



CORRIGENDUM

Stokes flows in three-dimensional fluids with odd and parity-violating viscosities – CORRIGENDUM

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The following corrections should be made to the paper Khain *et al.* (2022). The conclusions of the paper are not affected.

- (i) The labels D_∞ and $C_{\infty v}$ should be flipped in figure 1 (both sides), so that figure 1(d) corresponds to D_∞ and figure 1(e) corresponds to $C_{\infty v}$. The text describing the figure should read: ‘We call these fluids parity-preserving cylindrical, and examples include the aligned nematic particles ($D_{\infty h}$), aligned helices (D_∞), and dipolar molecules in an electric field ($C_{\infty v}$) shown in figure 1(c–e).’
- (ii) In figures 3(d), 4(e), and 4(j) and equations 4.24 to 5.14 and E3c, E4c, H5, I14, J10, J17, some of the viscosity coefficients are mislabeled or feature sign errors, as listed below:
 - (a) The label $\eta_R^o > 0$ should instead read $\eta_R^o < 0$ in figures 3(d), 4(e), and 4(j). The plots correspond to $\eta_R^o/\mu = -0.1$ (not $\eta_R^o/\mu = 0.1$). Additionally, the colored arrows that show rotational flow in figure 4(a, f) should be inverted, with red clockwise arrows for $z > 0$ and blue counterclockwise arrows for $z < 0$.
 - (b) All terms with $\epsilon_{R,1}$, $\epsilon_{R,2}$, and ϵ_R^o in Eq. 4.24 should flip sign. Additionally, the bottom right entry of Eq. 4.24 should read $q^2 + \epsilon_{R,1}(q_x^2 + q_y^2)$. In Eqs. 4.25, 4.26, and 4.27, ϵ_R should be $\epsilon_{R,1}$.
 - (c) In Eqs. 4.25, 4.26, and 4.27, all terms linear in ϵ_R^o and $\epsilon_{R,1}$ should acquire an additional minus sign. In Eqs. 4.28, 5.4, 5.5, 5.9, 5.10, and 5.14, terms corresponding to ϵ_R^o should acquire an additional minus sign. Equations E3c, E4c, H5, I14, J10, and J17 should all acquire an overall minus sign.
 - (d) In fig. 8, the labels of the subplots should be altered in the following way:

$$\eta_R^o > 0 \rightarrow \eta_R^o < 0, \quad \eta_{Q,2}^e \rightarrow \eta_{Q,2}^o, \quad \eta_{Q,2}^o \rightarrow \eta_{Q,2}^e, \quad \eta_{Q,3}^e \rightarrow \eta_{Q,3}^o, \quad \eta_{Q,3}^o \rightarrow \eta_{Q,3}^e$$

and

$$\eta_{R,1} > 0 \rightarrow \eta_{R,1} < 0, \quad \eta_{R,2} > 0 \rightarrow \eta_{R,2} < 0, \\ \eta_{Q,1}^e \rightarrow \eta_{Q,1}^o, \quad \eta_{Q,1}^o \rightarrow \eta_{Q,1}^e, \quad \eta_A^e \rightarrow \eta_A^o, \quad \eta_A^o \rightarrow \eta_A^e.$$

Notice that the coefficients in the second group do not give rise to an azimuthal flow, and so do not change the flows plotted in fig. 8. Since η_R^o flips sign, the third velocity field in the top row corresponds to $\eta_R^o/\mu = -0.01$.

(e) In fig. 10(c), the label and the caption should read $\eta_R^o/\mu = -0.01$.

(f) The code associated with this paper <https://github.com/talikhain/StokesletFFT> has been updated to reflect these changes.

These errors arose because we initially used a different convention ($\sigma_{ij} = \eta_{ijk\ell} \partial_k v_\ell$ instead of $\sigma_{ij} = \eta_{ijk\ell} \partial_\ell v_k$, notice the flipping of the k and ℓ indices). The second part of ii.b is an additional typo. With the above corrections, the equations and figures are consistent with the notation in Eq. 1.2 and Eq. 2.7.

(iii) Eq. B8 should read

$$\mathcal{P}_y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

This error arose because we initially used a different ordering of the rows in the viscosity matrix (Eq. 2.7).

REFERENCE

KHAIN, T., SCHEIBNER, C., FRUCHART, M. & VITELLI, V. 2022 Stokes flows in three-dimensional fluids with odd and parity-violating viscosities. *J. Fluid Mech.* **934**, A23.