beauty and power of mathematics that goes beyond the narrow confines of the curriculum. It would also make a nice tea table display copy for casual reading by anyone who might want to gain a better appreciation of the richness and variety of modern mathematics and its cultural significance.

My only criticism is that the book could have been even more useful to novices if it had contained some extended bibliography for interested students to explore some of the ideas further. Nowadays references could even have been made to the mathematical content given in *Wikipedia*.

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The art of mathematics – take two by Béla Bollobás, pp 334, £19.99 (paper), ISBN 978-1-108-97826-2, Cambridge University Press (2022)

When I was at school my wonderful maths teacher Ian Harris gave me a wonderful book called *Geometry for Sixth Forms* by Tuckey and Swan. In that book there is a section called 'SELECTED RIDERS' with the tempting headline 'Mostly *hard*, for those who like them *hard*.' Those italics have stayed with me ever since, and I think they apply very neatly to the book under review. The subtitle 'Tea time in Cambridge' mirrors that of the 'Take One' edition (2006) '*The Art of Mathematics – Coffee Time in Memphis*' (where the author holds a chair in combinatorics in addition to his Cambridge fellowship) and refers to the kind of problems which might be discussed among staff and students relaxing during a refreshment break. (Is this delightful practice dying out in these pressurised times?) This is certainly not a 'popular maths' book of puzzles; possibly some among the general mathematically literate public would be put off by Problem 1 which asks (the easy bit) 'Let $n \geqslant 1$ be a fixed natural number. Suppose that $0 < x_1 < x_2 <\ldots < x_N < 2n + 1$ are such that $|kx_i - x_j|$ for all natural numbers i, j and k with $1 \leqslant i < j \leqslant N$. At most how large is N?' Problem 2 on the existence of Egyptian fractions is much more down-to-earth, something of a relief to an ordinary mathematician like me.

There are 128 problems altogether, each with its own Hint and Solution; these are enlightening and often carry tangential information which is of significant interest in itself, including references to versions of the problems, anecdotes about those involved and pen sketches by Gabriella Bollobás. The extreme variety makes a general survey impossible but here are a few samples.

Geometry problems with 'adventitious angles' (which happen to allow others to be calculated without any use of trigonometry) are represented by two (one of which, for me, goes back to those wonderful SELECTED RIDERS), Both involve an isosceles triangle with extra lines in which so many angles are given that surely finding that angle should be easy—but isn't. There are problems with geometrical and combinatorial flavour, for example 'Let S be a set of $2n + 1 \geqslant 5$ points in the plane (no three on a line and no four on a circle). A circle C 'halves' S if 3 points are on C, and $n - 1$ inside and $n - 1$ outside C. Show that there are at least $n(2n + 1)/3$ halving circles.' This is done by showing that every pair of points is on a halving circle—'on the easy side for this volume' admits the author, but never mind, it is followed up by a tougher one, asking that for every n there is such a set of points with exactly n^2 halving circles. There are many linked problems, for example 'For a prime $p \geqslant 3$ the equation $p^m = r^n - 1$ has no solutions in positive integers m, r, n all > 1'.

This is the climax of a sequence of four problems starting with the more modest 'No perfect powers of 2 and 3 differ by exactly 1, except 2 and 3, 4 and 3, and 8 and 9'.

Several other problems are of an 'elementary number theory' kind (and some not-so-elementary), for example 'The number 1 is not a congruent number, i.e. there is no right-angled triangle with rational sides and area 1', a result due to Fermat; 'Find a large family of integer solutions of $A^4 + B^4 = C^4 + D^4$', a problem with a long history going back to Euler and including Peter Swinnerton-Dyer when he was a scholar at Eton; and 'Let a and b be positive integers such that $q = (a^2 + b^2)/(ab + 1)$ is also an integer. Then q is a square number.' This evidently comes from the 1988 International Mathematical Olympiad, where it did not have so much as a hint.

Some of the problems are decidedly odd (to be included in a problem book, that is) such as the proof that $\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$, where if you remember Euler's brilliant misuse of known results then you might have a chance, but a rigorous proof is not for the amateur. Other 'standard result' problems sit more happily, such as the Euler-Feuerbach result about the altitudes of a triangle: 'Let H be the intersection of the altitudes AD, BE, CF of a triangle ABC. Then the midpoints of the sides, the midpoints of the segments AH, BH, CH and the feet of the altitudes all lie on a circle'; or Sylvester's result that every rational number in (0, 1) is the sum of a finite number of reciprocals of distinct natural numbers. Inequalities and identities are scattered through the problems: Tepper's (factorial) identity, Dixon's identity, Hilbert's (square-summable sequence) inequality, Bessel's inequality, inequalities for the central binomial coefficient. Even the Monty Hall problem, the monkey and the coconuts and gambler's ruin find their place.

I have one complaint: I hope that the publisher and not the author is responsible for the sentence in the blurb on the back cover describing the author as 'teaching the very best undergraduates in England'. Brilliance arises everywhere and in any one year who knows where the 'very best undergraduates' are? Cambridge is not the centre of the universe.

So, a very individual problem book, with a huge range of subject matter and difficulty. There is material here for many purposes, even for enrichment material for advanced sixth formers or undergraduates, but you have to dig for it. Many of the problems are very specialised and (to me) not so attractive, but there is certainly something here, if not for everyone, then for a wide range of mathematical readers.

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Once upon a prime by Sarah Hart, pp 304, £14.95 (hard), ISBN 978-1-25085-088-1, Flatiron Books (2023)

Who is the ideal writer of a book for the lay reader that explores what maths and literature have in common? An expert mathematician, of course. Also, someone able to convey mathematical ideas to the general public—the sort of mathematician who might for example make a good Gresham Professor of Geometry, a chair established for that purpose. And a lover of literature, the sort of person who might sit down to read the entirety of the Booker Prize shortlist every year. Step forward Sarah Hart,