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Concepts of independence for bounded-sum variables

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Consider r non-negative random variables $X_1^{(t)}, \ldots, X_r^{(t)}$ satisfying the constraint $X_1^{(t)} + \ldots + X_r^{(t)} \leq t$. Because of this constraint, the variables virtually have to be dependent according to the usual definitions of independence. This thesis is largely concerned with the problem of defining a concept of independence for such "bounded-sum" variables, one which reflects no association among the variables except that due to the constraint on their sum.

Darroch [2] introduced such a concept, termed F-independence, for non-negative integer-valued variables. The definitions of F-independence required the invocation of a family of probability functions, one for each value of t from 1 to N say. In Chapter 2 we generalize his definitions by requiring the invocation of probability functions only for t between N_1 and N_2 say. An alternative multivariate model is also given.

Chapter 3 consists of the analogous F-independence theory for continuous variables, and the problem of determining when F-independence properties remain invariant under unequal scale changes of the variables is investigated. The theories of F-independence for integer-valued variables and continuous variables are similar, and some assorted results common to both are given in Chapter 4.

Connor and Mosimann [1] introduced the concept of neutrality for

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proportions, and gave a generalization of the Dirichlet distribution arising from complete neutrality. In Chapter 1 we show that the generality of this generalized Dirichlet distribution is curtailed, sometimes severely, when certain ratios of proportions are assumed to have beta distributions. Also in this chapter, different generalizations of the Dirichlet distribution are obtained from neutrality properties. In Chapter 5 it is shown that, for continuous variables, if the proportions $\left(X_1^{(t)}/t, \ldots, X_r^{(t)}/t\right)$ are distributed invariantly with respect to t, then F-independence properties imply neutrality properties. In this way, characterizations of the Dirichlet distributions are obtained via F-independence, and a similar characterization of the beta distribution is given.

The F-independence analogue of infinite divisibility, termed infinite F-divisiblity, is defined in Chapter 6. It is shown that, under regularity conditions, the limit distributions of sums of F-independent random variables are infinitely F-divisible.

References

[1] Robert J. Connor and James E. Mosimann, "Concepts of independence for proportions with a generalization of the Dirichlet distribution", *J. Amer. Statis. Assoc.* 64 (1969), 194-206.

[2] J.N. Darroch, "A definition of independence for bounded-sum, nonnegative, integer-valued variables", *Biometrika* 58 (1971), 357-368.