or $L(A \vee B) \rightarrow (LA \vee LB)$, which are part of what it is known as strong modal Łukasiewicz-type paradoxes. Because of this, this system was strongly criticized. On the other hand, in [Brady, 1982], R. T. Brady presents his relevant logic BN4, a four-valued version of the relevant implication system R. Taking this background into account, the main goal of this research is to build a system that works as a companion of BN4 (just like E does with respect to R) and lacks the paradoxes that can be found in Łukasiewicz's system. Firstly, we define the matrix M4, which is the base for all the systems that we develop later. We then introduce two different semantics, i.e., the four-valued semantics related to the matrix and a bivalent Belnap-Dunn type semantics, and we show that both semantics are equivalent. Next, the system that we have labeled FDF4, which is based on FDE, is defined. We prove that this system is both sound and complete in the strong sense and that it is indeed an axiomatization of the M4 matrix. Afterwards, we define a system based on E that we name EF4, for which we also prove strong soundness and completeness and how it originates from the M4 matrix, all of this based on the fact that EF4 is a system equivalent to FDF4. With respect to EF4, two different modalities are presented: the first one, which in this case is equivalent to the inherent modality of E, is developed from the interdefinitional extensions used by Łukasiewicz, and the second one, from the proposal of J. Y. Beziau related to the approach of J. M. Font and M. Rius that in its turn is linked to the Portuguese algebraic tradition led by A. Monteiro. In this way, we get two different modal systems, EF4-M and EF4-Ł. For the former, we give just one axiomatization, while for the latter, we supply up to four different ones. For both systems, EF4-M and EF4-Ł, we prove soundness and completeness. Furthermore, EF4 is provided with a reduced ternary relational semantics, as well as with a 2-set-up ternary relational semantics, and it is proven that it is sound and complete with respect to both semantics. Finally, it is shown that the system FDF4 is also sound and complete with respect to both aforementioned relational semantics and that the 2-set-up semantics is a particular case of the reduced semantics.

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Patrick Walsh, ***A Categorical Characterization of Accessible Domain***, Carnegie Mellon University, USA, 2019. Supervised by Wilfried Sieg. MSC: 00A30, 03G30, 03D70. Keywords: category theory, Hilbert's finitist program, inductive definitions, algebraic set theory.

Abstract

Inductively defined structures are ubiquitous in mathematics; their specification is unambiguous and their properties are powerful. All fields of mathematical logic feature these structures prominently: the formulas of a language, the set of theorems, the natural numbers, the primitive recursive functions, the constructive number classes and segments of the cumulative hierarchy of sets.

This dissertation gives a mathematical characterization of a species of inductively defined structures, called *accessible domains*, which include all of the above examples except the set of theorems. The concept of an accessible domain comes from Wilfried Sieg's analysis of proof-theoretic practices, starting with his dissertation (contributed to [1]). In particular, he noticed the special epistemological character of elements of an accessible domain: they can always be uniquely identified with their build-up. Generally, the unique build-up of elements justifies the principles of induction and recursion.

I use category theory to give an abstract characterization of accessible domains. I claim that accessible domains are all instances of *initial algebras for endofunctors*. Grounded in the historical roots of Sieg's discussions, this dissertation shows how the properties of initial algebras for endofunctors and accessible domains coincide in a satisfying and natural way.

Filling out this characterization, I show how important examples of accessible domains fit into this broad characterization. I first characterize some accessible domains by relatively

simple functors (e.g., finite, polynomial, those that preserve certain colimits) where we can see how iterating the functor can produce the accessible domain. Then I describe accessible domains that result from more involved specifications (e.g., ordinals and segments of the cumulative hierarchies associated with CZF, IZF, and ZF) by relying heavily on *algebraic set theory* [2]. I end with a discussion of some of the methodological features of category theory in particular that helped characterize accessible domains.

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Gianluca Basso, ***Compact Metrizable Structures via Projective Fraïssé Theory With an Application to the Study of Fences***, Università di Torino, Italy, and Université de Lausanne, Switzerland, 2020. Supervised by Riccardo Camerlo and Jacques Duparc. MSC: Primary 03E15. Secondary 54F50, 54F65. Keywords: Compact metrizable spaces, topological structures, projective Fraïssé limits.

Abstract

In this dissertation we explore projective Fraïssé theory and its applications, as well as limitations, to the study of compact metrizable spaces. The goal of projective Fraïssé theory is to approximate spaces via classes of finite structures and glean topological or dynamical properties of a space by relating them to combinatorial features of the associated class of structures.

Using the framework of *compact metrizable structures*, we establish general results which expand and help contextualize previous works in the field. Many proofs in the domain of projective Fraïssé theory are carried out in a context dependent fashion and have thus far eluded clean generalizations. A reason is to be found in the lack of a clear understanding of which spaces are amenable to be studied via projective Fraïssé limits. We give both positive and negative results in this direction.

We isolate a combinatorial condition which entails a correspondence between finite structures and regular quasi-partitions of compact metrizable spaces. This correspondence greatly aids the combinatorial-topological translation. We apply this machinery to study a class of one-dimensional compact metrizable spaces, which we call *smooth fences*. To this end, we isolate a class of finite structures—finite partial orders whose Hasse diagram is a forest—whose projective Fraïssé limit approximates a distinctive smooth fence with remarkable properties. We call it the *Fraïssé fence* and characterize it topologically by carefully exploiting the bridge between the combinatorial and topological worlds. We explore homogeneity and universality features of the Fraïssé fence and the properties of its spaces of endpoints, and provide some results on the dynamics of its group of homeomorphisms.

The dissertation is partially based on [1, 2].

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