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BOOK REVIEWS

JAMESON, G. J. O., *Summing and nuclear norms in Banach space theory* (London Mathematical Society Student Texts 8, Cambridge University Press, 1987), xii + 172 pp., hard cover 0 521 34134 5, £27.50; paper 0 521 34937 0, £8.50.

It is well known that an unconditionally convergent complex series Σz_n is absolutely convergent; that is, if $\Sigma z_{\pi(n)}$ converges for every permutation π of the positive integers, then $\Sigma |z_n|$ converges. However, this result does not extend to an infinite-dimensional setting. Indeed, A. Dvoretzky and C. A. Rogers showed in 1950 that, in any infinite-dimensional Banach space, there exists a conditionally convergent series Σx_n such that $\Sigma \|x_n\|$ diverges. With this fact in mind, it is natural to say that a linear operator $T:X \to Y$, where X and Y are Banach spaces, is absolutely summing if it maps unconditionally convergent series into absolutely convergent ones. A necessary and sufficient condition for T to be absolutely summing can be given in terms of the quantities

$$\mu_1(x_1,\ldots,x_k) \equiv \sup \sum_{n=1}^{k} |f(x_n)|,$$

defined for all k-tuples $x_1,\ldots,x_k \in X$, the supremum being taken over all f in the unit ball $U(X^*)$ of the dual space X^* of X. The operator T is absolutely summing if and only if there is a constant M such that, for all $x_1,\ldots,x_k \in X$ and all k,

$$\sum_{n=1}^{k} \|Tx_n\| \leqq M\mu_1(x_1,\ldots,x_k).$$

The least such constant M is called the absolutely summing norm of T and is denoted by $\pi_1(T)$. This led A. Pietsch in the 1960's to introduce the notion of a p-absolutely summing operator T as one for which there is a constant M such that

$$\left\{\sum_{n=1}^{k} \|Tx_n\|^p\right\}^{1/p} \leqq M \sup\left\{\left(\sum_{n=1}^{k} |f(x_n)|^p\right)^{1/p} : f \in U(X^*)\right\},$$

the least such constant M being the p-summing norm $\pi_p(T)$ of T.

These concepts have proved to be extremely important in the study of the geometry of classical Banach spaces. For instance, it follows from Grothendieck's celebrated inequality that all bounded operators from l^1 to l^2 are absolutely summing, whilst those from c_0 to l^p are 2-absolutely summing whenever $1 \leqq p \leqq 2$. Using these results, it can be shown that the spaces l^1 and c_0 have (essentially) unique unconditional bases. Likewise, 2-summing norms can be used to show that, if X is an arbitrary n-dimensional Banach space, then the Banach–Mazur distance from X to n-dimensional Hilbert space does not exceed $\sqrt{n}$ (this being the best possible result).

The present book gives an account of the theory of p-summing norms, together with the dual class of p-nuclear norms, concentrating almost exclusively on the most important cases when $p=1$ or 2. Much of the theory is developed in a finite-dimensional setting, which reflects the fact that, although usually applicable to infinite-dimensional spaces, it is finite dimensional in essence. Typically, what is at the heart of an infinite-dimensional result is often a dimensionally independent inequality in finite dimensions. There are eighteen chapters, the first eleven of which are devoted to core material (the summing and nuclear norms, Pietsch's fundamental theorem for p-absolutely summing operators, averaging techniques, the inequalities of Khinchin and Grothendieck, type and cotype) and the remainder with further selected ideas (basis constants, tensor products of operators, local reflexivity, cone-summing norms).

The book abounds with illustrations and exercises, and has been written in the author's characteristically enthusiastic style. He believes strongly that proofs of results should do more than merely establish their truth in a logical way; they should also give (as he says) a feel for *why* they are true. The only prerequisite is a knowledge of the basic theory of normed spaces and so it will prove an extremely useful text for a graduate student wishing to learn about an important area in modern Banach space theory.

T. A. GILLESPIE

ARROWSMITH, D. K. and PLACE, C. M. *An Introduction to Dynamical Systems* (Cambridge University Press, 1990), 433 pp., hard cover 0 521 30362 1, £50; paper 0 521 31650 2, £19.50.

There must be many university and college teachers who will grasp this book eagerly. The introduction of courses in modern dynamical systems theory is a growth industry, which is hampered only by a paucity of texts aimed at the right level. This one has its sights set on the interface between final-year undergraduates and first-year postgraduates. It does not aim to survey the field comprehensively, but the subjects on which it concentrates are taken at a measured pace which shows that the authors have kept this particular audience carefully in mind.

One of the great strengths of the book, at least as a teaching aid, is the abundance of exercises, over 300 in all. Most are in the style of ready-made examination questions, and, though there are no anwers, there is a helpful section of hints tucked at the back of the book. The 200-odd illustrations are also impressive, which is an important element in the teaching of such a subject as this where one diagram can convey a much better idea than pages of definitions. The figure illustrating the Melnikov function is just one of many beautifully executed diagrams.

The authors' fine sense of theatre will also appeal to readers fresh to this subject. For example, the earlier parts of Chapter 1 amble along, introducing ideas of diffeomorphisms, flows, invariant sets, conjugacy, equivalence, etc., just like any other text book, but then we turn the page and are struck with five pages of beautiful and intriguing illustrations of Hénon's quadratic map. And then down comes the curtain on Chapter 1. Familiar these pictures may be, but to a newcomer—and this is an introductory book—their appeal will be fresh and exciting, especially in the context of what has gone before.

It is hard to place this book on the applied-pure spectrum. It lacks the pervasive drive for abstraction which one would expect in a pure treatment. Many results are stated without proof, and sometimes quite informally. On the other hand there are conclusive signs that put it firmly in the pure camp. It is scrupulously unsensational, to the extent that chaotic behaviour is barely mentioned, though of course the publisher has ensured that the phrase appears in the very first sentence of the blurb on the cover. Another, more serious, sign is an almost complete lack of applications, as though all these problems of dynamical systems had nothing to do with mathematical models of convection, celestial mechanics, population growth, and so on, and are mere creations of the mathematician's imagination.

In fact the book concentrates on the "computational" aspects of dynamical systems, which here does not mean computer experiments, but rather such procedures as the calculation of normal forms, versal unfoldings, centre manifolds, etc., and the application of such techniques to the theory of bifurcations. For example, the beautiful kaleidoscopic diagrams of two-parameter bifurcations, which one associates so much with Arnold's books, are given quite spacious and illuminating treatment here.