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HOMOTOPY EQUIVARIANCE, STRICT EQUIVARIANCE AND INDUCTION THEORY

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An obvious question occurs at the very start of equivariant homotopy theory. What is the relationship between maps equivariant up to homotopy and strictly equivariant maps? This question has been studied by various people, usually away from the group order ([**8**, **11**, **22**, **25**, **26**]). We consider the problem stably and answer it by giving a spectral sequence proceeding from homotopy equivariant to strictly equivariant information. The form of the spectral sequence is not surprising, but there are three distinctive features of our approach: (1) we show that the spectral sequence may be viewed as an Adams spectral sequence based on nonequivariant homotopy, (2) we show how to exploit the product structure, and (3) we give a treatment showing how Dress's algebra of induction theory [**13**] applies to give non-normal subgroups equal status. As a spinoff from (3) we also obtain spectral sequences for calculating homology and cohomology of universal spaces (3.5).

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1. Introduction

Suppose that G is a finite group and that X and Y are pointed G-CW-complexes or G-spectra [**17**]. If H is a subgroup of G we may ask the following question.

Question 1.1. What does the group of stable H-equivariant maps $[X, Y]_*^H$ tell us about the group $[X, Y]_*^G$ of stable G-equivariant maps?

Of course if $H=1$ then G acts on $[X, Y]_*^1$ by conjugation and its fixed point set is the set of homotopy equivariant maps. More generally this applies whenever $H=N$ is normal and suggests we consider a spectral sequence based on the $\mathbb{Z}G$-module or comodule structure of $[X, Y]_*^N$. Professor J. F. Adams has asked (private communication) the precise form such a spectral sequence would take and how much information it would provide. In the special case that $H=N$ is normal our main theorem (3.4) states that there is a multiplicative spectral sequence

$$E_2^{s,t}=H^s(G/N;[X, Y]_t^N)\Rightarrow[X \wedge EG/N_+, Y]_*^G. \tag{1.2}$$

Comments.

(1) There are no conditions on X or Y.

(2) Convergence is in the usual conditional sense of Boardman appropriate to infinite decreasing filtrations (see [**2**]).

(3) The $s=0$ edge homomorphism is the forgetful map

$$[X \wedge EG/N_+, Y]_t^G \to H^0(G/N; [X, Y]_t^N) = \{[X, Y]_t^N\}^{G/N}$$

and so homotopy equivariant map $f: X \to Y$ represents an equivariant map $\hat{f}$: $X \wedge EG/N_+ \to Y$ iff it is an infinite cycle. This is clearly necessary for f to represent an equivariant map $\bar{f}: X \to Y$, but for each $\hat{f}$ there is one further obstruction in the composite $q \circ S\hat{f}$ of the following diagram

$$\begin{tikzcd}
X \wedge S^0 * EG/N \arrow[r, "q"] & X \wedge SEG/N_+ \arrow[r] \arrow[d, "S\hat{f}"'] & X \wedge S^1 \arrow[dl, dashed, "\exists ? \bar{f}"] \\
 & SY &
\end{tikzcd}$$

(4) If Y is finite $[X \wedge EG/N_+, Y]^G = [X, D(EG/N_+) \wedge Y]^G$ (where $D(\cdot)$ denotes functional duality). Thus if X is also finite the solution of the Segal conjecture in the form of an explicit identification of $D(EG/N_+)$ [**15**], [**16**] explains the relationship of the target to $[X, Y]^G$. In particular if G is a p-group and Y is p-complete Carlsson's theorem [**10**] shows that $[X \wedge EG/N_+, Y]^G = [X, Y]^G$ and the spectral sequence converges to the group of interest. Also if Y is localised away from the group order $[X \wedge EG/N_+, Y]^G$ is a summand of $[X, Y]^G$ and in this case we may also give a complete account of $[X, Y]^G$. (See Section 5).

(5) It is easy to construct the above spectral sequence by filtering EG/N_+ by skeleta and applying $[\cdot \wedge X, Y]^G$. However this does not make plain how to proceed for subgroups which are not normal, nor does it give ready access to the product structure.

(6) We prefer to regard the spectra sequence as the Adams spectral sequence based on H-equivariant homotopy applied to calculate $[S^0, F(X, Y)]^G$ as far as possible. From this point of view the identifications of both the E_2-term and the target for convergence are interesting results.

(7) For subgroups H which are not normal the algebra describing the E_2-term is the less familiar Amitsur–Dress cohomology [**13**], and we recall relevant factors in Section 2.

The paper is laid out as follows.

Section 0: Abstract
Section 1: Introduction
Section 2: Amitsur–Dress cohomology

Section 3: The spectral sequence
Section 4: The Adams spectral sequence for H-equivariant homotopy
Section 5: Away from the group order
Section 6: The action of the Burnside ring on cohomology
Section 7: Multiplicative structure
Section 8: Sample calculations.

It is a great sadness to me that Professor J. F. Adams had only seen an informal account of the present paper at the time of his tragic death. I will greatly miss his perceptive criticisms.

2. Amitsur–Dress cohomology

We will recall here the basic definitions from the work of Dress on induction theory [13]. Thus if A is any G-set we may form the Amitsur simplicial G-set $Am_{\bullet}(A)$

$$\begin{array}{ccccccc}
 & & & & & \leftarrow & \\
 & & & \leftarrow & & \leftarrow & \\
 & \leftarrow & & \leftarrow & & \leftarrow & \\
A & \leftarrow & A\times A & \leftarrow & A\times A\times A & \leftarrow & \cdots \\
 & \rightarrow & & \rightarrow & & \rightarrow & \\
 & & & \rightarrow & & \rightarrow & \\
 & & & & & \rightarrow &
\end{array} \tag{2.1}$$

Thus we take $Am_{\bullet}(A)$ to be defined by $Am_s(A)=A^{s+1}(s\geqq 0)$. We use the $(s+1)$-projection maps $A^{s+1}\to A^s$ as face maps, i.e. $d_i(a_0,\dots,a_s)=(a_0,\dots,a_{i-1},a_{i+1},\dots,a_s)$ for $0\leqq i\leqq s$ and the degeneracy maps $s_i\colon A^{s+1}\to A^{s+2}$ are defined by $s_i(a_0,\dots,a_s)=(a_0,\dots,a_{i-1},a_i,a_i,a_{i+1},\dots,a_s)$ for $0\leqq i\leqq s$. We remark that if $A=G$ and we form the free abelian group complex from $Am_{\bullet}(A)$ we obtain the bar resolution. Next we suppose given a contravariant additive functor M from the category of G-sets to the category of abelian groups. We may define further functors M_B for any G-set B by $M_B(C)=M(B\times C)$ and thus form the Amitsur complex

$$0\longrightarrow M_{A^1}\xrightarrow{\partial_0}M_{A^2}\xrightarrow{\partial_1}M_{A^3}\xrightarrow{\partial_2}\cdots \tag{2.2}$$

where $\partial_s\colon M_{A^{s+1}}\to M_{A^{s+2}}$ is defined to be

$$\sum_{i=0}^{s+1}(-1)^i d_i^*.$$

Finally we define the Amitsur–Dress cohomology group-functors by

$$H^i_{\mathscr{A}\mathscr{D}}(A;M)=\ker\partial_i/\operatorname{im}\partial_{i-1}. \tag{2.3}$$

We note that Dress uses the notation $H^i_A(M)$ for this functor.

More generally these cohomology group-functors may be calculated by constructing an A-split resolution of M by A-injective functors. Accordingly the groups $H^i_{\mathscr{A}\mathscr{D}}(A;M)$ vanish in positive dimensions if M is itself A-injective. In particular if there is a G-map $B\to A$ then M_B is A injective and so $H^i_{\mathscr{A}\mathscr{D}}(A;M)(B)=0$ if $i\geqq 1$.

We will be fundamentally concerned with the case $A=G/H$, and the following lemma relates this to more familiar things, and explains our choice of notation. We let $*$ denote the G-set G/G.

Lemma 2.4.

(a) *If N is a normal subgroup of G then*

$$H^i_{\mathscr{A}\mathscr{D}}(G/N;M)(*)=H^i(G/N;M(G/N)).$$

(b) *$H^i_{\mathscr{A}\mathscr{D}}(G/H;M)(*)$ depends only on $M(G/K)$ for subgroups K subconjugate to H (i.e. conjugate to a subgroup of H).*

Proof. Part (b) is clear since, applying (2.2) to $*$ we only have to consider $M((G/H)^k)$ for $k\geqq 1$, and $((G/H)^k)^K\neq\varnothing$ only if K is subconjugate to H. Part (a) follows from the fact that the free abelian group complex obtained from $Am_\bullet(G/N)$ is the bar resolution, once we prove the next lemma.

Lemma 2.5. *We have an isomorphism*

$$M((G/N)^k)=\operatorname{Hom}_{\mathbb{Z}G}(\mathbb{Z}(G/N)^k,M(G/N))$$

for $k\geqq 1$ which is natural in M and respects all projection maps $(G/N)^k\to(G/N)^{k-1}$.

Proof. The trick is to find a coordinate free isomorphism. For this we use the $|G/N|^k$ G-maps $G/N\to(G/N)^k$ to give us maps

$$M((G/N)^k)\to\prod_{\gamma\in(G/N)^k}M(G/N)$$

and

$$\operatorname{Hom}_{\mathbb{Z}G}(\mathbb{Z}(G/N)^k,M(G/N))\to\prod_{\gamma\in(G/N)^k}\operatorname{Hom}_{\mathbb{Z}G}(\mathbb{Z}G/N,M(G/N)).$$

Now we have a natural isomorphism $\operatorname{Hom}_{\mathbb{Z}G}(\mathbb{Z}G/N,M(G/N))\cong M(G/N)$ by using the standard generator $1.N\in\mathbb{Z}G/N$. Finally we pass to quotients of these products using the relation $(\gamma g,x)=(\gamma,gx)$. The composite is the required isomorphism and is clearly as natural as claimed. $\square$

Remark 2.6. The more general analogue of (2.5) is the statement

$$\mathrm{Hom}_{\mathscr{C}_G}(\underset{\sim}{\mathbb{Z}}S, M) = M(S).$$

Here $\mathscr{C}_G$ is Bredon's category of coefficient systems [**6**] or equivalently the category of additive contravariant functors from the category of G-sets to abelian groups and $\underset{\sim}{\mathbb{Z}}S(G/K) = \mathbb{Z}[S^K]$ is Bredon's projective coefficient system for the G-set S. Note however that the Amitsur resolution $\underset{\sim}{\mathbb{Z}}_* \leftarrow \underset{\sim}{\mathbb{Z}}A \leftarrow \underset{\sim}{\mathbb{Z}}A^2 \leftarrow \ldots$, although it consists of projective coefficient systems, is only exact upon evaluation at G/K if $A^K \neq \varnothing$.

3. The spectral sequence

In this section we construct the spectral sequence and identify its E_2-term using the algebra of Section 2. The convergence statements is clear by construction.

For any G-set A we let EA denote the geometric realisation of the Amitsur simplicial G-set (2.1) using face operators only. We note that if $A = G/N$ this notation accords with the usual usage of EG/N for a nonequivariantly contractible G/N-space on which G/N acts freely. Since we want a based G-space we add a distant basepoint to form EA_+. Now EA_+ comes with a natural filtration by skeleta and hence we may obtain a spectral sequence by applying $[X \wedge \cdot, Y]^G$ to the diagram

$$\begin{array}{ccccccc} * \longrightarrow & EA_+^{(0)} & \longrightarrow & EA_+^{(1)} & \longrightarrow & EA_+^{2} & \longrightarrow \ldots \\ & \downarrow & & \downarrow & & \downarrow & \\ & R_0 & & R_1 & & R_2 & \end{array} \tag{3.1}$$

where $EA_+^{(s)}$ denotes the s skeleton and $R_s = EA_+^{(s)}/EA_+^{(s-1)}$. By construction we have the following:

Lemma 3.2.

(a) $R_s \simeq S^s A_+^{s+1}$

(b) *The maps*

$$R_s \longrightarrow SR_{s-1}$$

inducing the differentials are alternating sums of the s-fold suspensions of the projection maps $A^{s+1} \to A^s$.

To identify the E_2-term we finally need:

Lemma 3.3. *The isomorphism*

$$[X \wedge G/K_+, Y]^G = [X, Y]^K$$

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is natural for stable G-maps of X and Y and for maps of G-sets $G/K \to G/K'$.

Proof of 3.3. Naturality in X and Y is clear. For naturality in K we must return to the definitions.

The isomorphism is obtained by combining the homeomorphism $h: G_+ \wedge_K X \xrightarrow{\cong} G/K_+ \wedge X$, natural for G-maps of X, with the stabilisation of the well known adjunction, which ([3, (5.1)], [17, (II. 4.7)])

$$[G_+ \wedge_K X, Y]_*^G = [X, Y]_*^K. \tag{3.4}$$

Now all G-maps $G/K \to G/K'$ are composites of those of the following two types

(a) Right multiplication by $g \in G$

$$R_g: G/K \to G/g^{-1}Kg.$$

(b) Quotient maps

$$\pi_{K,K'}: G/K \to G/K'$$

where $K \subseteq K'$.

Now if $f: X \to Y$ is a K-map the corresponding G-map is

$$I_K(f): G_+ \wedge_K X \xrightarrow{1 \wedge_K f} G_+ \wedge_K Y \xrightarrow{\varepsilon} Y$$

where ε is the G-map extending the identity, $1 \wedge_K Y \to Y$. Conversely if $F: G_+ \wedge_K X \to Y$ is a G-map, the corresponding K-map is obtained by composing with the K-map $1_+ \wedge_K X \to G_+ \wedge_K X$. Since $G_+ \wedge_{K'} (\cdot) = G_+ \wedge_{K'} K'_+ \wedge_K (\cdot)$, naturality for maps of type (b) is clear.

For maps of type (a) we let R_g^* denote the induced map

$$[G/K_+^g \wedge X, Y]_*^G \to [G/K_+ \wedge X, Y]_*^G.$$

We suppose the K^g-map $f: X \to Y$ corresponds to $F: G/K_+^g \wedge X \to Y$ and the K-map $f': X \to Y$ corresponds to $F \circ R_g: G/K_+ \wedge X \to Y$. Thus we have a diagram

$$\begin{tikzcd}
G/K_+^g \wedge X \arrow[r, "\cong", "h^{-1}"'] & G_+ \wedge_K gX \arrow[dr, "I(f)"] & \\
 & & Y \\
G/K_+ \wedge X \arrow[uu, "R_g \wedge 1"] & G_+ \wedge_K X \arrow[l, "\cong"', "h"] \arrow[ur, "I(f')"'] &
\end{tikzcd}$$

and hence

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$$
\begin{aligned}
f'(x) &= I(f')(1 \wedge_K x) \\
&= I(f) \circ h^{-1} \circ R_g \wedge 1(1 \wedge_K x) \\
&= I(f) \circ h^{-1}(g \wedge_K x) \\
&= I(f)(g \wedge_{K^g} g^{-1} x) \\
&= \varepsilon(g \wedge_{K^g} f(g^{-1} x)) \\
&= (gfg^{-1})(x).
\end{aligned}
$$

□

Combining (3.2), (3.3) and the definitions we have our main result.

Theorem 3.4. *We have a conditionally convergent spectral sequence for any G-set A*

$$E_2^{s,t} = H^s_{\mathscr{A}\mathscr{D}}(A; [X, Y]^{\bullet}_t)(*) \Rightarrow [X \wedge EA_+, Y]^G_{t-s}$$

where $[X, Y]^{\bullet}_t(G/K) = [X, Y]^K_t$.

We draw attention to Lemma 2.4 for the application of this result. Part (a) shows that if $A = G/N$ we obtain the spectral sequence (1.2) discussed in the introduction, and Part (b) shows that if $A = G/H$ the E_2-term only uses $[X, Y]^K_*$ for K subcongugate to H.

We also note a corollary to (3.4) in a slightly different direction. We may consider a family $\mathscr{F}$ of subgroups of G, closed under subconjugacy and consider the universal space $E\mathscr{F}$. It is of some interest to calculate the equivariant homology and cohomology of this space. Indeed if we consider the G-set $A(\mathscr{F}) = \Pi_{H \in \mathscr{F}} G/H$ we find $E\mathscr{F} \simeq EA(\mathscr{F})$ and hence by taking $X = S^0$ in (3.4) we have for the cohomology theory $Y^*_G(\cdot)$ represented by the G-spectrum Y:

Corollary 3.5. *There is a conditionally convergent spectral sequence*

$$E_2^{s,t} = H^s_{\mathscr{A}\mathscr{D}}(A(\mathscr{F}); Y^t_{\bullet})(*) \Rightarrow Y^{s+t}_G(E\mathscr{F}_+)$$

Remark 3.6. Similarly there is a strongly convergent spectral sequence

$$E^2_{s,t} = H^{\mathscr{A}\mathscr{D}}_s(A(\mathscr{F}); Y^{\bullet}_t)(*) \Rightarrow Y^G_{s+t}(E\mathscr{F}_+)$$

obtained by applying $Y^{\bullet}_*(\cdot)$ to the skeletal filtration of $EA(\mathscr{F})_+$.

Of course if Y^H_* satisfies the dimension axiom for $H \in \mathscr{F}$ the spectral sequences collapse. In particular we have:

Corollary 3.7. *For any Mackey functor M the Bredon homology and cohomology of* $E\mathscr{F}_+$ *is given by*

$$H^t_G(E\mathscr{F}_+; M) = H^t_{\mathscr{A}\mathscr{D}}(A(\mathscr{F}); M)(*);$$

$$H^G_t(E\mathscr{F}_+; M) = H^{\mathscr{A}\mathscr{D}}_t(A(\mathscr{F}); M)(*).$$

Remark. This allows us to find out something about the geometric fixed point spectrum $\Phi^N(HM)$ (see [**17**, (2.9)] and [**15**, §2]) of the Eilenberg–MacLane spectrum HM representing Bredon cohomology, for a normal subgroup N. Indeed by [**17**, (2.9.8)] if $[\not\supseteq N] = \{H \mid H \not\supseteq N\}$ we have

$$\pi^{G/N}_t(\Phi^N HM) = H^G_t(S^0 * E[\not\supseteq N]; M) = \tilde{H}^{\mathscr{A}\mathscr{D}}_{t-1}(A([\not\supseteq N]); M)(*) \tag{3.8}$$

where the reduced homology group is obtained by augmenting (2.1) with $*$ in dimension -1.

4. The Adams spectral sequence for H-equivariant homotopy

In this section we show that the spectral sequence of Section 3 coincides with the Adams spectral sequence for H-equivariant homotopy applied to calculate $[S^0, F(X,Y)]^G_*$ as far as possible. If one is interested solely in the Adams spectral sequence the present approach is still extremely efficient since the E_2 and convergence problems are so painlessly dealt with.

The minimal structure necessary for constructing an Adams spectral sequence is a ring spectrum representing the theory.

Lemma 4.1. *Stable H-equivariant homotopy $\pi^H_*(\cdot)$ is represented by G/H_+.*

Proof. The whole Adams spectral sequence can be regarded as a systematic exploitation of the Wirthmüller adjunction ([**13**, (5.2)], [**17**, (II.6.5)]). This states that for any G-spectrum X and H-spectrum Y we have

$$[X, G_+ \wedge_H Y]^G = [X, Y]^H. \tag{4.2}$$

In case Y itself a G-spectrum we then have the homeomorphism h: $G_+ \wedge_H Y \xrightarrow{\cong} G/H_+ \wedge Y$ that we saw in Section 3. Taking $X = S^0$ we have the desired result.

To make further progress we must recall the unit and counit of the adjunction ([**3**], [**17**]). The counit is easily described and arises unstably:

$$\varepsilon: G_+ \wedge_H Y \to Y$$

is given by

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$$1 \wedge_H y \mapsto y$$
$$g \wedge_H y \mapsto * \qquad (g \notin H).$$

The unit $\eta': X \to G_+ \wedge_H X$ is intrinsically stable. In fact we may let V be the representation $\mathbb{R}G/H$ in which G/H embeds with equivariant tubular neighbourhood W, and hence obtain a map $S^V \to V/(V\backslash W) \cong S^V \wedge G/H_+$; the Vth desuspension η of this is used to construct η' as the composite

$$X \cong S^0 \wedge X \xrightarrow{\eta \wedge 1} G/H_+ \wedge X \underset{\cong}{\xrightarrow{h^{-1}}} G_+ \wedge_H X.$$

Now we may make G/H_+ into a ring spectrum using the monad of the adjunction. Indeed the categorical unit and product map

$$\eta: 1 \to G_+ \wedge_H (\cdot) \quad \text{and} \quad \mu = G_+ \wedge_H \varepsilon: G_+ \wedge_H G_+ \wedge_H (\cdot) \to G_+ \wedge_H (\cdot)$$

give rise directly to spectrum level unit and product maps

$$\eta: S^0 \xrightarrow{\eta'} G_+ \wedge_H S^0 \xrightarrow{\cong} G/H_+ \quad \text{and} \quad \mu: G/H_+ \wedge G/H_+ \longrightarrow G/H_+.$$

Indeed a short and instructive calculation shows μ is represented unstably by the map

$$\mu: xH \wedge yH \mapsto \begin{cases} xH & \text{if } xH = yH \\ * & \text{if } xH \neq yH. \end{cases} \tag{4.3}$$

Corollary 4.4. *The spectrum G/H_+ with structure maps η and μ as above is a commutative associative ring spectrum.*

Remark. If G is a compact Lie group of positive dimension a suspension is involved in the Wirthmüller adjunction, and hence G/H_+ will not have a unit if H is not of finite index.

Proof of 4.4. The associativity and commutativity of μ are clear unstably from (4.3). The monadic identity diagrams for S^0 are

$$\begin{tikzcd} G_+ \wedge_H S^0 \arrow[r, "\eta'_{G_+ \wedge_H S^0}"] \arrow[dr, "1"'] & G_+ \wedge_H G_+ \wedge_H S^0 \arrow[d, "\mu"] & G_+ \wedge_H S^0 \arrow[l, "1 \wedge_H \eta_{S^0}"'] \arrow[dl, "1"] \\ & G_+ \wedge_H S^0 & \end{tikzcd}$$

The right hand triangle translates into the right unit axiom for G/H_+ since η_{S^0} is a

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G-map and the homeomorphism h^{-1} is natural. The left hand triangle is already the left unit axiom since η'_X was defined as $\eta'_{S^0} \wedge 1$. □

We may now form the canonical resolution of the 0-sphere S

$$\begin{array}{ccc} \downarrow & & \\ S_2 & \xrightarrow{k_2} & Q_2 \\ \downarrow & & \\ S_1 & \xrightarrow{k_1} & Q_1 \\ \downarrow & & \\ S=S_0 & \xrightarrow{k_0} & Q_0 \end{array} \tag{4.5}$$

by taking $Q_s = G/H_+ \wedge S_s$ and using the unit $\eta: S_s \to G/H_+ \wedge S_s$ to give the map k_s. As usual the canonical resolution of $F = F(X, Y)$ is obtained by smashing (4.5) with F.

Proposition 4.6. *The spectral sequence obtained by applying $[S^0, \cdot\,]^G_*$ to the canonical Adams resolution of F coincides with the spectral sequence of Section 3 obtained by applying $[X \wedge \cdot, Y]^G_*$ to the skeletal filtration of EG/H_+.*

From this we immediately deduce the E_2-term of the Adams spectral sequence and the fact that it converges to $[X \wedge EG/H_+, Y]^G_*$. The reader may find it instructive to establish the algebraic isomorphism between the usual comodule *Ext* description of the E_2 term of an Adams spectral sequence and the present one directly (he may be amused to learn this was our original approach.)

We take this opportunity to justify Remark (4.7) of **[14]**. Recall that the Borel spectrum b represents Borel cohomology $b^*(X) = H^*(EG_+ \wedge_G X)$ and that the coBorel spectrum spectrum $c = b \wedge EG_+$ has the property that $c_*(X) = H_*(EG_+ \wedge_G X)$ (since G is finite).

Corollary 4.7. (a) *The Bousfield completion of the G-spectrum Z with respect to $\pi^H_*(\,\cdot\,)$ is the function spectrum $F(EG/H_+, Z)$.*

(*b*) *The Bousfield completion of the coBorel spectrum c with respect to nonequivariant homology H is the Borel spectrum b.*

Proof. If completion is interpreted as G/H_+-localisation, Part (a) is clear from the definition: since EG/H is H-contractible the map $Z \to F(EG/H_+, Z)$ is a G/H_+-equivalence, and since EG/H_+ is constructed from cells G/K_+ with K subconjugate to H, $F(EG/H_+, Z)$ is G/H_+-local. If it is interpreted as nilpotent completion, Part (a) follows from the remarks above, or from (4.9) (b) below. Similarly since $c = b \wedge EG_+$ is nonequivariantly the Eilenberg–MacLane spectrum H it follows that the Bousfield completion is $F(EG_+, b \wedge EG_+)$. However since $F(EG_+, b \wedge S^0 * EG) \simeq *$ it follows that $F(EG_+, b \wedge EG_+) \simeq F(EG_+, b)$; since $EG_+ \wedge EG_+ \simeq EG_+$ this is in turn equivalent to b as required. □

Proof of (4.6). We observe that all terms of the canonical resolution (4.5) of S are finite. Hence the Adams spectral sequence, obtained by applying $[S^0, \cdot \wedge F]_*^G$ to (4.5) is the same as the spectral sequence obtained by applying $[\cdot, F(X, Y)]_*^G = [X \wedge \cdot, Y]_*^G$ to the Spanier–Whitehead dual of (4.5). It remains only to identify the dual resolution.

The fundamental ingredient is as follows.

Lemma 4.8. *Adams' unit $\eta: S^0 \to G/H_+$ is dual to the unstable collapse map $c: G/H_+ \to S^0$.*

Proof. Since the complexes are finite we need only show Dc qualifies as a unit.

In fact we may identify the V-dual of $c: G/H_+ \to S^0$. For this we must represent c as an inclusion of subcomplexes of $S^{1 \oplus V}$; G/H is contained in $V = \mathbb{R}G/H$ as a basis and S^0 is represented by the point ∞ together with segments joining 0 to each element of G/H.

If we now take the sphere of radius 2 as the complement of S^0, and spheres of radius 1/2 centred on points of G/H (together with hairs joining them to the basepoint) it is clear that $D_V c$ is homotopic to the collapse map $S^V \to V/(W \backslash V)$ defining η. □

Corollary 4.9. (a) *The dual of the* sth *Adams cover of S^0 is identified by*

$$DS_s \simeq (S^0 * G/H)^{\wedge s}.$$

(b) $\underset{\overrightarrow{s}}{\text{holim}}\, DS_s \simeq S^0 * EG/H$.

Proof. (a) Since $S_s \simeq (S_1)^{\wedge s}$ it is enough to consider the case $s=1$. In this case DS_1 occurs in the cofibration

$$G/H_+ \xrightarrow{D\eta} S^0 \longrightarrow DS_1$$

so the result follows from (4.8).

(b) We observe that $S^0 * EG/H$ is a *space* characterised up to homotopy equivalence by its fixed point sets, which are

$$(S^0 * EG/H)^K \simeq \begin{cases} S^0 & \text{if } K \not\subseteq H \\ * & \text{if } K \subseteq H. \end{cases}$$

By (4.8) the system $DS_0 \to DS_1 \to DS_2 \dots$ is realised by a sequence of spaces and continuous functions, hence

$$\underset{\overrightarrow{s}}{\text{holim}}\, DS_s = \underset{\overrightarrow{s}}{\text{holim}}\, (S^0 * G/H)^{\wedge s}.$$

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is a space, and its fixed points clearly coincide up to homotopy with those of $S^0 * EG/H$.
□ (4.9)

Now (4.6) follows by considering a cellular map

$$\underset{\overrightarrow{s}}{\text{holim}}\,(S^0 * G/H)^{\wedge s} \longrightarrow SEG/H_+. \qquad \square \quad (4.6)$$

5. Away from the group order

In this section we apply some results on idempotents fo the Burnside ring $A(G)$ due to Araki [4] and others to analyse the situation with the group order inverted. Similar analysis often works with fewer primes inverted.

We recall [12] that $A(G)[1/|G|]$ is isomorphic to a product of rings $\mathbb{Z}[1/|G|]$, one for each conjugacy class (H) of subgroups using the maps $\varphi_H: A(G) \to \mathbb{Z}$ defined by counting the number of H-fixed points in a G-set. In particular for every subgroup H of G, $A(G)[1/|G|]$ contains an idempotent $e[\lesssim H]$ determined by

$$\varphi_K(e[\lesssim H]) = 1 \text{ iff } K \text{ is subconjugate to } H.$$

Since the natural map $EG/H_+ \to S^0$ is an equivalence in K-fixed points iff K is subconjugate to H it follows that

$$e[\lesssim H]S^0\left[\frac{1}{|G|}\right] \simeq EG/H_+\left[\frac{1}{|G|}\right]. \tag{5.1}$$

From this we deduce that if Y is localised away from $|G|$

$$[X \wedge EG/H_+, Y]^G = e[\lesssim H][X, Y]^G.$$

Now each $e[\lesssim H] = \sum_{(K) \leqq (H)} e_K$ where the sum extends over G-conjugacy classes of subconjugates of H and e_K is determined by $\varphi_L(e_K) = 1$ iff L is conjugate to K. Using (2.5) and (4.7) of [4] we see

$$\begin{aligned} e_K[X, Y]^G &= e_K[X, Y]^N \\ &= e_1[\Phi^K X, \Phi^K Y]^W \\ &= [\Phi^K X \wedge EW_+, Y]^W \end{aligned}$$

where $N = N_G(K)$ is the normaliser of K, $W = W_G(K) = N_G(K)/K$ and where $\Phi^K X$

denotes the fixed point spectrum in the sense extending the usual fixed point set for pointed spaces (using the notation of Lewis–May [17]).

Thus

$$[X \wedge EG/H_+, Y]^G = \bigoplus_{(K) \leqq (H)} [\Phi^K X \wedge EW(K)_+, \Phi^K Y]^{W(K)}. \tag{5.2}$$

Now we will show in Section 6 that since we have inverted $|G|$, $H^i_{\mathscr{A}\mathscr{D}}(G/H; [X, Y]^{\bullet}_{*}) = 0$ for $i \geqq 1$ and so we take the following corollary.

Corollary 5.3. *Provided Y is localised away from $|G|$ we have*

$$[X \wedge EG/H_+, Y]^G = H^0_{\mathscr{A}\mathscr{D}}(G/H; [X, Y]^{\bullet})(*)$$

$$= \bigoplus_{(K) \leqq (H)} \{[\Phi^K X, \textstyle\sum \Phi^K Y]^1\}^{W(K)}.$$

6. The action of the Burnside ring on cohomology

The arguments of this section can be reformulated in terms of transfer, but the present approach seems more efficient.

We recall that any Mackey functor M is a module over the Burnside Mackey functor [13]. Specifically if U, A are G-sets $[U] \in A(G)$ acts on $M(A)$ as the composite

$$M(A) \xrightarrow{\pi^*} M(A \times U) \xrightarrow{\pi_*} M(A)$$

where $\pi: A \times U \to A$ is the projection.

It is clear that π induces a map

$$\pi^*: H^*_{\mathscr{A}\mathscr{D}}(A; M) \to H^*_{\mathscr{A}\mathscr{D}}(A; M_U)$$

but for π_* we need a lemma.

Lemma 6.4. *If M is a Mackey functor and $\alpha: A \to A'$ is surjective then the square*

$$\begin{array}{ccc} M(U \times A') & \xrightarrow{\pi_*} & M(A') \\ \downarrow{\scriptstyle (1 \times \alpha)^*} & & \downarrow{\scriptstyle \alpha^*} \\ M(U \times A') & \xrightarrow{\pi_*} & M(A) \end{array}$$

commutes, and hence in particular π induces a map

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$$\pi_*: H^*_{\mathscr{A}\mathscr{D}}(A; M_U) \to H^*_{\mathscr{A}\mathscr{D}}(A; M)$$

Proof. Since α is surjective

$$\begin{tikzcd} U \times A' \arrow[r, "\pi"] \arrow[d, "1\times\alpha"] & A' \arrow[d, "\alpha"] \\ U \times A' \arrow[r, "\pi"] & A \end{tikzcd}$$

is a pull back square. $\square$

Corollary 6.2. *The Burnside ring $A(G)$ acts on $H^*_{\mathscr{A}\mathscr{D}}(A; M)$ provided M is a Mackey functor.*

Lemma 6.3. *If there is a G-map $U \to A$ then $[U] \in A(G)$ acts as zero on $H^i_{\mathscr{A}\mathscr{D}}(A; M)$ for $i \geqq 1$.*

Proof. By definition the Amitsur complex is A-split and hence ([**13**, (1.1)]) U-split. Thus the central row in the diagram is exact.

$$\begin{tikzcd} M(A^i \times D) \arrow[r, "\partial_{i-1}"] \arrow[d, "\pi^*"] & M(A^{i+1} \times D) \arrow[r, "\partial_i"] \arrow[d, "\pi^*"] & M(A^{i+2} \times D) \arrow[d, "\pi^*"] \\ M(A^i \times D \times U) \arrow[r, "\partial_{i-1}"] \arrow[d, "\pi_*"] & M(A^{i+1} \times D \times U) \arrow[r, "\partial_i"] \arrow[d, "\pi_*"] & M(A^{i+2} \times D \times U) \arrow[d, "\pi_*"] \\ M(A^i \times D) \arrow[r, "\partial_{i-1}"] & M(A^{i+1} \times D) \arrow[r, "\partial_i"] & M(A^{i+2} \times D) \end{tikzcd}$$

Furthermore by (6.1) the diagram commutes. Now suppose $\alpha \in M(A^{i+1} \times D)$ is a cycle representing a cohomology class and chase the diagram to deduce $\pi_* \pi^* \alpha$ is a boundary. $\square$

Corollary 6.4. *If $[G/H]$ acts invertibly on $M(G/K)$ whenever K is subconjugate to H then*

$$H^i_{\mathscr{A}\mathscr{D}}(G/H; M) = 0 \quad \text{for} \quad i \geqq 1.$$

7. Multiplicative structure

It is useful in calculations to have available a multiplicative structure in the spectral sequence. In this section we pause to provide one. In fact Dress remarks ([**13**, (1.6)]) that a pairing of contravariant abelian group functors $M \times N \to P$ induces a map.

$$H^p_{\mathscr{A}\mathscr{D}}(A;M)\times H^q_{\mathscr{A}\mathscr{D}}(A;N)\to H^{p+q}_{\mathscr{A}\mathscr{D}}(A;P). \tag{7.1}$$

Furthermore it is easy to see that this coincides with the usual pairing in group cohomology when relevant.

In our case we are concerned with $M=[X,Y]^{\cdot}$, $N=[Y,Z]^{\cdot}$ and we use $P=[X,Z]^{\cdot}$ and the composition pairing. We note that this is induced by a map of function spectra.

Finally, to see that this corresponding pairing (7.1) is the map of E_2-terms in a pairing of spectral sequences converging to the composition pairing we have only to use an equivalence $EA_+\wedge EA_+\simeq EA_+$. In order to get a map of spectral sequences we use the canonical map $EA\times EA\to E(A\times A)$ induced at the level of simplicial sets, and then choose a map $A\times A\to A$ to induce the equivalence $E(A\times A)_+\simeq EA_+$.

8. Sample calculations

Because of the early interest of Bredon [7,8] we pay special attention to the case $G=C_2$, and in particular to the spectral sequences for $[S^{k\xi},S^0]^{C_2}_*$ where ξ is the nontrivial one dimensional real representation of G. In this particular case the Adams Tower over Y is simply

$$\to S^{-3\xi}Y\to S^{-2\xi}Y\to S^{-\xi}Y\to Y. \tag{8.1}$$

Accordingly all differentials in the spectral sequence

$$E_2^{**}(k)=H^*(C_2;[S^{k\xi},S^0]^1_*)\Rightarrow[S^{k\xi}\wedge EC_{2+},S^0]^{C_2}_*$$

are independent of k in the obvious sense that if $k'=k+\delta$ with $\delta>0$ the spectral sequences are related as follows:

Lemma 8.2. $E_2^{s,t}(k+\delta)=E_2^{s+\delta,t+\delta}(k)$ *for* $s>0$

$E_2^{0,t}(k+\delta)=E_2^{\delta,t+\delta}(k)$ *for* $k+\delta$ *is odd*

$E_2^{0,t}(k+\delta)/2=E_2^{\delta,t+\delta}(k)$ *for* $k+\delta$ *is even*

(b) *If* $x\in E_2^{s,t}(k+\delta)$ *is an* $(r-1)$*-cycle and* $d_rx=y$ *then* $\bar{x}\in E_2^{s+\delta,t+\delta}(k)$ *is an* $(r-1)$*-cycle and* $d_r\bar{x}=\bar{y}$.

Thus we may study all the spectral sequences $E_*^{**}(k)$ with $k\geqq 0$ within the single spectral sequence $E_*^{**}(0)$.

Now if we restrict attention to the 2-adic part, we find by Lin's Theorem [18] that 2-adically $[S^{k\xi}\wedge EC_{2+},S^0]^{C_2}_r=[S^{k\xi},S^0]^{C_2}_r$. For the remainder of this discussion all groups and spaces are completed at 2. For $r\leqq -2$ this is $[\mathbb{R}P_0^{k-1},S]^1_{r-1}$, and combining this with Atiyah's identification of functional duals [5] this is $[S^0,\mathbb{R}P^{-1}_{-k}]^1_r$. These groups have been calculated in a considerable range by Mahowald [19].

With this information to hand we can deduce information about which elements of

$$H^0(C_2;[S^{k\xi},S^0]_r^1)$$

are represented by strictly equivariant maps as follows. We know the target for $t-s=r-1$ for the $k+1$ spectral sequence and we know it for the k spectral sequence. The only changes must come from the passage from $E_2^{0,r-1}(k+1)$ to $E_2^{1,r}(k)$ (if any), from the addition of the infinite cycles in $E_2^{0,r-1}(k)$ and from the elements killed by differentials supported on $E_*^{0,r}(k)$. From the corresponding calculation for $r-2$ we know the infinite cycles in $E_2^{0,r-1}(k)$, so the only unknown fact is which part of $E_2^{0,r}(k)$ consists of infinite cycles. In general we will only be able to work out its order, but often the ambiguities can be resolved.

Summary 8.3. There is 2-adically an exact sequence for $r\leqq -1$

$$0\rightarrow K\rightarrow[S^0,\mathbb{R}P_{-k-1}^{-1}]_{r-1}\rightarrow[S^0,\mathbb{R}P_{-k}^{-1}]_{r-1}\rightarrow E_\infty^{0,r-1}(k)\rightarrow 0$$

and K has a filtration whose factors are (starting with the subgroup)

(i) The reverse of the factors in some filtration of $E_2^{0,r}(k)/E_\infty^{0,r}(k)=[S^{k+r},S^0]/\{$strictly equivariantly represented maps$\}$ if k is even

and

(ii) ${}_2[S^{k+r},S^0]$ and then the reverse of factors in some filtration of $E_2^{0,r}(k)/E_\infty^{0,r}(k)={}_2[S^{k+r},S^0]/\{$strictly equivariantly represented maps$\}$ if k is odd where ${}_2A$ denotes the elements of order 2.

We note in particular that if k is even every element of $2[S^{k+r},S^0]$ is strictly equivariently represented and that if k is odd only elements of order 2 in $[S^{k+r},S^0]$ can possibly be strictly equivalently represented.

From Mahowald's calculations we deduce the following. Obviously the results could be taken much further. These results are also deduced in [**20**], where much further information is provided. The deduction is independent of [**20**] except for the 6 entries marked with an asterisk, which depend on correcting the definition of Mahowald's groups A_k, and for this we rely on [**20**]. Our deductions seem to differ from [**20**] in the 3 entries marked with an obelus.

Proposition 8.4. *The following table shows which elements of $\pi_j=\pi_j(S^0)$ are represented by strictly equivariant maps $S^{k\xi}\rightarrow S^{k-j}$ for $j\leqq 14$, $k\geqq 0$ provided that $k-j\geqq 2$. If k is even the entry* a *means that the image of the group of strictly equivariantly represented maps in $\pi_j/2\pi_j$ is isomorphic to $(\mathbb{Z}/2)^a$. If k is odd the entry means that the subgroup of strictly equivariantly represented maps in π_j is isomorphic to $(\mathbb{Z}/2)^a$.*

For the purpose of defining the numbers d_j we define m by the condition $k-j-1\equiv 2^m (\mathrm{mod}\, 2^{m+1})$. Note that this really depends on k and not $k \bmod 16$. In this notation we have

k mod 16 / j	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
2	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
3	1	1	0	0	0	1	0	0	1	1	0	0	0	1	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1	0	1	1	0	0	0	0	1	0	1	1	0	0	0	0
7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	2	1	0	0	2	1	0	0	2	d_8^*	0	0	2	1	0	0
9	3	2	2	1	3	2	1	0	3	2	d_9^*	1	3	2	1	0
10	1	1	0	0	1	1	0	0	1	1	0	0*	1	1	0	0
11	1	1	1	0	1	1	1	0	1	1	1	0	d_{11}^*	1	1	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	2	0	0	0	1	0	1	0	1	0	0	0	0	1†	0	0*
15	2*	1	0	0	2	1	1	0	2	1	0	1†	2	0†	0	0

$$d_8=\begin{cases}1 & \text{if} \quad m=4\\ 0 & \text{if} \quad m\geqq 5\end{cases} \qquad d_9=\begin{cases}2 & \text{if} \quad m=4 \text{ or } 5\\ 1 & \text{if} \quad m\geqq 6\end{cases} \qquad d_{11}=\begin{cases}1 & \text{if} \quad m=4,5 \text{ or } 6\\ 0 & \text{if} \quad m\geqq 7\end{cases}.$$

We note that (8.4) improves on Bredon's published results ([8, Theorem C]) but his methods yield the same corollary of Mahowald's work. Perhaps more interesting is that we can use the product structure to enormously reduce the work involved. For example it is a small matter to calculate the groups $[\mathbb{R}P^n, S^{n-\varepsilon}]$ for $\varepsilon=0$ and 1. We find

$$[\mathbb{R}P^{k-1}, S^{k-1}]=\begin{cases}\mathbb{Z} & k \text{ even} \quad k\geqq 2\\ \mathbb{Z}/2 & k \text{ odd} \quad k\geqq 1\end{cases}$$

and

$$[\mathbb{R}P^{k-1}, S^{k-2}]=\begin{cases}\mathbb{Z}/2 & \text{for } k=2 \text{ or } 3\\ \mathbb{Z}/2\oplus\mathbb{Z}/2 & \text{for } k=0 \bmod 4 \quad k\geqq 4\\ 0 & \text{for } k=1 \bmod 4 \quad k\geqq 4\\ \mathbb{Z}/4 & \text{for } k=2 \bmod 4 \quad k\geqq 4\\ \mathbb{Z}/2 & \text{for } k=3 \bmod 4 \quad k\geqq 4.\end{cases}$$

In fact from $[\mathbb{R}P^4, S^3]=0$ we deduce that $\eta\in E_2^{0,3}(4)$ must be an infinite cycle in $E_*^{**}(4)$ and since it must die in $E_*^{**}(2)$ we deduce that $d_2\colon E_2^{0,2}(2)\to E_2^{2,5}(2)$ is an isomorphism. In $E_*^{**}(0)$ we therefore have

$$d_2(x\cdot\bar{\imath})=x^2\cdot\bar{\eta}$$

where $x \in H^2(C_2;\mathbb{Z})$ is the generator.

From this we deduce

$$d_2(x^{2k}\alpha)=0 \quad \text{if } \alpha\in\pi_*S^0 \text{ is of order 2}$$

and

$$d_2(x^{2k+1}\bar{\alpha})x^{2k+2}\bar{\eta}\bar{\alpha} \quad \text{for any } \alpha\in\pi_*(S^0).$$

In particular

$$d_2(x^{2k+1}\eta)=x^{2k+2}\eta^2 \qquad d_2(x^{2k+1}\sigma)=x^{2k+2}\eta\sigma$$

$$d_2(x^{2k+1}\eta\sigma)=x^{2k+2}\eta^2\sigma \qquad d_2(x^{2k+1}c_0)=\eta c_0$$

$$d_2(x^{2k+1}[Ph_1])=x^{2k+2}[Ph_1]\eta.$$

Next we observe that $H^*(C_2;\mathbb{Z}/2)$ is a free module over $\mathbb{F}_2[x]$ on two generators, $1\in H^0$, $t\in H^1$. We next consider $d_2\colon E_2^{1,1}(0)\to E_2^{3,2}(0)$; in fact we may see it is zero by considering the map $[S^{\xi}/S^0,S^0]^{C_2}\leftarrow[S^{2\xi}/S^0,S^2]^{C_2}$ and observing it is an isomorphism. Hence $d_2(t\eta)=0$. It therefore follows that

$$d_2(x^{2k+1}t\eta\alpha)=x^{2k+2}\eta^2\alpha.$$

Hence for example

$$d_2(x^{2k+1}t\eta)=x^{2k+2}\eta^2, \quad d_2(x^{2k+1}t\eta\sigma)=x^{2k+2}\eta^2\sigma.$$

In any case the above argument allows us to decide which elements of $\pi_j(S^0)$ are strictly equivariantly represented by maps $S^{k\xi}\to S^{k-j}$ provided $0\leqq j\leqq 2, k\geqq 0, k-j\geqq 2$.

In general the subgroup of $H^0(G;[S^U,S^0]_r^1)$ which are strictly equivariantly represented by elements of $[S^U\wedge EG_+,S^0]_r^G$ is the cokernel of the map

$$[S^U\wedge S^0{}_*G\wedge EG_+,S^0]_r^G\to[S^U\wedge S^0\wedge EG_+,S^0]_r^G.$$

This can of course be studied by nonequivariant means. The problem becomes more tractabled if G admits fixed point free representations, and U is a suitable one. Then if $r\leqq -2$ we may replace the above map by

$$[(S^U\wedge S^0{}_*G)/S^0,S^0]_r^G\to[(S^U\wedge S^0)/S^0,S^0]_r^G$$

which is much easier to study since the maps are out of finite complexes. For example if p is an odd prime we may complete at p and ask about the element α_1 of order p with $r=-|U|+2p-3$. Since α_1 is detected by the first Steenrod power and $(S^U/S^0)/G$ and

$((S^U \wedge S^0 * G)/S^0)/G$ are segments of SBG_+ we see that α_1 can only be strictly equivariantly represented if

$$P^1 : H^{|U|-2p+3}(SBG_+; F_p) \rightarrow H^{|U|+1}(SBG_+; \mathbb{F}_p)$$

is non-trivial. Since $\pi_j(S^0)_p^\wedge$ is zero for $0 < j < 2p-3$ we seen this condition is also sufficient.

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