STELLAR PERTURBATIONS ON COMETS

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ABSTRACT. The stochastic processes involved in the evolution of a hypothetical cloud of comets are investigated. The cloud is assumed to be randomly perturbed by passing stars approaching the Sun at distance less than 1 pc. Within the frame of the impulse approximation we derive analytical expressions for the probability distribution of the impulse imparted to comets for both close and distant approaches. An application to the rate of ejection of comets is presented.

I. INTRODUCTION

The existence of a reservoir of comets lying in the outskirts of the solar system is now largely admitted since its introduction by Oort in 1950. The dutch astronomer also thought of a possible mechanism to send comets from the cloud to the inner regions of the solar system where they become eventually active and visible. According to his views, close approaches with passing stars change the comet's orbital elements up to the point at which its perihelion distance may become small enough for the comet to be strongly influenced by planetary perturbations and turned into a short period comet. Within the framework of this model the role of the gravitational effect of stars is limited to the injection of comets to the sphere of influence of the planets, primarily that of Saturn and Jupiter. These comets are currently referred to as new comets in contrast with those comets repeating passages after they were perturbed by the planets.

A convenient way to test the efficiency of the model is to simulate the effect of cumulative perturbations due to passing stars on the orbital elements of comets within a cloud. Such a work has been in particular carried out by Weissman (1980, 1982, 1983). Weissman models the stellar perturbation on comets as a single impulse imparted to the comet at aphelion. This single impulse every orbital period is assumed to be the synthesis of the three or four perturbations by passing stars experienced by a comet during one revolution about the Sun. By selecting a cloud of initially very eccentric comets, Weissman follows the long term evolution of aphelion and perihelion distance. He shows that perihelion diffuses into the planetary region and fraction of the initial population may

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become visible. He demonstrates as well that a small fraction of the cloud is lost in the interstellar medium, mainly because of a diffusion of aphelia.

More recently, Remy and Mignard (1985) have refined the study by Weissman by allowing the perturbations to occur at random, whatever the location of the comet on its orbit. In addition the magnitude of the perturbation imparted to the comets during the passage of a star is evaluated through a modelling of the arrival of stars in the vicinity of the Oort cloud both in term of the distance to the Sun and to the comet and also by drawing at random the direction of the star's velocity vector.

Certainly the only way to follow properly the evolution of the Oort cloud during the past 4.5 billion years is to rely on numerical simulations. However since the passage of stars happens at random the impulse imparted to the comets by these stars retains also a certain randomness. As a consequence the evolution of the size of the comets orbits must random walk with the time. Thus it is possible to start from the probability distribution of the parameters related to the perturbing stars to derive intermediate results connected to the underlying stochastic process that ultimately rules the evolution of the Oort cloud.

It is the aim of this paper to present some properties of the probability distribution of the impulse generated by passing stars. In the limited space of this report we will restrict ourselves to the presentation of the results referring for the detailed computation to a paper in preparation (Mignard and Remy, 1985).

2. THE IMPULSE DISTRIBUTION

When a star crosses the region surrounding the Sun and the comet it passes at a certain time at a minimum distance from the Sun and later or before at a minimum distance from the comet. Let $\vec{R}_s$ and $\vec{R}_c$ the radius vectors Sun-star and comet-star at these closest approaches. It can be shown (Rickman, 1976) that the comet undergoes during the star passage an impulse,

$$\vec{I} = \frac{2GM}{V}\left[\frac{\vec{R}_c}{R_c^2} - \frac{\vec{R}_s}{R_s^2}\right] \qquad (1)$$

where G is the gravitational constant, M and V respectively the mass and the speed of the star. Typical values of I are of the order of some tens of centimeters per second.

The probability that a star has a closest approach to the Sun in the range (R, R+dR) and a velocity vector directed in a solid angle $d\Omega$ is,

$$dP = \frac{2}{R_M^2 - R_m^2}\, R\, dR\, \frac{d\Omega}{4\pi} \qquad (2)$$

where R_M and R_m are respectively the maximum and the minimum distance of

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passage of stars to the Sun allowed. The latter boundary is introduced to prevent a cloud from being disrupted by a single, but unlikely passage of star very close to the Sun. A close approach to the Sun would make R_S small in Eq.(1) and generate a large impulse for all the comets in the cloud. As for R_M it is introduced facilitate comparisons with numerical simulations in which we want to generate a substantial fraction of the passing stars likely to perturb the comets in the cloud. If we increase R_M we must also increase the rate of passing stars as R_M^2 and results as the probability of ejection of comets given in Eq. (8) is obviously independent of R_M. The probability distribution (e.g. Eq. 4) depends strongly of R_M but not the number of times a given perturbation occurs during a certain timespan, since the number of passing stars is proportional to R_M^2. So the introduction of R_M is a matter of mathematical conveniency for intermediate computations while it has no effect on the physical results. In numerical calculations we have used,

$$R_M = 2 \quad 10^5 \text{ a.u.}$$

$$R_m = 2 \quad 10^3 \text{ a.u.}$$

$$V = 20 \text{ km.s}^{-1}$$

From Eq.(2) it is in principle possible to derive the probability distribution for the impulse $\vec{I}$. Such an approach proved to be untractable. However we have derived simple expressions for two limiting cases : i) the central region of the distribution for impulses in the range of -1 to +1 m.s^{-1} ii) the tails of the distribution valid for larger impulses.

These two regions are connected, the first, to distant passage of stars, when the Sun-comet distance is much smaller than the Sun-star distance and the second to close approaches when the star comes very close to the Sun or to the comet in comparison with their mutual distance.

2.1. Close approaches

In this case Eq.(1) reduces to,

$$\vec{I} = \frac{2GM}{V} \frac{\vec{R}}{R^2} \tag{3}$$

where R is distributed according to Eq.(2). With the three components of $\vec{I}$, hereafter referred to as α , β , γ , it can be demonstrated that the probability distribution of I is isotropic and given by (Mignard and Remy, 1985) ,

$$P(I > I_o) = I_M^2/I_o^2 \tag{4}$$

where,

$$I_m = 2GM/VR_M \sim 40 \text{ cm.s}^{-1}$$

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and the probability law for the components is such that the probability that α is in the range $d\alpha$ is $f(\alpha)\,d\alpha$ with,

$$f(\alpha) = 1/3I_m \qquad \text{for } |\alpha| < I_m$$
$$f(\alpha) = I_m^2/3|\alpha|^3 \qquad \text{for } |\alpha| > I_m \qquad (5)$$

The above expression was obtained with the reduced impulse Eq.(3) and is likely to represent the tails of the impulse distribution. The distribution decreases as $1/\alpha^3$ instead of an exponential decrease for the normal distribution. The tails of the velocity distribution are then more pronounced that it would be for a gaussian law, as anticipated by Weissman (1982).

We have carried out a numerical simulation by using the reduced impulse to model the interaction between comets and passing stars. The result is shown in Fig.(1) and is in excellent agreement with the above theory.

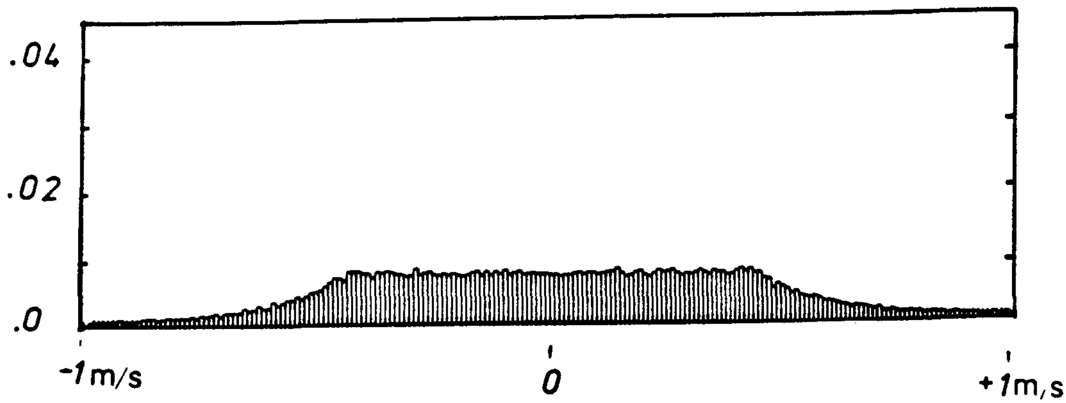

Figure 1. Simulated distribution for the impulses imparted to a comet by close encounters with a passing star.

Finally it must be pointed out that the distribution does not possess second order moment because of a slow decrease toward the large impulses. This fact prevents one from invoking the central limit theorem to synthesize several successive perturbations by a single impulse distributed according to the Maxwell law.

2.2. Distant approaches

In this case we have $R_C \sim R_S$ a fact which enhances the role of the differential effect expressed in the impulse equation. It is then reasonable to expand Eq.(1) up to its dipole term in,

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$$\vec{\delta} = \vec{R}_c - \vec{R}_s$$

By taking account of the fact the closest approach to the comet and to the Sun are not simultaneous we obtain with $\vec{R} = \vec{R}_s$,

$$\vec{I} = \frac{2GM}{V} \left[- \frac{\vec{r}}{R^2} + \frac{\vec{r}\,\vec{V}}{V^2 R^2} \vec{V} + 2 \frac{\vec{r}\,\vec{R}}{R} \vec{R} \right]$$

where $\vec{r}$ is the Sun-comet vector and is considered as a small parameter with respect to R (see Mignard and Remy, 1985 for a detailed derivation). The modulus of the impulse has a very compact expression,

$$I = \frac{2GM}{V} \frac{r}{R^2} \sin\theta$$

where θ is the angle between the comet's radius vector and the star velocity vector.

With the probability distribution given in Eq.(2) and by assuming a uniform distribution of points of closest approaches on the sphere of radius R we obtained the probability distribution for the modulus of the impulse and for the three components α , β , γ , in the central region of the distribution.

With $\bar{I}_m = I_m$ r r_m or R_M we have,

$$\begin{aligned} &\bar{I} < \bar{I}_m \qquad f(I) = \bar{I}_m \left[\frac{1}{3} \frac{I}{\bar{I}_m^3} + \frac{1}{10} \frac{I^3}{\bar{I}_m^5} + \left(\frac{\pi}{4} \frac{1}{3} \frac{1}{10}\right) \frac{I^5}{\bar{I}_m^7} \right] \\ &I > \bar{I}_m \qquad f(I) = \frac{\pi}{4} \frac{\bar{I}_m}{I^2} \end{aligned} \tag{6}$$

Typical values of $\bar{I}_m$ range between 5 to 15 $cm.s^{-1}$ for a comet's distance to the Sun between 2 10^4 to 6 10^4 a.u. For a very large R_M we must consider the number distribution nf(I) rather than the probability distribution f(I), where n is the number of star passages during a certain lapse of time. With $n \propto R_M^2$, only is the second branch of the frequency distribution meaningful since I_m goes to 0 while nI_m remains bounded.

The distribution of the components follows from the distribution of the modulus after an integration of the joint distribution for the three components over β and γ . The computation is straightforward and yields,

$$\begin{aligned} &|\alpha| < \bar{I}_m \qquad f(\alpha) = \frac{1}{\bar{I}_m} \left(A + B \frac{|\alpha|}{\bar{I}_m} + C \frac{|\alpha|^3}{\bar{I}_m^3} + D \frac{|\alpha|^5}{\bar{I}_m^5} \right) \\ &|\alpha| > \bar{I}_m \qquad f(\alpha) = \frac{\pi}{16} \frac{\bar{I}_m}{\alpha^2} \end{aligned} \tag{7}$$

with A = 0.415   B = -1/6   C = -1/60   D = -0.0352

With the variable $z = \alpha/\bar{I}_m$ the above distribution becomes dimensionless and well adapted for simulations.

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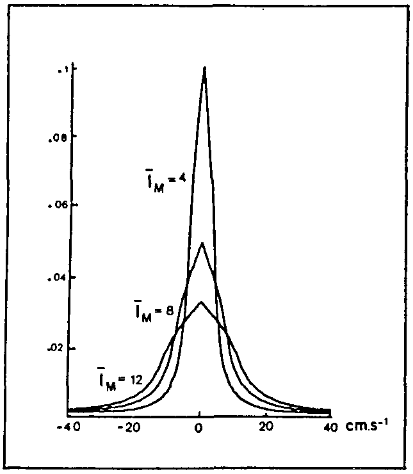

Figure 2. Computed impulse distribution for distant encounters with the passing stars.

The expansion of the impulse formula up to its dipole term makes the above results valid for a limit ranges of impulse, namely those generated by distant approaches. For a typical comet whose distance from the Sun is 3 10^4 a.u. we will restrict distant approaches to passing stars not approaching the Sun closer than twice the Sun-comet distance. Then the impulses to be considered are in the range -1 to +1 ms^{-1}. Then within this range we have the following moments for the frequency distribution,

$$E(\alpha) = 0, \; E(\alpha^2) = \sigma^2(\alpha) \sim 4 \; \bar{I}_m^2$$

The distribution differs from a normal law mainly because of its slow decrease reflected by the distribution function,

$$P(\alpha < \bar{I}_m) = 0.62$$

$$P(\alpha < 3\bar{I}_m) = 0.85$$

$$P(\alpha < 5\bar{I}_m) = 0.92$$

However in its central part the distribution can be matched with a gaussian of standard deviation $\sigma = \bar{I}_m$. But the tails become prominent as soon as $\alpha > \bar{I}_m$.

The distribution $f(\alpha)$ is plotted in Fig.(2) for three sample values of $\bar{I}_m$, corresponding to comets distance from the Sun, respectively 18, 36 and 54 10^4 a.u. We have also simulated the exact impulse distribution by using the complete equation (1) for the impulse to the comet. In the simulation comets have been maintained at fixed distance from the Sun but in different locations. Twenty thousand stars were drawn at random in distance and in orientation of the velocity vector. Results are plotted in Fig. (3) for $\bar{I}_m = 4$ and $8\ cm.s^{-1}$. A comparison with figure (2) shows that the fit to the theory is very good. A similar simulation was conducted for the modulus of the impulsion ans showed a similar agreement with Eq.(6).

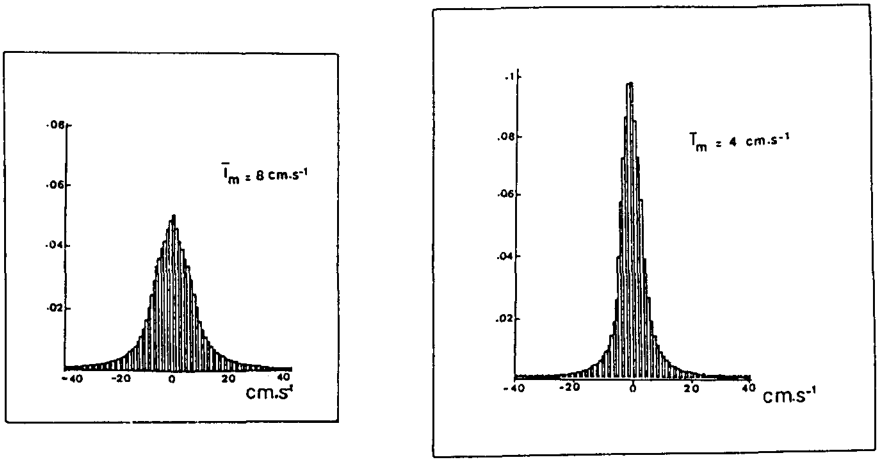

Figure 3. Simulated distribution for the impulses imparted to the comet during distant encounters with passing stars. The Sun-comet distance is 36 10^4 a.u. ($\bar{I}_m = 8\ cm.s^{-1}$) and 18 10^4 a.u. ($\bar{I}_m = 4\ cm.s^{-1}$).

3. APPLICATION : EJECTION OF COMETS

A comet is ejected from the solar system whenever its orbital velocity is larger than the escape velocity $V_e = (2GM/r)^{1/2}$ where r is the Sun-comet distance. For a typical comet we have $V_e \sim 200\ m.s^{-1}$ much larger than the comet's velocity in the vicinity of aphelion. Therefore an ejection would result from a single close encounter with a star, such that the impulse imparted to the comet is larger than V_e.

From Eq.(4) the probability for the impulse to be larger than V_e during one encounter is,

$$P_1\ (\text{ejection}) = I_m^2/V_e^2 \sim 6\ 10^{-6}$$

After N passages of stars the probability for a comet to be ejected is,

$$P_N\ (\text{ejection}) \sim 1 - \exp\ (-\ NI_m^2\ /\ V_e^2) \qquad (8)$$

Then if ρ denotes the number density of stars in the solar neighborhood we have for the probability of ejection during the time T,

$$P_N = 1 - \exp(-T/\tau)$$

where is the characteristic time,

$$\tau = V/(2\pi\rho\ GMr)$$

The waiting time for ejection follows an exponential distribution, hence the number of comets ejected per unit time follows a Poisson distribution. Over the age of the solar system the relative depletion of the Oort cloud by ejection is then,

$$P_N = 0.09 \quad \text{with} \quad \rho = 0.08\ \text{star/pc}^3, \quad M = 1M_\odot$$

for T = 4.5 billions years. This number is similar to the result first given by Weissman (1980).

4. CONCLUSION

In this paper we have obtained the probability distribution of the impulse imparted to comets within the Oort cloud by random passing stars. The main result is the fact that the tails of this distribution are much more extended than it would be for a gaussian distribution. In principle this distribution should be sufficient to allow the derivation of results more closely related to the dynamical properties of the Oort cloud, such as the frequency distribution of the orbital parameters after the passage of a certain number of stars. In practice an efficient method to achieve such results needs to be devised and it is our intent to advance in this direction.

REFERENCES

Mignard, F., Remy, F., 1985,'Dynamical evolution of the Oort cloud. II. A theoretical approach', submitted to *Icarus*.

Oort, J.H., 1950,'The structure of the cloud of comets surrounding the solar system and an hypothesis concerning its origin', *Bull. Astron. Neth.*, 11, 91-110.

Remy, F., Mignard, F., 1985, 'Dynamical evolution of the Oort cloud. I. A monte-Carlo simulation', submitted to *Icarus*.

Rickman, H., 1976, 'Stellar perturbations of orbits of long period comets and their significance for cometary capture', *Bull. of the Astron. Inst. of Czechoslovakia*, 27, 2, 92-105.

Weissman, P.R., 1980, 'Stellar perturbations of the cometary cloud', *Nature*, 288, 242-243.

Weissman, P.R., 1982, 'Dynamical history of the Oort cloud', in *Comets*, L.L. Wilkening Ed., Univ. of Arizona Press.