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Correction to "Irreducible modules of modular Lie superalgebras and super version of the first Kac-Weisfeiler conjecture"

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Abstract. In the article "Irreducible modules of modular Lie superalgebras and super version of the first Kac-Weisfeiler conjecture, Canad. Math. Bull. 67 (2024), no. 3, 554–573." The statement in Theorem 4.7 is improper, which is fixed here. Theorem 4.7 is an isolated result in the article. This correction does not influence any arguments and any main results after that in the original article.

Theorem 4.7 in [1] should be corrected as follows. All notations and assumptions are the same as in [1]. Especially, all Lie superalgebras are assumed to be finite-dimensional.

Theorem 0.1 *Let $\mathfrak{g}$ be a solvable Lie superalgebra. Any irreducible module V of $\mathfrak{g}$ is associated with some $\chi \in \mathfrak{g}_{\bar{0}}^*$, which has dimension $p^n 2^{\dim \mathfrak{g}_{\bar{1}}/\mathfrak{h}_{\bar{1}}}$ where $\mathfrak{h}$ is a subalgebra with $\chi(\mathfrak{h}^{(1)}) = 0$ for its derived subalgebra $\mathfrak{h}^{(1)} := [\mathfrak{h}, \mathfrak{h}]$, and V contains a one-dimensional $\mathfrak{h}$-module, and n depends on $\mathfrak{h}$.*

Proof For any given irreducible $\mathfrak{g}$-module (V, ρ), V becomes an irreducible $\mathfrak{g}_p$-modules associated with $\Upsilon \in (\mathfrak{g}_{\bar{0}})_p^*$ where $\mathfrak{g}_p$ is a minimal finite-dimensional p-envelope of $\mathfrak{g}$ with $\mathfrak{g}_p = (\mathfrak{g}_{\bar{0}})_p + \mathfrak{g}_{\bar{1}}$ such that $(\mathfrak{g}_{\bar{0}})_p$ is a p-envelope of $\mathfrak{g}_{\bar{0}}$ (see [1, Lemma A.3]). Moreover, V is associated with $\chi := \Upsilon|_{\mathfrak{g}_{\bar{0}}} \in \mathfrak{g}_{\bar{0}}^*$. By [1, Corollary 4.6], there exists a restricted subalgebra $\mathfrak{H}$ of $\mathfrak{g}_p$ with $\Upsilon(\mathfrak{H}^{(1)}) = 0$ such that $V \cong U_\chi(\mathfrak{g}_p) \otimes_{U_\chi(\mathfrak{H})} S$ where $S \subset V$ is a one-dimensional $\mathfrak{H}$-module. Correspondingly, $\dim V = p^{\dim (\mathfrak{g}_{\bar{0}})_p/\mathfrak{H}_{\bar{0}}} 2^{\dim \mathfrak{g}_{\bar{1}}/\mathfrak{H}_{\bar{1}}}$.

Take $\mathfrak{h} = \mathfrak{H} \cap \mathfrak{g}$. By definition, $\mathfrak{h}_{\bar{0}} = \mathfrak{H}_{\bar{0}} \cap \mathfrak{g}_{\bar{0}}$, and $\mathfrak{h}_{\bar{1}} = \mathfrak{H}_{\bar{1}} \cap \mathfrak{g}_{\bar{1}} = \mathfrak{H}_{\bar{1}}$. Then $\chi(\mathfrak{h}^{(1)}) = 0$, and $\mathfrak{h}$ has one-dimensional module S. Set $n = \dim (\mathfrak{g}_{\bar{0}})_p/\mathfrak{H}_{\bar{0}}$. Then we

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have

$$\dim V = p^n 2^{\dim \mathfrak{g}_{\bar{1}}/\mathfrak{h}_{\bar{1}}}.$$

The proof is completed. ■

Remark 0.2 When $\mathfrak{g}$ is a restricted solvable Lie superalgebra, n coincides with $\dim \mathfrak{g}_{\bar{0}}/\mathfrak{h}_{\bar{0}}$. In this case, the above theorem is actually a strengthened version of [1, Corollary 4.6] on the irreducible modules and their dimensions. So in this special case, the theorem implies [1, Proposition 5.4(1)] which states:

- Each irreducible $U_\chi(\mathfrak{g})$-module V for a restricted solvable Lie superalgebra $\mathfrak{g}$ is associated with certain restricted subalgebra $\mathfrak{h}$ with $\chi(\mathfrak{h}^{(1)}) = 0$, such that V has dimension

$$p^{\dim \mathfrak{g}_{\bar{0}}/\mathfrak{h}_{\bar{0}}} 2^{\dim \mathfrak{g}_{\bar{1}}/\mathfrak{h}_{\bar{1}}}$$

  and there is a one-dimensional $\mathfrak{h}$-submodule in V.

Reference

[1] B. Shu, *Irreducible modules of modular Lie superalgebras and super version of the first Kac-Weisfeiler conjecture*. Canadian Mathematical Bulletin **67**(2024), 554–573.

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