103	**USA**	+ 0.01239
Spain	− 0.00095	− 0.00198	+ 0.00103	**Spain**	+ 0.01221

Source: Own elaboration and official statistics. Escribá-Pérez et al. (2018, 2022, 2023).

First, we can observe that the US economy has shown a better adaptability to shocks than the Spanish economy. The absolute value of the US *index* of adaptability is lower than the absolute value of the Spanish index, actually it is 9.4% lower. In any case, both are close to zero, which implies an almost complete adaptation to the shocks experienced throughout the full sample period. Moreover, the number of times that the short-run series for capital stock crosses the long-run series, does not play any role in the previous comparative result because the frequency n is 3 for both Spain and the USA. Second, and at variance with the previous, the Spanish economy shows a lower value of the resistance index. The difference is not too high, about 1.5 percentage points, but it is enough to conclude that, for the entire sample period, the US economy has been less resistant to its shocks than the Spanish economy to its own. Then, putting the results of the two indexes together and comparing the two economies, we observe that it is not possible to reach a definite conclusion regarding resilience as a whole. There is not a clear dominance of one economy over the other simultaneously in the two attributes of resilience. While one dominates in resistance, the other dominates in adaptability.

Because of the full sample computation of the indexes in the previous table, a simplification of (18) when $t - t_0 = T$ may help to interpret the results under the columns corresponding to the depreciation and investment effects,

$$AID_T = \left(\frac{1}{1+n}\right)\left(\left(\frac{1}{T}\sum_{s=1}^{T}\gamma_{K_s^*}\right) - \bar{\gamma}_K\right) =$$

$$= \left(\frac{1}{1+n}\right)\left(\left(\frac{1}{T}\sum_{s=1}^{T}\delta_s - \frac{1}{T}\sum_{s=1}^{T}\delta_s^*\right) + \left(\frac{1}{T}\sum_{s=1}^{T}i_s^* - \frac{1}{T}\sum_{s=1}^{T}i_s\right)\right). \tag{20}$$

The adaptability index is then disaggregated into the depreciation and investment components. Abstracting from the correction term that involves the frequency n, the depreciation effect in equation (20) mainly measures the difference between the sample average of the long-run or statistical depreciation rate and the sample average of the short-run or economic depreciation rate. Similarly, the investment effect in the above equation mainly measures the difference between the sample average of the short-run or economic investment rate and the sample average of the long-run or statistical investment rate. Consequently, as was said in the previous section, a negative (positive) depreciation component of the adaptability index is representative of a high (low) economic depreciation rate with respect to its long-run reference level. In turn, a positive (negative) investment component is representative of a high (low) short-run investment-capital coefficient compared to the long-run value of the ratio.

The table with the comparative results reveals that, on average, there has been over-depreciation and over-investment in both the USA and Spanish economies. The adaptation process in both economies after their corresponding shocks, seems to have been made through transitional over-efforts in the destruction of old capital and the acquisition of new equipment. However, one important feature of this process that arises from the figures is that investment behavior in the two countries has been, on average, very similar, since the investment components of the adaptability index take the same numerical value. Consequently, the different ability to adapt in Spain and USA is due exclusively to the different behavior of depreciation in these two countries. For the entire period, on average, the endogenously determined economic deterioration and obsolescence has been proportionally lower in USA than in Spain.

6. Conclusions

In this paper, a new characterization of the concept of economic resilience has been provided. We address the notions of resilience, adaptability, and resistance to shocks focusing on the meaning given to them by the engineering and physics approach. These are best suited to describe the dynamic equilibrium in economic systems because of the key concepts of transitional and balanced growth paths to which they refer. Resilience is considered to be the result of merging the attributes of adaptability and resistance. The former stands for the economy's ability to absorb shocks as quickly as possible. The second accounts for the capacity to avoid or minimize the effects of shocks. Our proposal implies measuring resilience in an indirect way, from the separate measurement of its two main components. These can complement or contradict each other. We provide two independent indexes to capture the degree of adaptability and the degree of resistance of economic systems. These are built by comparing the capital growth rates of the trajectories corresponding to the short-run transition and the long-run balanced growth path. The adaptability and resistance indexes are somewhat related to the average and the variance, respectively, of the difference in capital growth rates. This is at odds with the conventional empirical literature, which focuses on employment and labor market outcomes by examining short-term dynamics and ignoring long-term structural trends. Our approach is imperfect but, since it takes a state variable as a referent instead of a flow variable, it is more in line with the original concept of resilience discussed in the natural sciences than any of the alternatives handled in other economic studies that have preceded us.

We assume that the short-run trajectory of the economy is subject to all kinds of impacts but also that the long-run trajectory represents its evolution once the shocks have been completely absorbed. Our indexes of adaptability and resistance capture the extent to which the economy stays close to its balanced growth path or moves away. Going further, we also propose a decomposition of the ability to absorb shocks in two complementary terms that depend, respectively, on depreciation and investment. That is, we provide the tools to find out whether the adaptation has been based mainly on the destruction of old capital or on the acquisition of new equipment. In summary, an economy that is better (worse) adapted and more (less) resistant may be classified as being more (less) resilient. However, it is not always possible to establish a clear relationship of simultaneous dominance both in terms of adaptability and resistance.

A case study concerning two different geographic areas, the USA and the Spanish economies, is implemented to check the scope of our adaptability and resistance indexes in measuring economic resilience. Our quantitative measures are computed for the full sample period and provide a static global characterization of these economies. We find that the US economy shows a better result than the Spanish economy as regards the ability to adapt to shocks, but it shows a worse result regarding the capacity to resist and avoid the consequences of impacts. Our results also show that the higher adaptability of the US economy is based on a lower overreaction of its short-run or economic depreciation rate compared to the long-run or statistical depreciation rate. This means that, on average, economic deterioration and obsolescence have played a more important role in Spain than in USA. In any case, the lack of a clear pattern involving unidirectional dominance in both attributes, adaptability and resistance, prevents us from answering the question of which of the two economies is more resilient.

In this paper, we have proposed a series of ideas, concepts, and tools related to the phenomenon of economic resilience and its measurement, and we have done so by adopting an alternative approach to the standard one. However, without detracting from the achievements of the work, at least two extensions are pending for future research. First, an additional effort should be made to study the resilience, adaptability, and resistance of the economy in other OECD countries, applying our measurement proposals. Second, we recognize that our results are limited in scope because the state variable we use, together with the corresponding investment and depreciation flows, is the stock of private and productive physical capital. Therefore, it is expected that future research will seek to integrate other components of capital, such as infrastructure and human capital, into the analysis.

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Notes

1 Instead, Pike et al. (2010) highlight the weaknesses of the equilibrium-based approach, and Simmie and Martin (2010) propose going beyond the standard use of the concepts of equilibrium, unique or multiple, and dynamic stability. These authors emphasize the concepts of structural instability and bifurcation to analyze resilience from an evolutionary perspective. They focus on the ability of a system to adapt from one regime of stability to another in a context characterized by strongly nonlinear dynamics.

2 Optimal growth models are often stated with an objective functional in which only the individual's instantaneous utility function enters the current integrand. This case corresponds to what is called average utilitarianism or the Millian case. It is different from Benthamite or classical utilitarianism in which the total population also appears explicitly in the current integrand.

3 Martin (2012) argues for a different point of view: "*a regional or local economy may resume output growth following a recession without a corresponding recovery in employment, thereby creating major problems of adjustment for local unemployed workers. How far and in what ways regional employment rebounds following recession is thus arguably a more insightful indicator of a regional economy's resilience.*"

4 The empirical literature that investigates regional resilience is based on the series of the current level of employment. As Fingleton et al. (2012) pointed out, researchers are mainly interested in studying resistance to and recovery from recessions. Their standard computations include (i) the regional percentage decline in employment relative to the national percentage decline in employment during the recession period (resistance index); ii) the post-recession percentage growth in employment in a region relative to the percentage growth in national employment until the onset of the next recession (recovery index); and iii) the number of quarters elapsed until the previous highest employment levels are recovered (recovery index).

5 We need an absolute, reference for the coming computational definitions of resilience, adaptability, and resistance. Our choice has been to define the corresponding extreme cases associated with the perfectness of each of the above concepts. Subsequently, we can use them as a pattern for comparisons.
6 It is probably true that, as in most economic models with a BGP, the variable $KS(t)$ will eventually converge to $KL(t)$, $\lim_{t\to\infty} KS(t) = \lim_{t\to\infty} KL(t)$. However, this property of convergence at infinity cannot be used to characterize resilience in mathematical terms because in our context $\lim_{t\to\infty} KL(t) = \infty$, which poses a major problem to handle it algebraically.
7 This is an obvious reductionism, given that different growth models propose different categories of capital as the engine of growth. However, pragmatism leads us to start from the available data on physical capital and leave the window open to new developments and new research on economic resilience. Future research should focus, in the first stage, on the possibility of measuring economic resilience by handling two state variables, that is, human capital and physical capital combined in a single capital index. Subsequently, even new additions to the capital index could be considered.
8 Here we use the following transformations $\ln\left(\prod_{t_0}^{t}(1+x_\tau)\right) - \ln\left(\prod_{t_0}^{t}(1+y_\tau)\right) = \sum_{t_0}^{t} \ln\left(\frac{1+x_\tau}{1+y_\tau}\right) \simeq \sum_{t_0}^{t}(x_\tau - y_\tau)$.
9 For a given T, the new term $\tilde{\imath}_\tau$ is variable because of the variability of δ_τ. The latter is related to the statistical measures of depreciation and capital associated with the Perpetual Inventory Method. The variability of the statistical depreciation rate along the BGP only reflects the changes in capital composition. In the case of a constant capital composition, we would get $A\tilde{I}D_t^d \equiv AID_t = AID_t^d = \frac{1}{(1+n)}\frac{1}{(t-t_0)}\sum_{t_0}^{t}\left(\frac{1}{T}\sum_{s=1}^{T}\delta_s - \delta_\tau^*\right) + \frac{1}{(1+n)}\frac{1}{(t-t_0)}\sum_{t_0}^{t}\left(i_\tau^* - \frac{1}{T}\sum_{s=1}^{T} i_s\right)$.
10 It is worth noting that over- (infra-) depreciation is more likely to occur in periods of low (high) maintenance activity and high (low) obsolescence.

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