BOOK REVIEWS

LANG, S., *Diophantine Geometry* (Interscience Tracts in Pure and Applied Mathematics, No. 11, John Wiley & Sons Ltd., 1962), 170 pp., 57s.

This book is best regarded as the third of a series. The first (*Introduction to Algebraic Geometry*, Interscience Tract No. 5) is an introduction to algebraic geometry over an arbitrary groundfield (not necessarily algebraically closed and possibly with a prime characteristic). The second (*Abelian varieties*, Interscience Tract No. 7) gives the theory of abelian varieties over such groundfields. Abelian varieties are not only interesting objects in themselves, but they provide the natural tool for studying such topics as linear and algebraic equivalence. In *Abelian varieties* the author applies the theory in its simplest case to give an illuminating proof of the so-called " Riemann Hypothesis for Function-fields " about the number of points on a curve defined over a field with finitely many elements.

The volume under review is devoted to showing that not merely does the theory of abelian varieties cast considerable light on some of the deepest parts of the theory of rational points on algebraic varieties (i.e. the major part of the theory of " diophantine equations "), greatly simplifying the proofs, but it suggests far-reaching generalisations (in part due to Lang himself) and unsuspected connections with other parts of algebraic geometry.

Great simplifications in the exposition are also made possible by a break-through in another field, namely Roth's theorem about the approximation of algebraic numbers by rationals. The original treatment of some of the topics of the book had to work with the much weaker theorem of Thue and Siegel.

The three main topics are as follows:

(i) The *Mordell-Weil Theorem*. As formulated here, this states that if A is an abelian variety defined over a field K which is finitely generated over the prime field, then the group A_K of points on A defined over K is finitely generated. When A is of dimension 1 and K is the ration field then this is just Mordell's theorem that the set of rational points on a curve of genus 1 can be obtained from a finite number of them by repeated use of the so-called chord-and-tangent process. Lang also gives a " relative version " of this theorem in which K is a function field over a constant field k, and this implies the geometrical " theorem of the base " for an algebraically closed field of arbitrary characteristic. If V is a variety nonsingular in codimension 1, then its group of divisors modulo those algebraically equivalent to 0 is finitely generated.

(ii) *Siegel's theorem*. In its original form this decides when a curve in affine space defined over an algebraic numberfield k can have infinitely many points whose coordinates lie in the ring o of integers of k. This can happen only when the genus of the curve is 0, and then only in specified cases. In this book o is replaced by any ring R which is finitely generated over the ring of rational integers and k by its field of quotients. As with the Mordell-Weil theorem there is also a " relative version ".

(iii) *Hilbert's irreducibility theorem*. This is concerned with fields k with the following property: Let $f(T_1, \ldots, T_n, X_1, \ldots, X_m)$ be a polynomial with coefficients in k which is irreducible considered as a polynomial in $X_1, \ldots, X_m$ with coefficients in the field $k(T_1, \ldots, T_n)$. Then there exist values $t_1, \ldots, t_n$ in k of the variables $T_1, \ldots, T_n$ such that $f(t_1, \ldots, t_n, X_1, \ldots, X_m)$ is irreducible (as a polynomial in $X_1, \ldots, X_m$ with coefficients in k). The complex numbers, for example, lack this property, but Hilbert

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showed that the rational field possesses it. Lang shows how results of this kind are tied up with Siegel's theorem. He also gives a treatment of the case when k is the rational field from first principles (essentially a streamlined version of the original).

" In the large " the book is first class. Lang has the gift of dissecting out the concepts which play the leading role in a theory and exhibiting them with full clarity. For example, the concept of the " height " of a point on a variety (which has been distilled from Fermat's " infinite descent ") is analysed and its formal (" functorial ") properties displayed. The strategies of the various proofs and of the book as a whole are brought out clearly and there are most enlightening discussions of the relations of the results to one another and to others in the literature (I was particularly struck by the discussion of the various generalisations of Roth's theorems to include p-adic valuations).

" In the small " the effect is less satisfactory. To quite a large extent this is hardly the author's fault, but is caused by the chaotic state of algebraic geometry. Every few years there is a new formulation of the elements of algebraic geometry, and although the professional geometer can doubtless transpose a theorem or a theory without effort from the style of " the Italian School " to that of Weil's " Foundations " or to the language of schemas, this all increases the difficulties of the mere number theorist, who should also be vitally interested in this book. Lang writes for geometers and, besides assuming the facility of transposition mentioned above, is apt to justify his steps by a vague reference to some whole theory (e.g. " by the theory of Chow co-ordinates " without even a reference to where it may be found in one formulation or another). Difficulties of comprehension are increased because one is discouraged from a too curious analysis of mysterious arguments by the lurking suspicion that what is in the book is not quite what was meant: there are some strategically placed misprints, and the muddle near the top of page 136 can hardly be blamed on the printer.

But one would infinitely prefer to have the book, even with these blemishes, than no book at all, particularly in these days when so many new ideas either circulate for years in restricted circles before (if ever) they appear in print, or appear only in arid unappetising form. Professor Lang has greatly added to our already deep debt of gratitude by producing this fascinating account of a topical and important field.

J. W. S. CASSELS

WHITEHEAD, A. N. AND RUSSELL, B., *Principia Mathematica* to *56 (Students' Edition) (Cambridge University Press, 1962), xlvi+410 pp., 17s. 6d.

Here we have a book which is widely known about—but little known. This students' edition makes the book much more readily accessible; it is to be hoped that it will be more widely read.

This book is a paperback edition of a part of the first volume of *Principia Mathematica*; the whole of Part I and Section A of Part II are included, with Appendices A and C. This restriction on the size of the book appears to be a very reasonable compromise between weight, expense and usefulness.

R. M. DICKER

GÖDEL, K., *On Formally Undecidable Propositions of Principia Mathematica and Related Systems*, translated by B. MELTZER with an Introduction by R. B. BRAITHWAITE (Oliver and Boyd, 1962), viii+72 pp., 12s. 6d.

This little book is one half a translation of Gödel's famous paper and the other half an introduction to this paper. The translation itself is very good; the symbolism

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