<u>Diane</u> <u>M</u>. <u>Johnson</u>, Representations of the symmetric group, presented at the University of Toronto, December 1958 (Supervisor, G. de B. Robinson).

A representation of a finite group over a field of characteristic zero is completely reducible but this is no longer necessarily the case if the representation is considered over a field of characteristic p, where p divides the order of the group. In this latter case the indecomposable representations must be distinguished from the modular irreducible representations. The decomposition matrix introduced by R. Brauer and C.J. Nesbitt gives the relationship between the ordinary irreducible representations and these indecomposable and modular irreducible representations. The rows of the matrix yield the modular irreducible components of the ordinary representations and the columns the indecomposables of the regular representation. O.E. Taulbee developed a method for constructing the decomposition matrix for the symmetric group on n elements by making use of D.E. Littlewood's concept of the admitted permutations of an indecomposable.

In this thesis a method was found for constructing the transformation matrix for any two-rowed diagram of the symmetric group on n elements which would demonstrate the modular splitting, predicted by the decomposition matrix, of the transposition matrices of that diagram. This construction was based on the transformation matrices associated with the diagrams of the symmetric group on n-1 elements and led to the result that if $[\alpha]$ and $[\beta]$ are any pair of two-rowed diagrams some of whose tableaux belong to the same column of the decomposition matrix modulo p for the symmetric group on n elements, then the modular components of their representation matrices, associated with the tableaux in the same column, are identical.

<u>M</u>. <u>Eisen</u>, Piecewise continuous solutions of mixed boundary value problems, presented at the University of Toronto, October 1959 (Supervisor, G.F.D. Duff).

In this thesis we find solutions of systems of analytic partial differential equations which assume prescribed values

https://doi.org/10.1017/S0008439500025431 Published online by Cambridge University Press

on two adjoining surfaces. In physical problems these are taken as initial and boundary surfaces.

The linear system of equations we first consider is

$$
(1)\quad
\begin{aligned}
\frac{\partial \omega_r}{\partial t} &= \lambda_r \frac{\partial \omega_r}{\partial x} + \sum b^{\rho}_{rs} \frac{\partial \omega_s}{\partial x^{\rho}} + \sum c^{\rho}_{rq} \frac{\partial u_q}{\partial x^{\rho}} + \sum d_{rs}\, \omega_s + \sum e_{rq} u_q \\
\frac{\partial u_q}{\partial t} &= \mathcal{L}_q (u_p, \omega_s) \quad \rho = 3, \ldots, N;\ r, s = 1, \ldots, R;\ p, q = 1, \ldots, L.
\end{aligned}
$$

All the coefficients in (1) are analytic functions of t, x, and x^{ρ}. The $\mathcal{L}_q$ are linear differential operators involving differentiations with respect to x^{ρ} only. The boundary conditions are

$$(2)\quad \omega_r = \sum c_{ri}\, \omega_i + g_r \qquad r = 1, \ldots, k_o;\ i = k_o + 1, \ldots, R$$

while the initial conditions are

$$(3)\quad U_s = 0, \quad \omega_r = 0 \qquad s = 1, \ldots, L;\ r = 1, \ldots, R$$

A surface is called characteristic with respect to a system of first order differential equations if we cannot find the derivative of the solutions across this surface when the solutions are given on the surface. In general, the above system (1) has many characteristic surfaces passing through the intersection of the initial and boundary surfaces. We select any k_o of these in the first quadrant. These surfaces divide this region into $k_o + 1$ sectorial domains. By expanding the discontinuities which are generated by corresponding characteristic surfaces in a series, we obtain the various analytic terms of the solution. The sum of these terms makes up the complete solution. These facts lead to the following theorem.

Let a non-characteristic surface S:t=0 and a characteristic surface T:x = 0 relative to the analytic system (1) intersect in an edge from which issue into a quadrant at least k_o distinct characteristic surfaces. Then there exists a solution continuous in the first quadrant and analytic except across the characteristic surfaces which takes given initial values on S and for which the selected variables ω_r take values on T determined by the boundary conditions (2).

We show that under certain conditions this solution is unique and use these results to solve related problems concerning singularities on an initial surface.

Next we consider a mixed problem for linear systems of higher partial differential equations. Here certain linear combinations of the solution and its derivatives are given on the initial and boundary surfaces. By reducing the given system to a first order system, similar to that studied previously, we show that a solution exists.

Then we consider systems of non-linear equations. In particular, we study quasi-linear systems. For such equations the characteristic surfaces are not known in advance since they depend upon the solution. Thus in addition to the expansion of the solution about the characteristic surfaces we must carry along an expansion in power series for the characteristic surfaces. It is shown that we can calculate the coefficients in these series recursively and that the series converge.

Next we prove that a first order non-linear system can be reduced to a first order quasi-linear system. Finally, we show that a high order non-linear system can be reduced to a first order system of non-linear equations. Thus by successively applying the theorems derived for less complex systems we deduct analogous theorems for high order non-linear systems of partial differential equations.

W. R. Knight, Exponential and subexponential distributions in statistical life testing, presented at the University of Toronto, October 1959 (Supervisor, D. A. S. Fraser).

Statistical life testing is concerned with the statistics of controlled tests upon length of life. Two common testing schemes are called replacement tests and non-replacement tests. In a replacement test n objects are continually under test and an object is replaced upon failure. In non-replacement tests n objects are tested and an object is not replaced upon failure. A non-replacement test can be shortened by using the first k observations only, and the statistic,
$\frac{1}{R} T = \frac{1}{R} \left[\sum x_i + (n-k)x_k \right]$, where x_i is the i-th order statistic, is used as a substitute for the mean.

The exponential distribution seems particularly fitted to life testing and considerable work has been done on it. It is known that in a replacement test the number of failures up to a given time, t,has mean nt/μ, where μ is the expected life on an object, and has variance equal to the mean. In a non-replace-

https://doi.org/10.1017/S0008439500025431 Published online by Cambridge University Press

ment test T has mean $k\mu$ and variance $k\mu^2$. An approach more general than any known to the author, including replacement and non-replacement tests as special cases, is developed for the exponential distribution.

A distribution is subexponential if the failure rate,

$$g(t) = f(t)/(1-F(t)) ,$$

is non-decreasing. Properties of subexponential distributions are investigated. Several conditions assuring subexponentiality are described, and it is shown that subexponential distributions can be decomposed in terms of censored exponential distributions. The expected number of failures up to a given time, t, in a replacement test is equal to or less than nt/μ, and the variance is equal to or less than the mean. In a non-replacement test the expectation of T is equal to or greater than $k\mu$, and the coefficient of variation is equal to or less than $\sqrt{1/k}$. Estimators of μ based on the exponential distribution will tend to overestimate μ in both cases, but if it is possible to compensate for the bias the spread will be less (or to be exact, not more). When testing two populations for equality of mean the biases will cancel, and thus such tests will be conservative, at least for large samples.