9

Wall-string junctions

In Chapter 8 we reviewed the construction of D-brane prototypes in field theory. In string theory D branes are extended objects on which fundamental strings can end. To make contact with this string/brane picture one may address a question whether or not solitonic strings can end on the domain wall which localizes gauge fields. The answer to this question is yes. Moreover, the string endpoint plays a role of a charge with respect to the gauge field localized on the wall surface. This issue was studied in [215] in the sigma-model setup and in [217] for gauge theories at strong coupling. A solution for a 1/4-BPS wall-string junction in the $\mathcal{N} = 2$ supersymmetric U(1) gauge theory at weak coupling was found in [142], while [37] deals with its non-Abelian generalization. Further studies of the wall-string junctions were carried out in [172] where all 1/4-BPS solutions to Eqs. (4.5.5) were obtained, and in [218, 231] where the energy associated with the wall-string junction (boojum) was calculated, and in [232, 228] where a quantum version of the effective theory on the domain wall world volume which takes into account charged matter (strings of the bulk theory) was derived. Below we will review the wall-string junction solutions and then briefly discuss how the presence of strings in the bulk modifies the effective theory on the wall.

9.1 Strings ending on the wall

To begin with, let us review the solution for the simplest 1/4-BPS wall-string junction in $\mathcal{N} = 2$ SQED obtained in [142]. As was discussed in Chapter 8, in both vacua of the theory the gauge field is Higgsed while it can spread freely inside the wall. This is the physical reason why the ANO string carrying a magnetic flux can end on the wall [11, 213].

Assume that at large distances from the string endpoint which lies at $r = 0$, $z = 0$ the wall is almost parallel to the (x_1, x_2) plane while the string is stretched along the z axis. As usual, we look for a static solution assuming that all relevant

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fields can depend only on x_n, $(n = 1, 2, 3)$. The Abelian version of the first-order equations (4.5.5) for various 1/4-BPS junctions in the theory (8.1.1) is [142]

$$\begin{aligned} & F_1^* - i F_2^* - \sqrt{2}(\partial_1 - i\partial_2)a = 0\,, \\ & F_3^* - \frac{g^2}{2}\left(\left|q^A\right|^2 - \xi\right) - \sqrt{2}\,\partial_3 a = 0\,, \\ & \nabla_3 q^A = -\frac{1}{\sqrt{2}} q^A (a + \sqrt{2} m_A)\,, \\ & (\nabla_1 - i\nabla_2) q^A = 0\,. \end{aligned} \tag{9.1.1}$$

These equations generalize the first-order equations for the wall (8.2.2) and the Abelian ANO string.

Needless to say, the solution of the first-order equations (9.1.1) for a string ending on the wall can be found only numerically especially near the endpoint of the string where both the string and the wall profiles are heavily deformed. However, far away from the string endpoint, deformations are weak and we can find the asymptotic behavior analytically.

Let the string be on the $z > 0$ side of the wall, where the vacuum is given by Eq. (8.1.5). First note that in the region $z \to \infty$ far away from the string endpoint at $z \sim 0$ the solution to (9.1.1) is given by an almost unperturbed ANO string. Now consider the domain $r \to \infty$ at small z. In this domain the solution to (9.1.1) is given by a perturbation of the wall solution. Let us use the *ansatz* in which the solutions for the fields a and q^A are given by the same equations (8.2.5), (8.2.8) and (8.2.9) in which the size of the wall is still given by (8.2.7), and the only modification is that the position of the wall z_0 and the phase σ now become slowly varying functions of r and α, the polar coordinates on the (x_1, x_2) plane. It is quite obvious that z_0 will depend only on r, as schematically depicted in Fig. 9.1.

Substituting this *ansatz* into the first-order equations (9.1.1) one arrives at the equations which determine the adiabatic dependence of the moduli z_0 and σ on r and α [142],

$$\partial_r z_0 = -\frac{1}{\Delta m r}, \tag{9.1.2}$$

$$\frac{\partial \sigma}{\partial \alpha} = 1, \qquad \frac{\partial \sigma}{\partial r} = 0\,. \tag{9.1.3}$$

Needless to say our adiabatic approximation holds only provided the r derivative is small, i.e. sufficiently far from the string, $\sqrt{\xi} r \gg 1$. The solution to Eq. (9.1.2) is straightforward,

$$z_0 = -\frac{1}{\Delta m} \ln r + \text{const.} \tag{9.1.4}$$

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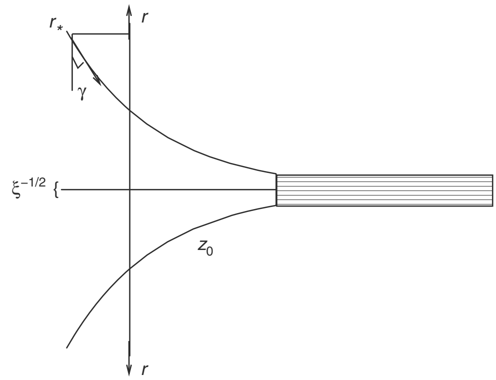

Figure 9.1. Bending of the wall due to the string-wall junction. The flux tube extends to the right infinity. The wall profile is logarithmic at transverse distances larger than $\xi^{-1/2}$ from the string axis. At smaller distances the adiabatic approximation fails.

We see that the wall is logarithmically bent according to the Coulomb law in $2+1$ dimensions (see Fig. 9.1). This bending produces a balance of forces between the string and the wall in the z direction so that the whole configuration is static. The solution to Eq. (9.1.3) is

$$\sigma = \alpha\,. \tag{9.1.5}$$

This vortex solution is certainly expected and welcome. One can identify the compact scalar field σ with the electric field living on the domain wall world volume via (8.3.10). Equation (9.1.5) implies

$$F_{0i}^{(2+1)} = \frac{e^2}{2\pi}\frac{x_i}{r^2} \tag{9.1.6}$$

for this electric field, where the $(2+1)$-dimensional coupling is given by (8.3.9).

This is the field of a point-like electric charge in $2+1$ dimensions placed at $x_i = 0$. The interpretation of this result is that the string endpoint on the wall plays a role of the electric charge in the dual U(1) theory on the wall. From the standpoint of the bulk theory, when the string ends on the wall, the magnetic flux it brings with it spreads out inside the wall in accordance with the Coulomb law in $(2+1)$ dimensions.

From the above discussion it is clear that in the world volume theory (8.3.13), the fields (9.1.4) and (9.1.6) can be considered as produced by classical point-like charges which interact in a standard way with the electromagnetic field A_n and the scalar field a_{2+1},

$$S_{2+1} = \int d^3x \left\{ \frac{1}{2e^2}(\partial_n a_{2+1})^2 + \frac{1}{4e^2}(F_{mn}^{(2+1)})^2 + A_n\, j_n - a_{2+1}\,\rho \right\}, \tag{9.1.7}$$

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where the classical electromagnetic current and the charge density of static charges are

$$j_n(x) = n_e\{\delta^{(3)}(x), 0, 0\}, \quad \rho(x) = n_s \delta^{(3)}(x). \tag{9.1.8}$$

Here n_e and n_s are electric and scalar charges associated with the string endpoint with respect to the electromagnetic field A_n and the scalar field a, respectively [228],

$$\begin{aligned} n_e &= +1, \quad \text{incoming flux,} \\ n_e &= -1, \quad \text{outgoing flux,} \end{aligned} \tag{9.1.9}$$

while their scalar charges are

$$\begin{aligned} n_s &= +1, \quad \text{string from the right,} \\ n_s &= -1, \quad \text{string from the left.} \end{aligned} \tag{9.1.10}$$

These rules are quite obvious from the perspective of the bulk theory. The anti-string carries the opposite flux to that of a string in (9.1.6) and the bending of the wall produced by the string coming from the left is opposite to the one in (9.1.4) associated with the string coming from the right.

9.2 Boojum energy

Let us now calculate the energy of the wall-string junction, the boojum. There are two distinct contributions to this energy [231]. The first contribution is due to the gauge field (9.1.6),

$$\begin{aligned} E^G_{(2+1)} &= \int \frac{1}{2e^2_{2+1}} (F_{0i})^2 \, 2\pi r \, dr \\ &= \frac{\pi \xi}{\Delta m} \int \frac{dr}{r} = \frac{\pi\xi}{\Delta m} \ln\left(g\sqrt{\xi} L\right). \end{aligned} \tag{9.2.1}$$

The integral $\int dr/r$ is logarithmically divergent both in the ultraviolet and infrared. It is clear that the UV divergence is cut off at the transverse size of the string $\sim 1/g\sqrt{\xi}$ and presents no problem. However, the infrared divergence is much more serious. We introduced a large size L to regularize it in (9.2.1).

The second contribution, due to the z_0 field (9.1.4), is proportional to $\int dr/r$ too,

$$\begin{aligned} E^H_{(2+1)} &= \int \frac{T_{\rm w}}{2} \left(\partial_r z_0\right)^2 2\pi r \, dr \\ &= \frac{\pi\xi}{\Delta m} \ln\left(g\sqrt{\xi} L\right). \end{aligned} \tag{9.2.2}$$

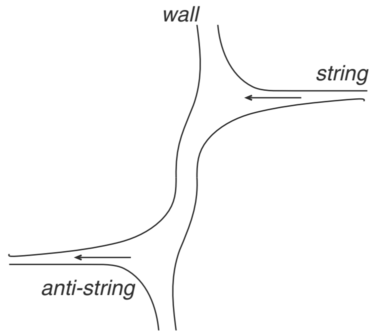

Figure 9.2. String and anti-string ending on the wall from different sides. Arrows denote the direction of the magnetic flux.

Both contributions are logarithmically divergent in the infrared. Their occurrence is an obvious feature of charged objects coupled to massless fields in $(2+1)$ dimensions due to the fact that the fields A_n and a_{2+1} do not die off at infinity, which means infinite energy.

The above two contributions are equal (with the logarithmic accuracy), even though their physical interpretation is different. The total energy of the string junction is

$$E^{G+H} = \frac{2\pi\xi}{\Delta m} \ln\left(g\sqrt{\xi}L\right). \tag{9.2.3}$$

We see that in our attempt to include strings as point-like charges in the world volume theory (9.1.7) we encounter problems already at the classical level. The energy of a single charge is IR divergent. It is clear that the infrared problems will become even more severe in quantum theory.

A way out was suggested in [231, 228]. For the infrared divergences to cancel we should consider strings and anti-strings with incoming and outgoing fluxes as well as strings coming from the right and from the left. Clearly, only configurations with vanishing total electric and scalar charges have finite energy (see (9.1.9) and (9.1.10)).

In fact, it was shown in [231] that the configuration depicted in Fig. 9.2 is a non-interacting 1/4-BPS configuration. All logarithmic contributions are canceled; the junction energy in this geometry is given by a finite negative contribution

$$E = -\frac{8\pi}{g^2}\,\Delta m, \tag{9.2.4}$$

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which is called the boojum energy [218]. In fact this energy was first calculated in [172]. A procedure allowing one to separate this finite energy from logarithmic contributions described above (and make it well-defined) was discussed in [231].

9.3 Finite-size rigid strings stretched between the walls. Quantizing string endpoints

Now, after familiarizing ourselves with the junctions of the BPS walls with the semi-infinite strings, the boojums, we can ask whether or not the junction can acquire a dynamical role. Is there a formulation of the problem in which one can speak of a junction as of a particle sliding on the wall?

The string energy is its tension (4.2.12) times its length. If we have a single wall, all strings attached to it have half-infinite length; therefore, they are infinitely heavy. In the wall world volume theory (9.1.7) they may be seen as classical infinitely heavy point-like charges. The junctions are certainly non-dynamical objects in this case.

In order to be able to treat junctions as "particles" we need to make strings "light" and deprive them of their internal dynamics, i.e. switch off all string excitations. It turns out a domain in the parameter space is likely to exist where these goals can be achieved.

In this section we will review a quantum version of the world volume theory (9.1.7) with additional charged matter fields. The latter will represent the junctions on the wall world volume [232, 228] (of course, the junctions have strings of the bulk theory attached to them; these strings will be rigid).

Making string masses finite is a prerequisite. To this end one needs at least two domain walls at a finite distance from each other with strings stretched between them. This scenario was suggested in [232]. A quantum version of the wall world volume theory in which the strings were represented by a charged chiral matter superfield in $1+2$ dimensions was worked out. However, in the above scenario the strings were attached to each wall from one side. From the discussion in Section 9.2 it must be clear that this theory is not free from infrared problems. The masses of $(1+2)$-dimensional charged fields are infinite.

To avoid these infinities we need a configuration with strings coming both from the right and from the left sides of each wall. This configuration was suggested in [228], see Fig. 9.3.

Let us describe this set-up in more detail. First, we compactify the $x_3 = z$ direction in our bulk theory (8.1.1) on a circle of length L. Then we consider a pair "wall plus antiwall" oriented in the $\{x_1, x_2\}$ plane, separated by a distance l in the perpendicular direction, as shown in Fig. 9.3. The wall and antiwall experience attractive forces. Strictly speaking, this is not a BPS configuration –

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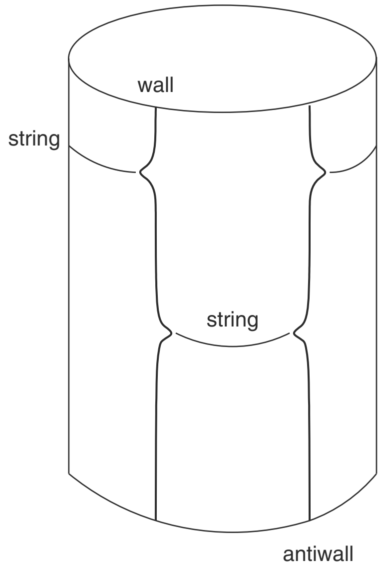

Figure 9.3. A wall and antiwall connected by strings on the cylinder. The circumference of the circle (the transverse slice of the cylinder) is L.

supersymmetry in the world volume theory is broken. However, the wall-antiwall interaction due to overlap of their profile functions is exponentially suppressed at large separations,

$$L \sim l \gg R, \tag{9.3.1}$$

where R is the wall thickness (see Eq. (8.2.7)). In what follows we will neglect exponentially suppressed effects. If so, we neglect the effects which break supersymmetry in our $(2+1)$-dimensional world volume theory. Thus, it continues to have four conserved supercharges ($\mathcal{N}=2$ supersymmetry in $(2+1)$ dimensions) as was the case for the isolated single wall.

Let us denote the wall position as z_1 while that of the antiwall as z_2. Then

$$l = z_2 - z_1.$$

On the wall world volume $z_{1,2}$ become scalar fields. The kinetic terms for these fields in the world volume theory are obvious (see (8.3.11)),

$$\frac{T_{\rm w}}{2}\left[(\partial_n z_1)^2 + (\partial_n z_2)^2\right] = \frac{1}{2e^2}\left[\left(\partial_n a^{(1)}_{2+1}\right)^2 + \left(\partial_n a^{(2)}_{2+1}\right)^2\right], \tag{9.3.2}$$

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where we use (8.3.12) to define the fields $a^{(1,2)}_{2+1}$. The sum of these fields,

$$a_+ \equiv \frac{1}{\sqrt{2}}\Big(a^{(2)}_{2+1} + a^{(1)}_{2+1}\Big),$$

with the corresponding superpartners, decouples from other fields forming a free field theory describing dynamics of the center-of-mass of our construction. This is a trivial part which will not concern us here.

An interesting part is associated with the field

$$a_- \equiv \frac{1}{\sqrt{2}}\Big(a^{(2)}_{2+1} - a^{(1)}_{2+1}\Big). \tag{9.3.3}$$

The factor $1/\sqrt{2}$ ensures that a_- has a canonically normalized kinetic term. By definition, it is related to the relative wall-antiwall separation, namely,

$$a_- = \frac{2\pi\xi}{\sqrt{2}}\, l\,. \tag{9.3.4}$$

Needless to say, a_- has all necessary $\mathcal{N}=2$ superpartners. In the bosonic sector we introduce the gauge field

$$A^-_n \equiv \frac{1}{\sqrt{2}}\Big(A^{(1)}_n - A^{(2)}_n\Big), \tag{9.3.5}$$

with the canonically normalized kinetic term. The strings stretched between the wall and antiwall, on both sides, will be represented by two chiral superfields, S and $\tilde{S}$, respectively. We will denote the corresponding bosonic components by s and $\tilde{s}$.

In terms of these fields the quantum version of the theory (9.1.7) is completely determined by the charge assignments (9.1.9) and (9.1.10) and $\mathcal{N}=2$ supersymmetry. The charged matter fields have the opposite electric charges and distinct mass terms, see below. A mass term for one of them is introduced by virtue of a "real mass," a three-dimensional generalization [96] of the twisted mass in two dimensions [32]. It is necessary due to the fact that there are two inter-wall distances, l and $L-l$. The real mass breaks parity. The bosonic part of the action has the form

$$S_{\rm bos} = \int d^3x\,\Big\{\frac{1}{4e^2}F^-_{mn}F^-_{mn} + \frac{1}{2e^2}\big(\partial_n\, a_-\big)^2 + |\mathcal{D}_n s|^2 + |\tilde{\mathcal{D}}_n\tilde{s}|^2 + 2a_-^2\,\bar{s}\, s + 2(m-a_-)^2\,\bar{\tilde{s}}\,\tilde{s} + e^2\big(|s|^2 - |\tilde{s}|^2\big)^2\Big\}. \tag{9.3.6}$$

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According to our discussion in Section 9.1, the fields s and $\tilde{s}$ have charges $+1$ and -1 with respect to the gauge fields $A_n^{(1)}$ and $A_n^{(2)}$, respectively. Hence,

$$\mathcal{D}_n = \partial_n - i\left(A_n^{(1)} - A_n^{(2)}\right) = \partial_n - i\sqrt{2}A_n^-,$$
$$\tilde{\mathcal{D}}_n = \partial_n + i\left(A_n^{(1)} - A_n^{(2)}\right) = \partial_n + i\sqrt{2}A_n^-. \tag{9.3.7}$$

The electric charges of boojums with respect to the field A_n^- are $\pm\sqrt{2}$. The last term in (9.3.6) is the D term dictated by supersymmetry.

So far, m is a free parameter whose relation to L will be determined shortly. Moreover, $F_{mn}^- = \partial_m A_n^- - \partial_n A_m^-$. The theory (9.3.6) with the pair of chiral multiplets S and $\tilde{S}$ is free of IR divergences and global Z_2 anomalies [96, 98] (see also Section 3.2.1). At the classical level it is clear from our discussion in Section 9.2. A version of the world volume theory (9.3.6) with a single supermultiplet S was considered in Ref. [232] but, as was mentioned, this version is not free of IR divergences.

It is in order to perform a crucial test of our theory (9.3.6) by calculating the masses of the charged matter multiplets S and $\tilde{S}$. From (9.3.6) we see that the mass of S is

$$m_s = \sqrt{2}\,\langle a_-\rangle. \tag{9.3.8}$$

Substituting here the relation (9.3.4) we get

$$m_s = 2\pi\xi\, l. \tag{9.3.9}$$

The mass of the charged matter field S reduces to the mass of the string of the bulk theory stretched between the wall and antiwall at separation l, see (4.2.12). A great success! Of course, this was expected. Note that this is a nontrivial check of consistency between the world volume theory and the bulk theory. Indeed, the charges of the strings' endpoints (9.1.9) and (9.1.10) are unambiguously fixed by the classical solution for the wall-string junction.

Now, imposing the relation between the free mass parameter m in (9.3.6) and the length of the compactified z-direction L in the form

$$m = \frac{2\pi\xi}{\sqrt{2}}\,L \tag{9.3.10}$$

we get the mass of the chiral field $\tilde{S}$ to be

$$m_{\tilde{s}} = 2\pi\xi\,(L - l)\,. \tag{9.3.11}$$

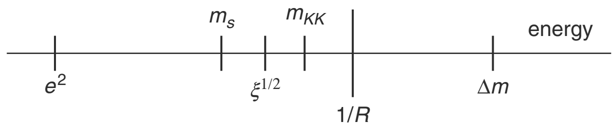

Figure 9.4. Mass scales of the bulk and world volume theories.

The mass of the string $\tilde{S}$ connecting the wall with the antiwall from the other side of the cylinder is the string tension times $(L - l)$, in full accordance with our expectations, see Fig. 9.3.

The theory (9.3.6) can be considered as an effective low-energy $(2 + 1)$-dimensional description of the wall-antiwall system dual to the $(3+1)$-dimensional bulk theory (8.1.1) under the choice of parameters specified below (Fig. 9.4). Most importantly, we use the quasiclassical approximation in our bulk theory (8.1.1) to find the solution for the string-wall junction [142] and derive the wall-antiwall world volume effective theory (9.3.6). This assumes weak coupling in the bulk, $g^2 \ll 1$. According to the duality relation (8.3.14) this implies strong coupling in the world volume theory.

In order to be able to work with the world volume theory we want to continue the theory (9.3.6) to the weak coupling regime,

$$e^2 \ll \frac{1}{R}, \tag{9.3.12}$$

which means strong coupling in the bulk theory, $g^2 \gg 1$. The general idea is that at $g^2 \ll 1$ we can use the bulk theory (8.1.1) to describe our wall-antiwall system while at $g^2 \gg 1$ we better use the world volume theory (9.3.6). In [228] this set-up was termed bulk–brane duality. In spirit – albeit not in detail – it is similar to the AdS/CFT correspondence.

In order for the theory (9.3.6) to give a correct low-energy description of the wall-antiwall system the masses of strings (including boojums) in this theory should be much less than the masses of both the wall and string excitations. These masses are of order of $1/R$ and $m_{KK} = k/l \sim k/(L - l)$, respectively, where k is an integer. The high mass gap for the string excitations make strings rigid.

These constraints were studied in [228]. It was found that for the constraints to be satisfied different scales of the theory must have a hierarchy shown in Fig. 9.4.

The scales Δm, $\sqrt{\xi}$ and $e^2_{2+1} \sim \xi/\Delta m$ are determined by the string and wall tensions in our bulk theory, see (4.2.12) and (8.2.4). In particular, the $(2 + 1)$-dimensional coupling e^2 is determined by the ratio of the wall tension to the square of the string tension, as follows from Eqs. (8.3.11) and (8.3.12). Since the strings and walls in the bulk theory are BPS-saturated, they receive no quantum corrections.

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Equations (4.2.12) and (8.2.4) can be continued to the strong coupling regime in the bulk theory. Therefore, we can always take such values of the parameters Δm and $\sqrt{\xi}$ that the conditions

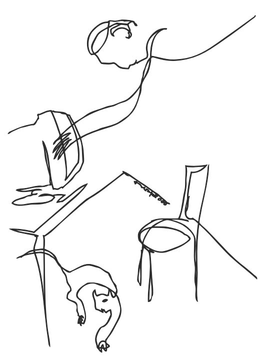
$$e^2 \ll \sqrt{\xi} \ll \Delta m \tag{9.3.13}$$

are satisfied.

To actually prove duality between the bulk theory (8.1.1) and the world volume theory (9.3.6) we only need to prove the condition

$$\sqrt{\xi} \ll \frac{1}{R}, \tag{9.3.14}$$

which ensures that strings are lighter than the wall excitations. This will give us the hierarchy of the mass scales shown in Fig. 9.4. With the given values of the parameters Δm and $\sqrt{\xi}$ we have another free parameter of the bulk theory to ensure (9.3.14), namely, the coupling constant g^2. However, the scale $1/R$ (the mass scale of various massive excitations of the wall) is not protected by supersymmetry and we cannot prove that the regime (9.3.14) can be reached at strong coupling in the bulk theory. Thus, the above bulk–brane duality is in fact a conjecture essentially equivalent to the statement that the regime (9.3.14) is attainable under a certain choice of parameters. Note, that if the condition (9.3.14) is not met, the wall excitations become lighter than the strings under consideration, and the theory (9.3.6) does not correctly describe low-energy physics of the theory on the walls.



9.4 Quantum boojums. Physics of the world volume theory

What is a boojum loop?

Let us integrate out the string multiplets S and $\tilde{S}$ and study the effective theory for the U(1) gauge supermultiplet at scales below m_s. As long as the string fields enter the action quadratically (if we do not resolve the algebraic equations for the auxiliary fields) the one-loop approximation is exact.

Integration over the charged matter fields in (9.3.6) leads to generation of the Chern–Simons term [94, 95, 96] with the coefficient proportional to

$$\frac{1}{4\pi}[\text{sign}(a) + \text{sign}(m-a)]\varepsilon_{nmk}A_n^-\partial_m A_k^- . \tag{9.4.1}$$

Another effect related to the one in (9.4.1) by supersymmetry is generation of a nonvanishing D-term,

$$\frac{D}{2\pi}[|m-a_-| - |a_-|] = \frac{D}{2\pi}(m-2a_-), \tag{9.4.2}$$

where D is the D-component of the gauge supermultiplet. As a result we get from (9.3.6) the following low-energy effective action for the gauge multiplet:

$$S_{2+1} = \int d^3x \left\{ \frac{1}{2e^2(a_-)}(\partial_n a_-)^2 + \frac{1}{4e^2(a_-)}(F_{mn}^-)^2 \right.$$
$$\left. + \frac{1}{2\pi}\varepsilon_{nmk}A_n^-\partial_m A_k^- + \frac{e^2(a_-)}{8\pi^2}(2a_- - m)^2 \right\}, \tag{9.4.3}$$

where we also take into account a finite renormalization of the bare coupling constant e^2 [233, 234, 98],

$$\frac{1}{e^2(a_-)} = \frac{1}{e^2} + \frac{1}{8\pi|a_-|} + \frac{1}{8\pi|m-a_-|}. \tag{9.4.4}$$

This is a small effect since $1/e^2$ is the largest parameter (see Fig. 9.4). Note that in Eq. (9.4.3) the coefficient in front of the Chern–Simons term is an integer number (in the units of $1/(2\pi)$), as required by gauge invariance.

The most dramatic effect in (9.4.3) is the generation of a potential for the field a_-. Remember a_- is proportional to the separation l between the walls. The vacuum of (9.4.3) is located at

$$\langle a_-\rangle = \frac{m}{2}, \quad l = \frac{L}{2}. \tag{9.4.5}$$

There are two extra solutions at $a_- = 0$ and $a_- = m$, but they lie outside the limits of applicability of our approach.

We see that the wall and antiwall are pulled apart; they want to be located at the opposite sides of the cylinder. Moreover, the potential is quadratically rising with the deviation from the equilibrium point (9.4.5). As was mentioned in the beginning

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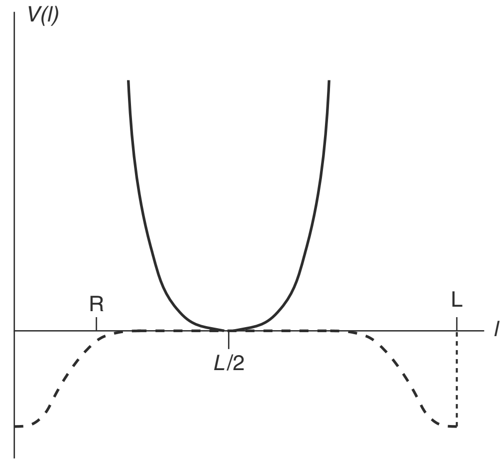


Figure 9.5. Classical and quantum wall-antiwall interaction potential. The dashed line depicts the classical exponentially small potential while the solid line the quantum potential presented in Eq. (9.4.3).

of this section, the wall and antiwall interact with exponentially small potential due to the overlap of their profiles. However, these interactions are negligibly small at $l \gg R$ as compared to the interaction in Eq. (9.4.3). The interaction potential in (9.4.3) arises due to virtual pairs of strings ("boojum loops") and it pulls the walls apart.

Clearly, our description of strings in the bulk theory was purely classical and we were unable to see this quantum effect. The classical and quantum interaction potential of the wall-antiwall system is schematically shown in Fig. 9.5. The quantum potential induced by virtual string loop is much larger than the classical exponentially small $W\bar{W}$ attraction at separations $l \sim L/2$. The quantum effect stabilizes the classically unstable $W\bar{W}$ system at the equilibrium position (9.4.5).

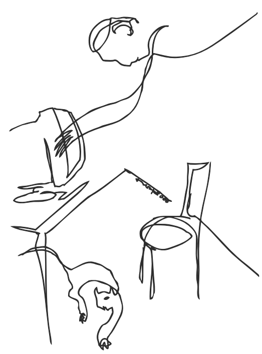
Note, that if the wall-antiwall interactions were mediated by particles they would have exponential fall-off at large separations l (there are no massless particles in the bulk). Quadratically rising potential would never be generated. In our case the interactions are due to virtual pairs of extended objects – strings. Strings are produced as rigid objects stretched between walls. The string excitations are not taken into account as they are too heavy. The fact that the strings come out in our treatment as rigid objects rather than local particle-like states propagating between the walls is of a paramount importance. This is the reason why the wall-antiwall potential does not fall off at large separations. Note that a similar effect, power-law interactions between the domain walls in $\mathcal{N} = 1$ SQCD, was obtained via a two-loop calculation in the effective world volume theory [235].

The presence of the potential for the scalar field a_- in Eq. (9.4.3) makes this field massive, with mass

$$m_a = \frac{e^2}{\pi}. \tag{9.4.6}$$

By supersymmetry, the photon is no longer massless too, it acquires the same mass. This is associated with the Chern–Simons term in (9.4.3). As it is clear from Fig. 9.4, $m_a \ll m_s$. This shows that integrating out massive string fields in (9.3.6) to get (9.4.3) makes sense.

Another effect seen in (9.4.3) is the renormalization of the coupling constant which results in a non-flat metric. Of course, this effect is very small in our range of parameters since $m_s \gg e^2$. Still we see that the virtual string pairs induce additional power interactions between the walls through the nontrivial metric in (9.4.3).



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