measurable function is defined by subdivision of the range and the definition is extended in the usual way to unbounded functions. The original Russian text contained no treatment of measure or integrals on unbounded sets, but, in this American edition, the editor, Edwin Hewitt, has added appendices to the appropriate chapters in order to rectify this omission. There is a chapter on the space L^2 and the book ends with two chapters on functions of bounded variation, absolute continuity and the differentiation properties of the integral.

Throughout, the book is very easy to read; proofs are given clearly and in full. The author has not confined himself to the bare bones of the subject, but has clad them with a wealth of additional material. As well as the main subjects outlined above, many other topics are introduced. For example, proofs are given of Weierstrass's approximation theorem, Helly's choice principle and Riesz's representation theorem for continuous linear forms on the space C. Within the limitations imposed by restriction to real variables only, the contents form an excellent account of integration theory, and a useful introduction to functional analysis.

A. P. ROBERTSON

ZAANEN, A. C., *An Introduction to the Theory of Integration* (North Holland Publishing Company, Amsterdam, 1958), ix+254 pp., 50s.

It is now customary to develop the general theory of integration either by means of measure theory or by extending an elementary integral to a larger class of functions. The author, feeling that it is important to be familiar with both approaches, has combined them in this book. After an introductory chapter containing some set theory and topology, the book starts with measure theory. Given a measure on a semi-ring of sets, it is shown how to extend it (by means of the corresponding exterior measure and Carathéodory's definition of measurability) to a larger σ-ring of sets. This extension process is then used to develop the theory of the Daniell integral. One starts with a vector lattice of bounded functions on which there is a positive linear functional, continuous under monotonic convergence to zero. The ordinate sets of the positive functions in the lattice generate a semi-ring on which the functional is a measure, and its extension is made to yield the classes of measurable and integrable functions. If the vector lattice contains $\min(f, 1)$ whenever it contains f, every integral so obtained corresponds to a measure in the usual way.

Subsequent chapters deal with such topics as Fubini's theorem, the Radon-Nikodym theorem and differentiation of the integral; in addition to the usual differentiation theory for the Lebesgue integral on a Euclidean space, there is an account of differentiation of set functions relative to a monotone sequence of nets. The author has also included certain parts of functional analysis which are relevant to integration theory; the sections on Banach spaces and Hilbert space form a useful introduction to these subjects, and they are applied to the study of the spaces L_p and the Fourier transformation in L_2. The book ends with a chapter on ergodic theory.

Numerous exercises are scattered throughout the book; many contain further results in the theory and are accompanied by condensed solutions. A reader with the minimum preparation may find the going hard in places, but he will be doubly rewarded, by having mastered the most useful parts of integration theory and also by becoming acquainted with some other important branches of modern analysis.

A. P. ROBERTSON

NIVEN, I., *Irrational Numbers* (Carus Mathematical Monographs No. 11, 1956), 164 pp., 24s.

This book covers a wide field, beginning with Cantor series and the countability of the rationals in Chapter I, then giving an elementary treatment of trigonometric and exponential functions in Chapter II, finishing in the last two chapters with the general theorem of Lindemann and with Gelfond and Schneider's theorem on the transcendence of α^{β}. There are chapters on Diophantine approximation, continued fractions and normal numbers.

One could hardly expect to exhaust the topic in a monograph of less than 200 pages. Yet the material has been so skilfully and concisely arranged that a very great deal is included. Many of the proofs are new in form and some in essentials also, and the whole book gives the impression of careful work, both in outline and in detail. Much of the book can be understood by a reader with only elementary knowledge, yet it covers a substantial part of what is known. Those chapters (such as the ones dealing with Diophantine approximation and algebraic numbers) which necessarily cover only a part of their topic are supplemented by bibliographical notes.

Professor Niven is to be congratulated on having produced an excellent little book.

A. M. MACBEATH

ZYGMUND, A., *Trigonometric Series* (Cambridge University Press, 2nd ed., 1959). Two Volumes. Volume I, xii+383 pp., Volume II, vii+354 pp., 84s. per volume.

The first book bearing this title and written by Professor Zygmund appeared in 1935, and it has now been replaced by this considerably more comprehensive work. To appreciate fully the labours involved in producing what will certainly be regarded for many years to come as the standard treatise on trigonometric series, it is only necessary to note the amount of space in mathematical periodicals devoted to the subject during the past fifty years. The author has had to synthesise this material, simplify proofs where possible, and present a coherent account without submerging his arguments in tiresome detail. Analysts will agree that he has most successfully carried out this task.

Comprehensive though it is, the book does not, and can hardly be expected to, deal with all the recent advances in the subject. Thus, during the past few years, extended definitions for both absolute and strong summability have appeared, and have produced some interesting results when applied to Fourier Series. Further, to the vast field of multiple Fourier series, the author only devotes one chapter, but he justifies this brief treatment on the grounds that most of the theory of such series is deducible from the corresponding theory of single Fourier series. The reader, therefore, who wishes to use the book as a work of reference may not, on occasions, discover what he is seeking, but he will certainly find his subject matter dealt with from the basic point of view.

It would be pointless in this brief review to present even an outline of the contents of the book, but I feel that the attention of readers should be especially drawn to the first chapter. There the author sets out clearly the results, borrowed from the theory of sets and of integrals, on which the subsequent development rests. A grasp of this chapter, together with a knowledge of the definition and simple properties of summability by the Abel and Cesàro methods, will enable the reader to take up almost any later chapter at will without feeling obliged to become acquainted with the intervening work. Wherever essential groundwork does not appear in this first chapter, a sketch is provided in that chapter where it is to be applied, so that the book may be regarded as being almost completely self-contained.

At the end of each chapter there is a varied set of examples, so that anyone sufficiently interested may be encouraged to acquire, not only a reading, but also a working knowledge of the subject. They range from those which illustrate points not fully dealt with in the main development to some which, if space permitted, could well be classified as theorems. In short, the book will commend itself to analysts on

https://doi.org/10.1017/S0013091500025086 Published online by Cambridge University Press