

ERRATUM

Bruce E. Hansen, Strong Laws for Dependent Heterogeneous Processes. *Econometric Theory* 7(1992): 213–221.

There were some errors made in [2]. The results of Section 2 are stated to hold for L^r -mixingales, $r > 1$. They hold, however, only for $r \geq 2$. The proof of Lemma 1 on page 219 uses Minkowski's inequality in the $r/2$ norm, which requires that $r/2 \geq 1$. The author is grateful to Myoung-jae Lee for pointing out this error.

For $1 < r \leq 2$, $x \geq 0$ and $y \geq 0$, the inequality $(x + y)^{r/2} \leq x^{r/2} + y^{r/2}$ can be used in place of Minkowski's inequality to establish the following results for L^r -mixingales satisfying $\|E_{i-m}X_i\|_r \leq c_i\psi_m$. Set $S_j = \sum_{i=1}^j X_i$, $\bar{K} = 18[r/(1-r)]^{3/2}$, and $\Psi = \sum_1^\infty \psi_m$.

LEMMA 1.

$$\|\max_{j \leq n} |S_j|\|_r \leq \bar{K} \sum_{m=-\infty}^{\infty} \left(\sum_{i=1}^n E|E_{i-m}X_{im} - E_{i-m-1}X_i|^r \right)^{1/r}. \quad \blacksquare$$

LEMMA 2.

$$\|\max_{j \leq n} |S_j|\|_r \leq 2\bar{K}\Psi \left(\sum_{i=1}^n c_i^r \right)^{1/r}. \quad \blacksquare$$

COROLLARY 1. If $\Psi < \infty$ and $\sum_1^\infty c_i^r < \infty$, then S_n converges almost surely. \blacksquare

COROLLARY 2. If $\Psi < \infty$ and $\sum_1^\infty (c_i/i)^r < \infty$, then $S_n/n \rightarrow 0$ almost surely. \blacksquare

Section 3 concerned zero-mean sequences $\{Y_i\}$ which are L^q near-epoch dependent ($q > 1$) upon some strong-mixing sequence $\{X_i\}$ with mixing coefficients α_m satisfying

$$\|E(Y_i | \mathcal{F}_m) - Y_i\|_q \leq d_i v_m, \quad \text{where } \mathcal{F}_m = \sigma(X_i : i - m \leq t \leq i + m).$$

For the case $1 < q < 2$ the following theorem follows directly from Corollary 2 and the near-epoch dependent inequality given in [1]:

THEOREM 2. If for some $p > q$, $\sum_1^\infty v_m < \infty$, $\sum_1^\infty d_i^{q-1} < \infty$, $\sum_1^\infty \alpha_m^{1/q-1/p} < \infty$ and $\sum_1^\infty d_i^{q-1} \|Y_i\|_q^p < \infty$, then $n^{-1} \sum_1^n Y_i \rightarrow 0$ a.s. \blacksquare

It should also be noted that the original proof of Theorem 2 in [2] worked with the truncated sequence $Y_{1i} = Y_i 1(|Y_i| \leq 1)$, implicitly assuming that Y_{1i} is NED with the same coefficients as Y_i . This is not obviously true, and the author is grateful to Don Andrews for pointing out this error.

REFERENCES

1. Andrews, D.W.K. Laws of large numbers for dependent non-identically distributed random variables. *Econometric Theory* 4 (1988): 458–467.
2. Hansen, B.E. Strong laws for dependent heterogeneous processes. *Econometric Theory* 7 (1991): 213–221.