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BOOK REVIEWS

HATCHER, A. *Algebraic topology* (Cambridge University Press, 2002), 556 pp., 0 521 79540 0 (softback), £20.95, 0 521 79160 X (hardback), £60.

This is a carefully written and very detailed introduction to algebraic topology. Probably, one may call it an introduction to the 'Introduction to algebraic topology' as the author's prime goal is a readable and self-contained account of the three most traditional themes of this branch of mathematics—homotopy groups, homology and cohomology—without using such instruments as, say, spectral sequences and the (closed model) category theory, and hence without even attempting to give an overview of the most recent developments. This is a pure 'back to basics' approach, which some readers, and not only beginners, may actually find very useful and interesting as it allows space for a wider than usual selection of examples, illustrations and concrete calculations. In my opinion, the author's selection is very informative and nice, and the author's emphasis on CW complexes throughout the book pays back well. I should also note that almost every section offers a wide range of exercises (with no solutions, however).

Rather unusually, even after the book's release in print its text is still available on the web. Those readers with access to the internet are advised to put this review aside and go instead to

http://www.math.cornell.edu/~hatcher

The book consists of five chapters and one appendix. The latter is a self-contained introduction to CW complexes and the compact-open topology. Each chapter (except the zeroth one) is divided in turn into two sections: 'compulsory' material and additional topics which can be skipped on the first reading.

Here is a brief description of the book's content.

Chapter 0. Some underlying geometric notions

A down-to-earth explanation of homotopy types and homotopy extensions, cell complexes, suspensions, cones and joins.

Chapter 1. The fundamental group

1.1. Basic constructions: the main notion of Chapter 1 is derived from a well-illustrated idea of paths and homotopy of paths; next the author carefully computes the fundamental group of the circle and discusses a few applications and further examples.

1.2. Van Kampen's Theorem is an important method for computing the fundamental groups of spaces by decomposing them into something simpler; lots of examples and concrete calculations.

1.3. Covering spaces: this central notion of Chapter 1 is very widely illustrated and analysed, and then classified.

Additional topics include graphs, free groups, $K(G, 1)$ spaces and graphs of groups.

Chapter 2. Homology

2.1. Simplicial and singular homology: their definitions and basic properties are followed by proofs of several important facts such as their equivalence and the Excision Theorem.

2.2. Computations and applications include among other things cellular homology, Mayer–Vietoris sequences and homology with coefficients.

2.3. Categories and functors: a very short note on the language, which is not used in the book.

Additional topics include interrelations between homology and fundamental groups, classical applications and simplicial approximation.

Chapter 3. Cohomology

3.1. Cohomology groups: the universal coefficients theorem and (relative) cohomology of spaces.

3.2. Cup product is a very important structure making the cohomology into a ring; other themes include a Künneth formula and spaces with polynomial cohomology (such as $\mathbb{C}P^\infty$, etc.).

3.3. Poincaré duality and other dualities between homology and cohomology are explained and, as usual, nicely illustrated.

Additional topics include the universal coefficients for homology, the general Künneth formula, H-spaces and Hopf algebras, the cohomology of $SO(n)$, Bockstein homomorphisms, limits and Ext, transfer homomorphisms and local coefficients.

Chapter 4. Homotopy theory

4.1. Homotopy groups are natural generalizations of the fundamental group; their basic properties are studied and then the question of when two spaces have isomorphic homotopy groups is addressed; in particular Whitehead's Theorem is proved; finally cellular and CW approximations of maps are discussed.

4.2. Elementary methods of calculations: a number of concrete and important examples are studied; the Hurewicz theorem is proved; fibre bundles and stable homotopy groups are introduced.

4.3. Connections with cohomology: the homotopy construction of cohomology, fibrations, Postnikov towers, and an elementary overview of the main iterative idea of obstruction theory.

Additional topics include homotopy versus base points, the Hopf invariant, minimal cell structures, cohomology of fibre bundles, the Brown representability theorem, spectra, gluing constructions, Eckmann–Hilton duality, stable splitting of spaces, the loop space of a suspension, the Dold–Thom theorem, Steenrod squares and powers.

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