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A FLOW LAW FOR TEMPERATE GLACIER ICE*

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ABSTRACT. Uniaxial compressive creep tests were performed on polycrystalline samples of glacier ice at stresses ranging from 0.06 bar to 1.0 bar under conditions similar to those actually occurring in a temperate glacier. Tests were conducted in an ice tunnel on the Blue Glacier, Mt Olympus, Washington, U.S.A., thus ensuring that the temperature remained at the pressure melting point.

The flow law proposed as an appropriate one-dimensional flow law for temperate glacier ice is a polynomial expression containing linear, cubic and fifth-order terms. This law provides a good fit to the data and is also consistent with the quasi-viscous creep data of Glen (1955).

RÉSUMÉ. *Une loi d'écoulement pour la glace de glacier tempéré.* Des essais de déformation à la compression uniaxiale furent entrepris sur des échantillons polycristallins de glace de glacier sous des efforts allant de 0.06 bar à 1.0 bar sous des conditions semblables à celles effectivement réalisées dans un glacier tempéré. Les essais furent conduits dans un tunnel dans la glace sur le Blue Glacier, Mt Olympus, Washington, U.S.A., en s'assurant que la température restait au point de fusion correspondant à la pression.

La loi d'écoulement proposée comme une loi convenable pour l'écoulement unidimensionnel de la glace de glacier tempéré est une expression polynomiale contenant des termes linéaires, cubiques et du 5^e^ ordre. Cette loi s'ajuste bien aux résultats obtenus et est également cohérente avec les données du fluage quasi-visqueux de Glen (1955).

ZUSAMMENFASSUNG. *Ein Fliessgesetz für Eis temperierter Gletscher.* Versuche zum einachsigem Kriechen unter Druck wurden mit polykristallinen Gletschereisproben in einem Spannungsbereich von 0.06 bar bis 1.0 bar unter ähnlichen Bedingungen angestellt, wie sie tatsächlich in temperierten Gletschern vorkommen. Die Versuche wurden in einem Eistunnel am Blue Glacier, Mt Olympus, Washington, U.S.A., durchgeführt, womit gesichert war, dass die Temperatur auf dem Druckschmelzpunkt blieb.

Das Fliessgesetz, das als eindimensionales Fliessgesetz für temperierte Gletscher vorgeschlagen wird, ist ein Polynom, das lineare Glieder sowie solche dritter und fünfter Ordnung enthält. Es liefert eine gute Angleichung an die Messdaten und zeigt auch Übereinstimmung mit den Werten für quasi-viskoses Kriechen von Glen (1955).

NOMENCLATURE

$\dot{\epsilon}$	uniaxial strain-rate
$\bar{\dot{\epsilon}}$	average strain-rate over the length of the sample
$\dot{\epsilon}_0$	uniaxial strain-rate as stress vanishes
$\dot{\epsilon}_c$	strain-rate at $x = 0$
ρ	density of the samples
σ	uniaxial stress
$\bar{\sigma}$	average stress over the sample width
σ_c	stress of the center of the sample
A, B, C	constants
a	area of the sample
b	a measure of the tilting of the upper platen
E	error due to tilting
E_2	error in the strain-rate
g	acceleration due to gravity
L	length of the sample

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l one-half sample width
n exponent
Q activation energy
R gas constant
T absolute temperature

1. Introduction

The development of theoretical models for the analysis of glacier flow is dependent on the availability of a meaningful mechanical constitutive law. For normal flow conditions a constitutive law for steady creep is adequate. The application to field conditions of data obtained from laboratory-prepared polycrystalline ice samples is by no means a straightforward task, however, because of the differences in the manner by which the ice crystals form as well as differences in temperature and grain size. In addition, stress states occurring in Nature are far more complex than the simple states under which most laboratory experiments are carried out. The purpose of the experiments described in this paper was to investigate the uniaxial flow law under conditions pertinent to temperate glacier flow, i.e. to test temperate glacier ice at the pressure melting point.

A pragmatic approach was taken to the experiments and their interpretation. A single flow law was sought which would be applicable for the entire range of stress likely to be found in a glacier. The mechanisms of deformation were not of prime interest; in particular, no attempt was made to measure directly any intergranular regelation which might occur. Such regelation would be both measured and interpreted as "creep", provided that the overall deformation followed a creep-type law.

The first creep tests performed on polycrystalline ice under controlled laboratory conditions were those of Glen (1952) who determined the flow law of polycrystalline ice in simple compression to be of the form

$$\dot{\epsilon} = A\sigma^n \tag{1}$$

where n is equal to about 4. The range of stresses used was from 0.9 to 9.3 bar. Glen's tests did not last long enough to reach a steady-state creep rate so the value of the exponent n depends upon how these tests are interpreted. Also the magnitudes of the stresses used in the experiments were mostly larger than those which generally occur in glaciers. Butkovich and Landauer (1960) found the response to be nearly linearly viscous at stresses between 0.02 and 0.2 bar. Mellor and Smith (1967) reported the results of creep tests on polycrystalline ice at sub-freezing temperatures. Their results are described by the two-term flow law,

$$\dot{\epsilon} = A\sigma + B\sigma^n, \tag{2}$$

where A and B are constants which vary with temperature and type of ice. This form of the flow law was first suggested by Meier (1960) and supports the idea that two distinct mechanisms are responsible for the creep of ice. The governing creep mechanisms cannot be determined from these experiments but the results strongly suggest that two (or more) mechanisms operate when ice deforms. Mellor and Smith suggested each mechanism may have its own characteristic activation energy hence the apparent activation energy may also vary with stress.

Bromer and Kingery (1968) stressed polycrystalline samples of ice with low tensile stresses at sub-freezing temperatures. They found a linearly viscous response for ice with the viscosity proportional to the square of the grain size. Jellinek and Brill (1956) have shown that the rheological properties of ice depend upon age.

The most recently reported steady creep tests were those by Mellor and Testa (1969[b]) who compressed laboratory-prepared ice samples at 0° C. Unusual results were found at stresses below 1 bar, their data being represented by the exponential flow law,

$$\dot{\epsilon} = \dot{\epsilon}_0 \exp(\sigma/A),$$

which predicts a finite strain rate at zero stress. They attribute this result to rapid recrystallization due to the high temperature.

In addition, Mellor and Testa (1969[a]) reported the results of simple compression tests performed on polycrystalline samples of ice at low stresses (0.093 and 0.43 bar) and at a temperature of $-2.06°$ C. They found that a power law with an exponent of 1.8 described their results. They combined their data with some of the data of Mellor and Smith to show that the exponent in the power law increases at larger stresses.

The effect of temperature on the flow law is an important consideration, particularly with regard to temperate glacier flow. Glen (1955) showed that the rheological properties of ice depended upon the temperature of the ice and proposed the law

$$\dot{\epsilon} = A \exp(-Q/RT)\, \sigma^n. \tag{3}$$

In Glen's experiments, however, the ice was always below the melting temperature and the possibility of a marked change in the properties of ice at the melting temperature remained unresolved. Mellor and Testa (1969[b]) also investigated the effect of temperature on creep rates and found that the effective viscosity of ice decreases more rapidly than is predicted by the exponential relationship proposed by Glen. The hardness measurements of Barnes and Tabor (1966) are also relevant to the temperature dependence of rheological properties. These workers investigated the indentation hardness of polycrystalline ice samples near the melting temperature. They found that the indentation hardness changes much more rapidly above $-1.2°$ C than it does below $-1.2°$ C. Their experiments suggested that the flow law for ice at its melting temperature might not be predictable by extrapolation using Equation (3).

Steinemann (1958[b]) considered the thermodynamics of ice at its melting temperature by using Verhoogen's (1951) theory of equilibrium between a liquid and a solid. Steinemann concluded that the rheological properties of ice would not suddenly change at the melting temperature and, hence, that the flow law at the melting temperature could be predicted by extrapolating the results of experiments at sub-freezing temperatures. Verhoogen's model, however, has been shown to be incorrect by Kamb (1961) and by Nye (1967).

2. The experiments

The University of Washington's research station on Mount Olympus is located on a rock ridge adjacent to the snow dome of the Blue Glacier. A tunnel was excavated horizontally into the snow dome (LaChapelle, 1968) during the summer of 1966. This part of the glacier moves relatively slowly (about 2 m per year) and, being in the accumulation zone, the ice is relatively young and has experienced little deformation. The interior of the tunnel provided an environment in which to perform compressive creep tests at the pressure melting temperature and these tests were conducted under conditions as close as possible to those occurring within the temperate glaciers by using glacier ice at deviator stress levels comparable with those to which glaciers are normally subjected (i.e. up to 1 bar).

In the tunnel, five locations were prepared to install compression testing machines (see Fig. 1). Because of their position some 30 m from the tunnel entrance, these locations provided an essentially constant-temperature environment. At this depth in the tunnel the ambient pressure melting temperature (with 15 m of overburden and very low deviator stresses) was calculated to be $-0.01°$ C. The pressure melting temperature of the ice samples should have been slightly higher than this but temperature differences of this magnitude could not be measured in the tunnel. Each location was partitioned from the rest of the tunnel so that, when a sample was being installed at one location, any temperature increase of the air would not affect the other samples. Other important temperature considerations are discussed later.

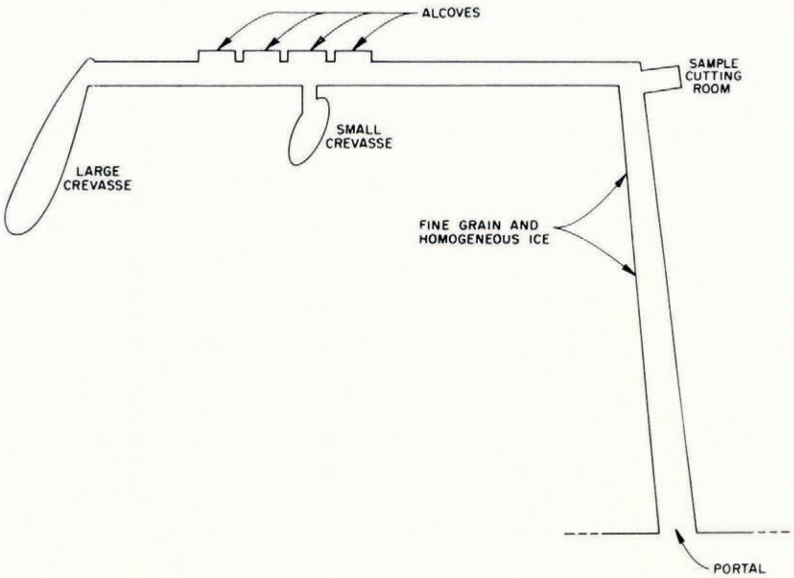

Fig. 1. Plan view of the ice tunnel. Scale 1 : 300.

(i) *Preparation of samples*

The structure of the ice on the inside of the tunnel varied. Bands of fine-grained clear ice, bands of fine-grained bubbly ice and bands of coarse-grained clear ice were present. The crystal sizes in the tunnel walls ranged from large crystals (30 mm in diameter) to small crystals (less than 1 mm in diameter). Most areas had a definite structure with small bands of clear crystals intersecting the generally cloudy appearance but large areas of bubbly ice with no visible inhomogeneities were present.

Large blocks of ice were cut from the tunnel walls with an electric chain saw. The blocks were cut down to 0.52 m by 0.13 m by 0.13 m, weighed, and the final dimensions were measured. The samples were then placed in the testing machines and surrounded (but not constrained) by many layers of plastic wrap in order to minimize evaporation and possible melting losses.

It was desired to test ice specimens which were uniform and isotropic, yet as representative as possible of typical glacier ice. Accordingly, ice samples were chosen to be uniformly fine-grained and to contain evenly distributed air-bubble inclusions. Average density was 0.89 Mg m^{-3}. Because of the relatively short strain history of the ice, simple grain shapes predominated. Isotropy was tested by determining 200 crystal orientations from numerous thin sections. The data were reduced according by the method of Kamb (1959), and no significant anisotropy was found. Average grain size was 2 mm, thus ensuring samples to be polycrystalline. This grain size was basically determined by the availability of such ice in the tunnel walls. Because the total strains were small, no significant changes occurred in fabric and texture of the samples during the experiments. Each sample was weighed and its dimensions were measured before and after each experiment.

(ii) *The test apparatus*

The five compression testing machines were constructed at the station. The field location required these machines to be extremely simple. The machines had bases made of thick plywood and heavy lumber. Two of the machines (for low stress tests) used dead weights, two used beams held on one end by turnbuckles and pulled on the opposite end by springs, and one machine used a beam with springs attached to both ends (see Fig. 2).

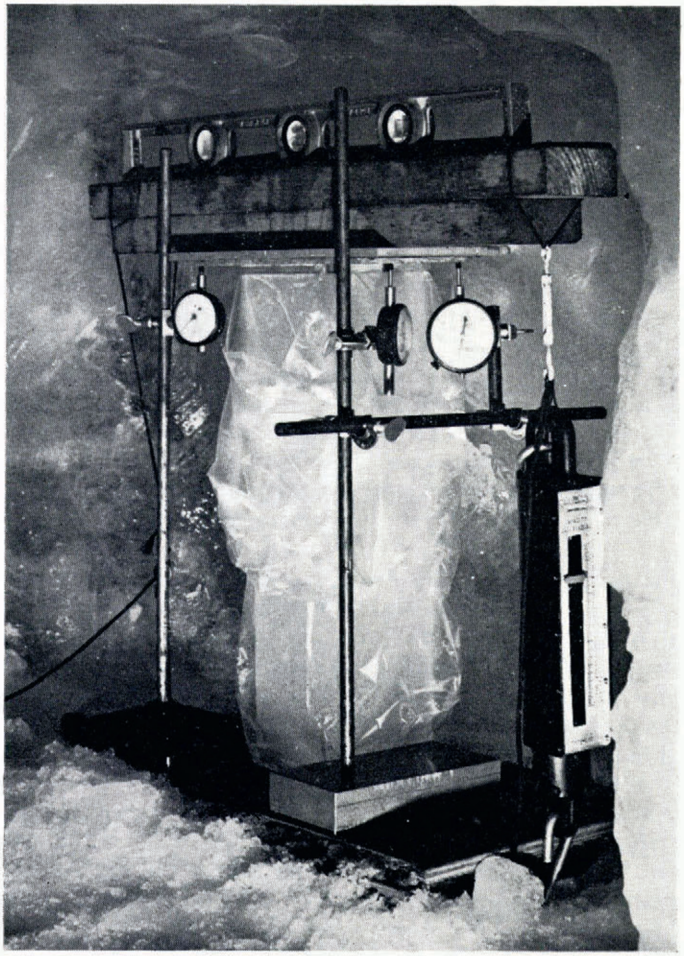

Fig. 2. Compression test apparatus (photograph by E. R. LaChapelle).

The platens were made of heavy plywood with pieces of 5 mm "lucite" polymethylmethacrylate plate next to the ice surfaces. Reinforcing aluminum plates were used to prevent warping of the platens. The lucite plates provided additional insulation against heat flow through the platens (the problem of melting at the sample ends is discussed later) and a smooth surface to minimize lateral constraint. The deformations in each experiment were measured by three micrometer dial gages which were accurate to 10^{-4} in (2.54 μm), three gages per experiment being used in order to determine the exact motion of the upper platen. Tilting was minimized by differential adjustment of the turnbuckles on the high-stress machines and by adjusting the position of weights on the dead-weight machines. On all five of the testing machines a small movable weight was placed on the upper platen for making small balance adjustments.

3. Errors

Some tilting was present in almost all tests. This was reduced to a small amount as described above but, because of the nonlinearity of the flow law, an error in computing mean strain-rate was introduced. An estimation of the tilting error is given in the Appendix. Tilting errors were found to be usually less than 1%. When the error was over 3%, the sample was rejected.

Thermometers with an accuracy of ± 0.05 deg were placed between the insulating layers of plastic wrap and the ice during the experiments. The air temperature next to the ice was never measurably different from 0° C. The ambient air temperature in the tunnel was in fact never measurably above 0° C except when a sample was installed in one of the machines at which time the air temperature in the immediate vicinity of the machine increased to 0.1° or 0.2° C. The air temperatures in the other locations were not affected by this and, within a few hours after installing a sample, the air temperature returned to 0° C.

Clearly the most significant point to be resolved is to establish that the samples were, in fact, at the pressure melting point, but at the same time, that strain readings were not made erroneous by melting or by regelation at the sample ends. Both qualitative and quantitative observations verified this to be the case. A simple heat-flow model was constructed under the assumption of the least favorable conditions and it was found that the amount of apparent strain-rate due to pressure melting under the platens could have been no more than $0.0088\ \sigma^2$ year^{-1}. Also, the resulting measurements of strain-rate show that the amount of apparent strain-rate due to pressure melting under the platens is negligibly small. That the ice samples were always at their melting temperature was indicated by the presence of liquid water on their surfaces.

In addition, the possibility of errors due to regelation or melting under the platens was directly checked. The method for this was to duplicate the experiments with an additional set of samples 0.13 m by 0.13 m by 0.26 m. Since regelation or end melting effects must be independent of sample length, melting would have been indicated if computed flow laws appeared to depend on sample length. In fact no such difference was found. Small weight losses were measured; these amounted to less than 0.2% for the duration of the tests and were attributed to evaporation from the sample sides. Consequently, in the analysis described below, data from both long and short samples are used. A length-to-width ratio of 4 is common for compression tests. Use of shorter samples in these tests was considered acceptable, however, since the presence of the water film lubricated the ends of the samples and reduced the end effects normally associated with compression tests. It should be noted that the above remarks do not refer to intergranular regelation. If such regelation occurred it would be measured as contributing to the deformation.

The possibility of errors due to relaxation from an initial stress state was considered. Because the tunnel averaged some 15 m in depth below the snow-dome surface, the blocks of ice from which samples were prepared were under an initial state of stress. Because of the non-linear character of relaxation (Jellinek and Brill, 1956), however, any relaxation effects must have become insignificant by the time steady creep developed.

Possible errors from other sources should be considered. Because of the limitations of the dial micrometer, calculated strain-rates below 0.01 year^{-1} were open to error. All of the dial micrometers were either read directly or interpolated to the nearest 10^{-4} inch (2.54×10^{-3} mm). At a strain-rate of 0.01 year^{-1}, the micrometers advanced only 2.5×10^{-4} inch per day (6.35×10^{-3} mm per day) and only 8×10^{-4} inch (20.3×10^{-3} mm) per average period of steady-state response. The percentage variations in the measured strain-rates were larger for the low stress tests than for the higher stress tests (see Fig. 5). The results with strain-rates less than 0.01 year^{-1} were included in the analysis of the data but are considered to be less accurate than the results with higher strain-rates. We feel, however, that the large scatter of the data, which is typical of such tests, is mostly attributable to variations within the specimens and not to random measurement errors.

No measurements of impurities were made. The purpose of the experiments was to obtain a flow law applicable to glacier ice and the use of specimens taken from within the glacier ensured that purity levels of the ice specimens were representative of the glacier.

4. Results

A typical curve of measured deformation versus time is shown in Figure 3. Transient creep usually lasted 20 to 30 h and then the creep became steady. This steady creep was allowed to continue for sufficient time to ensure an accurate calculation of its rate. The tests were usually terminated after 100 h. Some samples were stressed for up to 200 h in order to detect any further decrease in the strain-rate (see Fig. 4). No decrease in the strain-rate occurred during these additional 100 h.

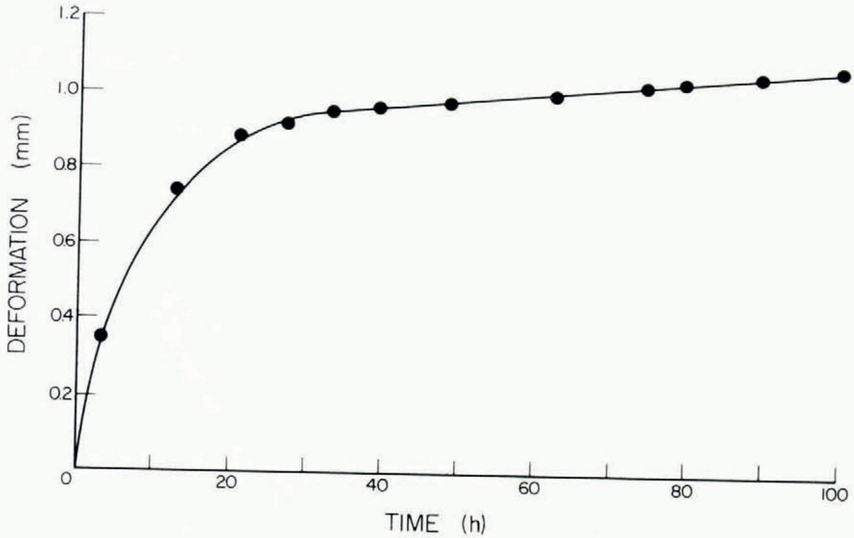

Fig. 3. Typical deformation versus time curve (σ = 0.1 bar).

Strain was computed by averaging the total displacement over the sample length while the stress was computed at the center of the sample, i.e. the average stress taking into account the weight of the specimen was found. Because of the non-linear relationship between strain-rate and stress, this averaging technique does introduce an error into the results. This error is estimated in the Appendix where it is shown to be less than 1%.

A total of 90 experiments were completed during the summers of 1968 and 1969. 26 of these experiments were rejected* and 64 were used for the flow-law computations. Many experiments were needed in order to obtain statistically significant results, the number of experiments completed being limited by the necessary duration of each experiment and the number of testing machines available. 28 of these 64 samples were 0.52 m long, the remaining 36 being short samples. Several forms of flow law were used to fit the data. Two of these

* Twelve were rejected because of faulty equipment or experimental procedures, eight were rejected due to an uneven deformation and six were rejected because they showed an irregular steady-state response.

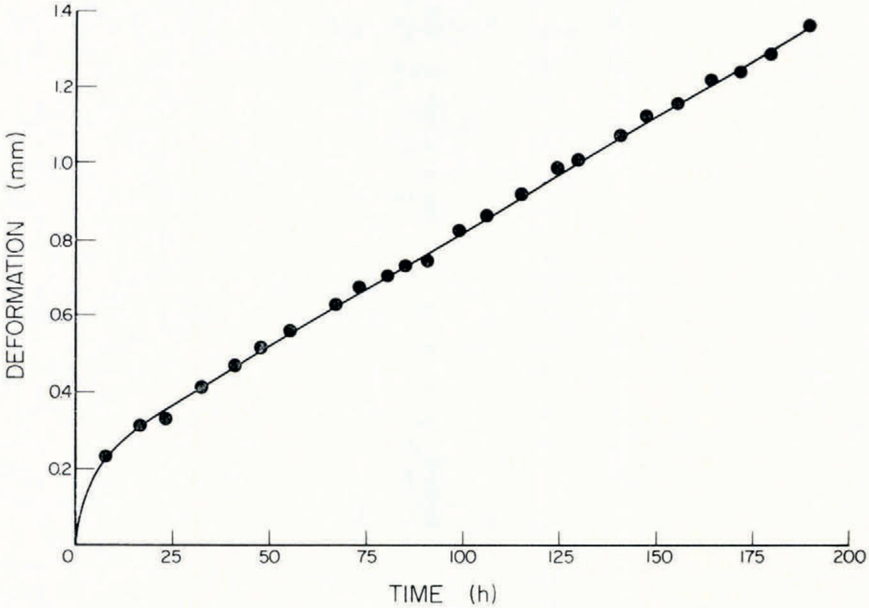

Fig. 4. Typical deformation versus time curve ($\sigma = 0.58$ bar).

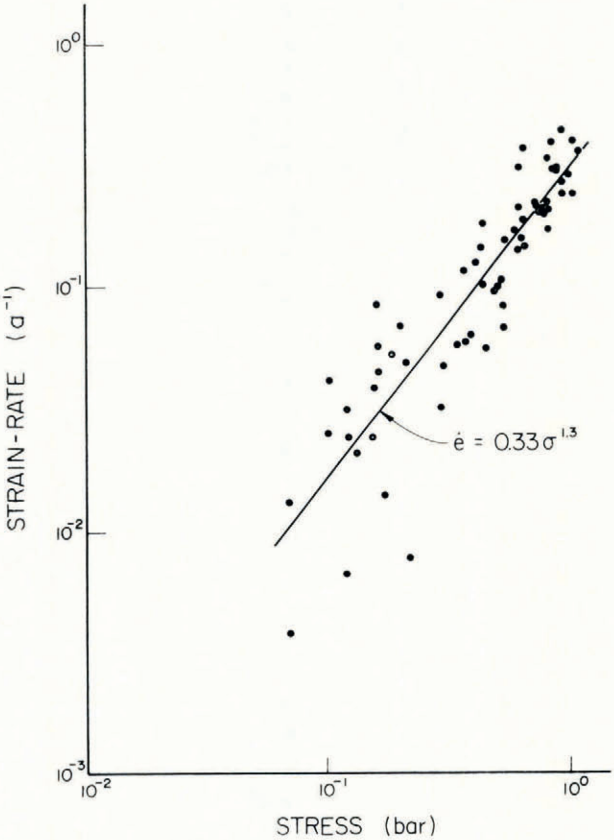

Fig. 5. Stress versus strain-rate.

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laws are discussed separately below. For details of other forms of flow law as well as a description of statistical methods used and confidence bounds on the results the reader is referred to Colbeck (unpublished).

(i) *Power law*

The best fit of the data to Equation (1) was found by the method of least squares. Initially the short and long samples were analyzed separately and the best fit for each case was

$$\dot{\epsilon} = 0.33\sigma^{1.3}. \tag{4}$$

Because no difference between the specimens of different length was found (their separate consideration enabled the effect of pressure melting under the platens to be discounted, as discussed above), for subsequent analysis, no distinction between sample lengths was made. Equation (4) is shown in Figure 5 together with data from both the long and short samples.

(ii) *Polynomial power law*

Equation (4) is not entirely satisfactory since, as may be seen from Figure 5, the exponent in the flow law appears to increase with stress. Other workers, including Glen (1955), Jellinek and Brill (1956), Butkovich and Landauer (1958, 1960), and Mellor and Smith (1967), have noted this apparent increase of the exponent with stress. Therefore, the flow law is best represented by a power law if only a narrow range of stresses is considered.

Such laws, however, are impractical for subsequent analytic use in glacier flow problems where a wide range of stress may be encountered. For such purposes a single law applicable everywhere throughout the glacier is desirable. These considerations make polynomial laws attractive and it was decided to consider the form used by Lliboutry (1969), i.e.

$$\dot{\epsilon} = A\sigma + B\sigma^3 + C\sigma^5. \tag{5}$$

The presence of the linear term was suggested from analysis of data when n was made stress dependent (Colbeck, unpublished). In this case n was found to approach 0.98 as stress approached zero. The fifth-order term was included to take into account the decrease in viscosity with increasing stress.

The form of Equation (5) obtained from the experimental data was

$$\dot{\epsilon} = 0.21\sigma + 0.14\sigma^3 + 0.055\sigma^5. \tag{6}$$

Equation (6) is shown in Figure 6 together with the experimental data.

5. Interpretation and application of flow law

(i) *Relation to other experiments*

As stated previously, in order for a single law to be applicable to glacier flow it should at least be applicable over a wide range of stresses, in particular higher than those at which the present tests were conducted. Equation (6) is thus compared with the data of Glen (1955) whose experimental conditions corresponded more closely to those of the present data than any other available work.* Glen used ice with small differences in texture and density from the ice used here, a temperature just below the melting temperature (−0.02° C) and compressive stresses from 0.9 to 9.2 bar. Thus, in the present experiments and in those of Glen, the ice was similar, the temperatures were nearly equal and the two ranges of compressive stresses were just overlapping, the essential difference being the fact that Glen's experiments were at temperatures below the pressure melting point.

* For other comparisons see Colbeck (unpublished).

Figure 6 shows Equation (6) and Glen's quasi-viscous flow law at $-0.02°$ C. The highly non-linear behavior of Glen's law over the region from 0.9 to 9.2 bar is in close agreement with the behavior of Equation (6) except Equation (6) predicts strain-rates which are about an order of magnitude larger than Glen found. These large differences in the strain-rates are probably due to the fact that, in contrast to the present experiments, Glen's experiments were performed below the pressure melting temperature. Apparently there is an accelerating decrease in the effective viscosity of ice as the melting temperature is approached.

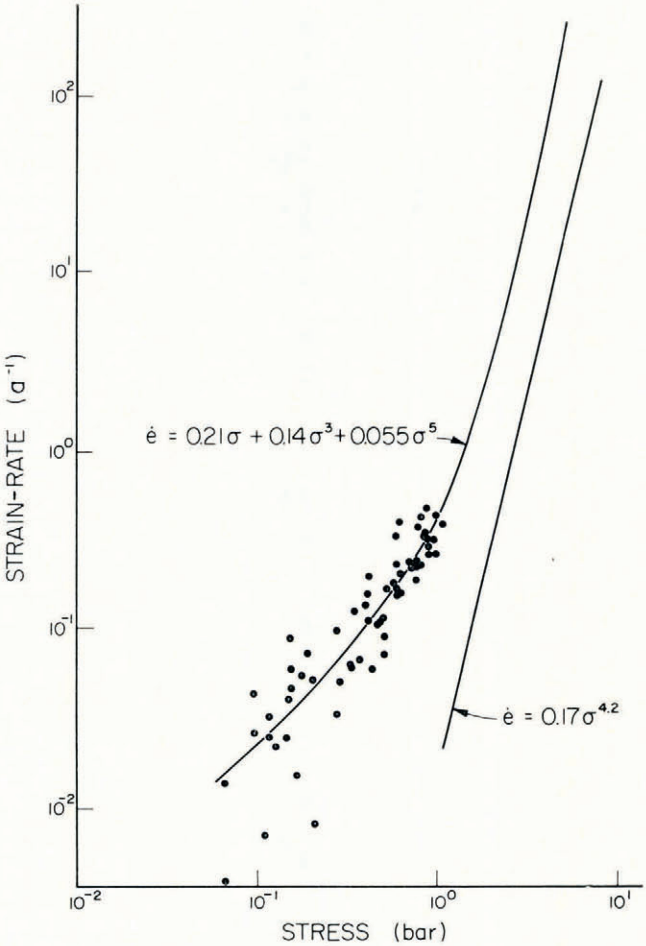

Fig. 6. Equation (6) and Glen's quasi-viscous flow law.

On Figure 7, Steinemann's (1958) results in simple compression are shown together with Equation (6). This is a favorable comparison showing the increasing exponent with stress and the reduced strain-rates at sub-freezing temperatures.

Thus, because Equation (6) accurately represents the response of glacier ice up to 1 bar and is compatible with Glen's quasi-viscous results in the stress range 0.9 to 9.2 bars, this equation is proposed as the uniaxial compressive law for glacier ice at its pressure melting temperature.

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(ii) *Temperature effects near the pressure melting point*

The question of whether or not the flow law changes abruptly at the melting temperature is still open. To resolve this experimentally, it would be necessary to test in compression specimens taken from the tunnel walls at temperatures slightly below freezing. This has not yet been done. It is useful, however, to compare the results obtained here with those of Glen and Steinemann obtained at 1 bar at various temperatures. In Figure 8 Glen's minimum

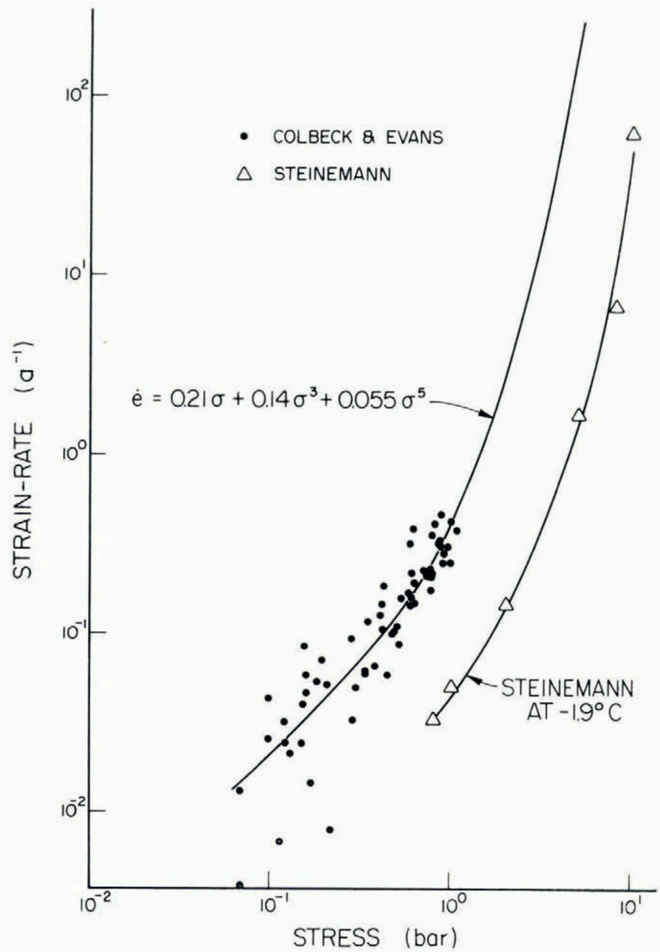

Fig. 7. Equation (6) and Steinemann's results.

observed creep rate and Steinemann's steady-state creep rate are plotted against the reciprocal of absolute temperature and, in addition, the strain rate at 1 bar as given by Equation (6) is shown. The relationship between the logarithm of strain-rate and the reciprocal of absolute temperature is clearly not linear at a stress of 1 bar in the temperature range considered. This is consistent with the conclusions reached by Mellor and Testa (1969[b]) and Barnes and Tabor (1966). It is significant to note, however, that there does not appear to be a discontinuity in the relationship when it is extrapolated to the melting temperature.

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6. Conclusions

A uniaxial compressive flow law applicable to temperate glacier ice at the pressure melting temperature has been determined experimentally. The flow law is compatible with existing theories of creep which suggest that a linear-viscous mechanism dominates at low stresses (leading to linear flow in the limiting case as stress goes to zero) and that intracrystalline glide dominates at high stress, the viscosity thus decreasing with increasing stress.

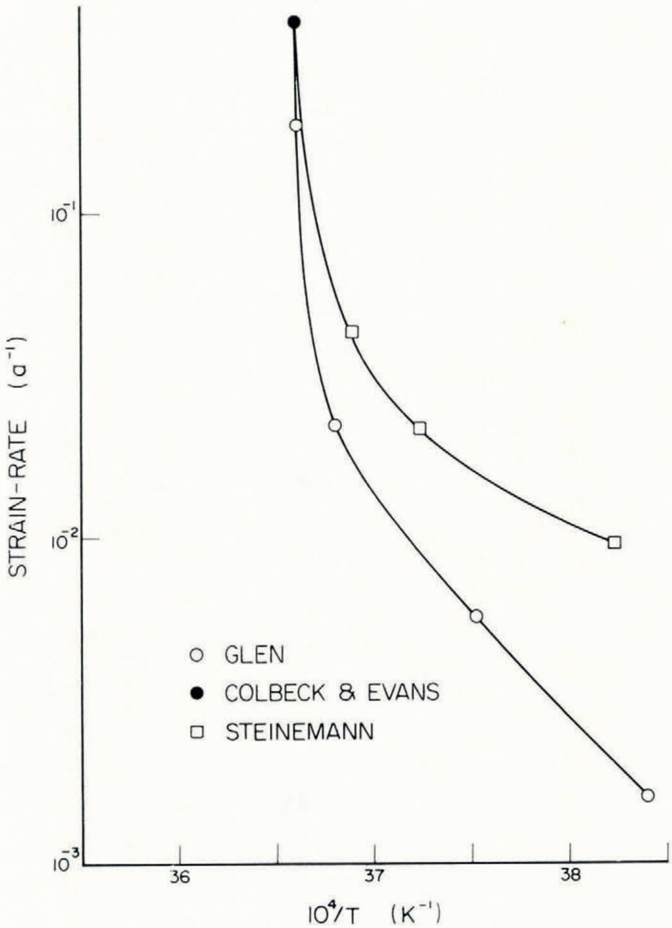

Fig. 8. Strain-rate versus reciprocal of absolute temperature at 1 bar.

No definite conclusions can be drawn with regard to an abrupt change in the flow law at the pressure melting temperature but it appears that no such abrupt change takes place. To investigate this further, compressive tests on samples taken from the ice tunnel should be made at temperatures slightly below freezing. Additional experimental data on large-grained samples from the actively flowing region of the glacier would be valuable in finding the effect of grain size on the flow law.

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APPENDIX

(i) *Estimation of the tilting error*

Assuming the sample to be homogeneous, then tilting occurs because the load is not placed exactly at the center of the platen. The platens restrict the deformation so that horizontal plane sections remain plane and the distribution of strain is thus

$$\dot{\epsilon} = \dot{\epsilon}_c(1 + bx/l) \tag{A1}$$

where $2l$ is the width of the sample. In this calculation we are considering a unit thickness and assuming tilting to be in the plane of this thickness (see Fig. A1).

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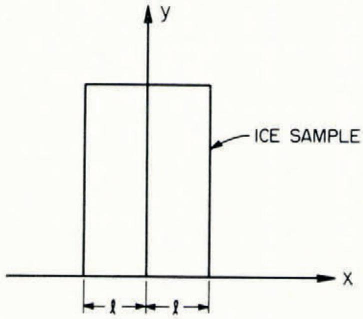

Fig. A1. Compression test specimen.

Assuming that the stress state is uniaxial and that a power law describes steady creep then

$$\dot{\epsilon}(x) = A[\sigma(x)]^n$$

or

$$\sigma(x) = [\dot{\epsilon}(x)/A]^{1/n}. \quad \text{(A2)}$$

Combining Equations (A1) and (A2),

$$\sigma(x) = (\dot{\epsilon}_c/A)^{1/n}(1+bx/l)^{1/n} \quad \text{(A3)}$$

and the average stress $\bar{\sigma}$ on the sample is given by

$$\bar{\sigma} = \frac{1}{2l}\int_{-l}^{l} \sigma(x)\,dx$$

so that

$$\bar{\sigma} = \frac{n(\dot{\epsilon}_c/A)^{1/n}}{2b(n+1)}[(1+b)^{n+1/n}-(1-b)^{n+1/n}]. \quad \text{(A4)}$$

In these experiments we have calculated the strain-rate at the center of the platen and have related this to stress. Thus, assuming the power law to hold, we have determined the flow law from

$$\sigma_c = (\dot{\epsilon}_c/A)^{1/n}. \quad \text{(A5)}$$

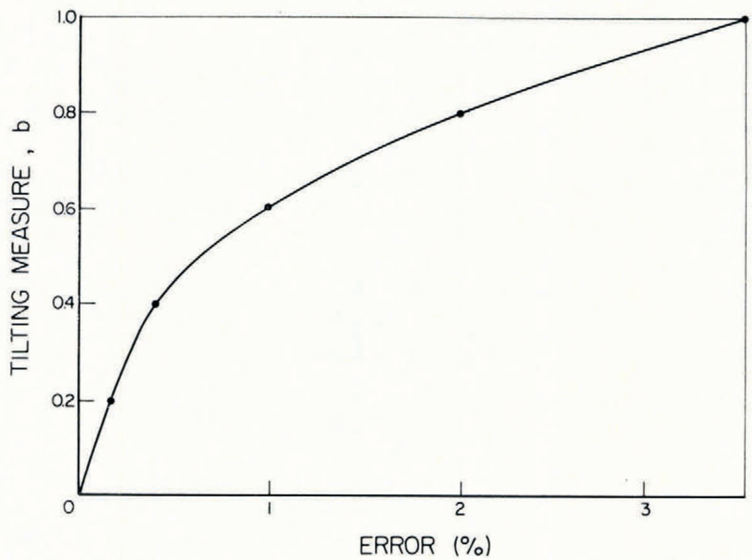

Fig. A2. Tilting error versus tilt.

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A measure of the tilting error E is thus

$$E = \frac{100(\sigma_c - \bar{\sigma})}{\sigma_c}\ \%$$

which, from Equations (A4) and (A5), is

$$E = 100\left[\frac{n}{2b(n+1)}\{(1-b)^{n+1/n} - (1-b)^{n+1/n}\} + 1\right]\ \%. \tag{A6}$$

Knowing n, the error can be found in terms of the tilt b. Thus for $n = 1.3$, E is plotted against b in Figure A2. Tests where the tilting error was over 3% according to Equation (A6) were rejected. In only three tests were errors over 2% present; in the majority of tests the error according to Equation (A6) was less than 1%.

It should be noted that the above calculation is only an estimate of the error since it is based on a presupposed flow law. The error needs to be estimated since E is always non-negative.

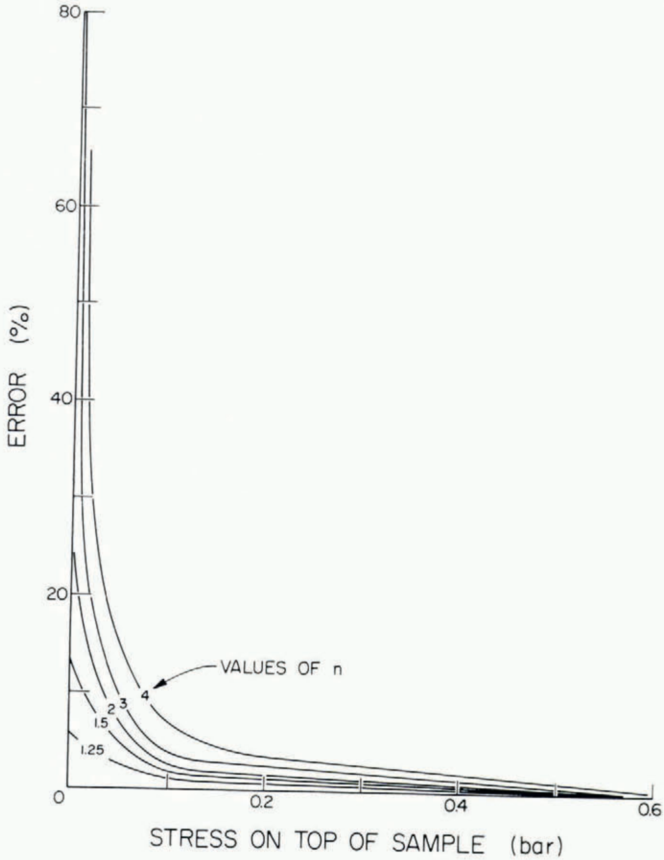

Fig. A3. Flow-law error versus stress.

(ii) *Estimation of average strain-rate*

The average strain-rate, over the length of the sample, $\bar{\dot{\epsilon}}$, is given by

$$\bar{\dot{\epsilon}} = \frac{1}{L}\int_0^L \dot{\epsilon}(y)\,dy \tag{A7}$$

where $\dot{\epsilon}(y)$ is the strain-rate of any section.

For uniform stress

$$\sigma = F/a - \rho g L - \rho g y \tag{A8}$$

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whence, for a power flow law,

$$\dot{\epsilon} = A(F/a + \rho g L - \rho g y)^n. \tag{A9}$$

Substituting Equation (A9) into Equation (A7) and integrating we obtain

$$\bar{\dot{\epsilon}} = \frac{A}{\rho g L(n+1)}\{(F/a+\rho g L)^{n+1} - (F/a)^{n+1}\}.$$

The error associated with strain measurement is thus

$$E_2 = \frac{100\{\bar{\dot{\epsilon}} - \dot{\epsilon}(L/2)\}}{\dot{\epsilon}(L/2)}\ \%.$$

This error is shown in Figure A3 for various values of n and F/a. For values of interest here, i.e. $n = 1.3$ and $F/a \geqslant 0.04$, no correction is considered necessary. In general the error is less than 1%. Only for very low loads can the error amount to 4%. Also, because n has been found to approach unity as F approaches zero, the above calculation over-estimates the possible error for small loads.

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