AN INTERMEDIARY PERIODIC ORBIT FOR HYPERION

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Abstract. We have seen (Stellmacher, 1996) that the long-period terms for Hyperion's motion are very well represented by a second kind and second genius periodic orbit, after Poincaré's classification.

We have shown how to construct such an orbit with, as only data, the observed periods, which characterise the resonance of the Titan-Hyperion couple, and the Titan's motion which is an elliptical one. The physical quantities as the masses and the J_2 term of Saturn's flattening are given. We will present the results that we obtained, and compare them with those that other authors obtained by fitting the series to the observations.

1. Description of the method

Let T_H, T', T_ω be the observed periods for Hyperion's and Titan's mean anomalies and for the longitude of Hyperion's pericenter and n_H, n', n_ω the corresponding frequencies. We have $4n_H - 3n' + 3n_\omega = 0$, $-n_H/n_\omega \equiv N_H = 332$ and $-3n/n_\omega \equiv N_T = 1325$. We put : $T_H = k\overline{T}/N_H, T' = 3k\overline{T}/N_H$ and $T_\omega = k\overline{T}$ with $k \in N$ and $\overline{n} = 2\pi/\overline{T}$. We have then $4N_H - N_T - 3 = 0$ and N_H, N_T and k characterise the resonance.

If n' is known, all the other frequencies can be calculated and must fit to the observations. With $n' = 22.57697385^0/j$ (see Garcia, 1972) then $n_\omega = -3n'/N_T = -0.05111755^0/j$, $n_H = 3n'N_H/N_T = 16.971068645^0/j$ and if $k = 11, \overline{n} = 0.5622944^0/j$. These calculated data are very close to the observed ones.

In order to determine the semi-major axis, we define: $n_H = n_0(1 + \varepsilon\gamma_H)$, n_0 the mean motion corresponding to the observed semi major axis of Hyperion a_{0H}, a_H the semi major axis of a keplerian orbit corresponding to n_H, a' the semi major axis of Titan's orbit, m', M the masses of Titan and Saturn, $\varepsilon = m'/M$, $n'^2a'^3 = n_H^2a_H^3 = n_0^2a_{0H}^3 = \mu$, $n_H^2a_{0H}^3 = \mu(1 + \varepsilon\gamma_H)^2 \neq \mu$ and $\rho_{0H} = a'/a_{0H}, \rho_H = a'/a_H$. Hyperion's perturbed motion does not verify the third Keplerian law and the semi major axis a_{0H} of the real orbit will be determinated after having calculated the quantity $\varepsilon\gamma_H$.

Let $R_1 = Gm'/\Delta = (Gm'/a_{0H}) \cdot (a_{0H}/\Delta)$ (with Δ = mutual distance Hyperion-Titan), be the principal part of the disturbing function. Let $r, v, 1, \omega$ be the radius vector, the true and mean anomalies, and the longitude of the pericenter of Hyperion (the primed quantities are those for Titan), and put $\rho = r/r'; \theta = \omega - \omega'$ and $\psi = 4\ell - 3\ell' + 3\theta$.

As we explained in (Stellmacher, 1996), the disturbing function is developped by taking Saturn's flattening and Titan's action upon Hyperion into account. As far as Titan's action upon Hyperion is concerned, we only consider the $n\psi \pm m\theta$ ($n \leq 12$ if $m = 0; n \leq 9$ if $m = 1$ or 2).

2. Results

Let $\tau = \overline{n}(t - t_0), \xi = n_\omega t + \omega_0 - \omega_0', \rho_{0H} = a'/a_{0H}$ and a, e, ω, λ the semi major axis, the eccentricity, the longitude of the pericenter, and the longitude.

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In the first case (I) we have calculated ρ_{0H}, e_0 with only the $n\psi$ arguments, β_0 and $\overline{\pi}$ are calculated by replacing θ by ξ for the $n\psi \pm \theta$ and $n\psi \pm 2\theta$ terms. In the second case (II), we have calculated $\rho_0, e_0\beta_0$ and $\overline{\pi}$ with $n\psi, n\psi \pm \theta; n\psi \pm 2\theta$ arguments with $\theta = \xi - \beta_0 \sin\xi$. The quantities in brackets (III) are the minimal and maximal values determined by different authors (see references).

(I) $\rho_{0H} = 0.824846$

$$a = a_0(1 - 0.00327 \cos\tau)$$
$$e = 0.10440 - 0.00395 \cos\tau + 0.0245 \cos\xi - 0.00183 \cos 2\xi$$
$$\omega = n_\omega t + \omega_0 - 13^0 47 \sin\xi + 1^0 64 \sin 2\xi - 0^0 49 \sin\tau - 0^0 32 \sin(\xi-\tau) - 0^0 27 \sin(\xi+\tau)$$
$$\lambda = (n_\omega + n_H)t + \lambda 0 + 9^0 05 \sin\tau + 0^0 25 \sin(\xi-\tau) + 0^0 20 \sin(\xi+\tau)$$

(II) $\rho_{0H} = 0.824825$

$$a = a_0(1 - 0.00332 \cos\tau)$$
$$e = 0.10384 - 0.00403 \cos\tau + 0.0260 \cos\xi - 0.00160 \cos 2\xi$$
$$\omega = n_\omega t + \omega_0 - 13^0 09 \sin\xi + 1^0 50 \sin 2\xi - 0^0 50 \sin\tau - 0^0 31 \sin(\xi-\tau) - 0^0 26 \sin(\xi+\tau)$$
$$\lambda = (n_\omega + n_H)t + \lambda 0 + 9^0 18 \sin\tau + 0^0 25 \sin(\xi-\tau) + 0^0 21 \sin(\xi+\tau)$$

(III) $\rho_{0H} = 0.824942$

$$a = a_0(1 - [0.00323, 0.00354] \cos\tau)$$
$$e = [0.10346, 0.10473] - [0.00389, 0.00410] \cos\tau + [0.0230, 0.0245] \cos\xi - [0.00110, 0,0014] \cos 2\xi$$
$$\omega = n_\omega t + \omega_0 - [12^0 87, 13^0 58] \sin\xi + [1^0 56, 2^0 16] \sin 2\xi - [0^0 43, 0^0 45] \sin\tau - [0^0 354, 0^0 360] \sin(\xi-\tau) - [0^0 263, 0^0 265] \sin(\xi+\tau)$$
$$\lambda = (n_\omega + n_H)t + \lambda 0 + [9^0 089, 3^0 142] \sin\tau + [0^0 210, 0^0 220] \sin(\xi-\tau) + [0^0 190, 0^0 228] \sin(\xi+\tau)$$

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