Multiple Periodic Variable Stars
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THE MAGNETIC VARIABLE STARS

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1. Introduction

The peculiar A stars were first noted as peculiar because they exhibit anomalously strong lines of one or more of the elements Si, Cr, Mn, Sr, and Eu. These diverse objects are all called Ap stars because their assigned spectral types fall predominantly in the range B9-F0. Recent studies have suggested, however, that the Ap stars should be subdivided into two groups. The stars characterized by enhanced lines of Mn, and usually Hg as well, differ from the remaining Ap stars in composition and binary frequency; there are no confirmed spectrum, photometric, or magnetic variations in any Hg-Mn stars; and the upper limit on the magnetic field strengths of the Hg-Mn stars is about 200 gauss. Because of these differences, many observers (e.g. Sargent and Searle 1967; Preston 1971b; Wolff and Wolff 1974) have suggested that the Hg-Mn stars are fundamentally different in their properties, and possibly in their origin and evolution, from the SiCrEuSr stars. We shall be concerned today only with this latter group of stars, and in the review that follows only members of the SiCrEuSr class of stars will be referred to as Ap stars.

The problem of the Ap stars, as Bidelman (1967) has stated, is that "stars of unusual spectrum are doing unusual things," and indeed the peculiarities of these objects have been recognized for about three quarters of a century. The star α^2 CVn, which exhibits most of the properties typical of Ap stars, is quite bright (V = 2.9) and can be easily observed. The spectrum of α^2 CVn was first classified as peculiar by Maury (1897), who commented on the weakness of the K-line and the strength of the Si II doublet at $\lambda\lambda$4128,31. That this star was of particular interest became

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apparent when Ludendorff (1906) reported that several lines in the spectrum of α^2 CVn varied in intensity. It is curious, in view of the subsequent analyses of α^2 CVn, that the features reported by Ludendorff to be variable, including lines of Fe, Cr, and Mg, are among the lines that vary least in this star. He noted no variations in the Eu lines λ4129 or λ4205, but it is possible that these lines were outside the range of his instrument.

An extensive analysis of α^2 CVn was carried out by Belopolsky (1913), who showed that the line at λ4129 varied in a period of 5.5 days. This feature, and several other prominent lines in the spectrum of α^2 CVn, were attributed to Eu by Baxandall (1913). Belopolsky (1913) also derived the radial velocities of the Eu lines, and his discussion of the measurements is surprisingly close to the modern interpretation. After demonstrating that the radial velocity of the line at λ4129 varied in quadrature with the changes in intensity, Belopolsky commented (translation by Struve 1942):

> "It is difficult to decide wherein to see the cause of this phenomenon. An obvious hypothesis suggests itself, namely that the central body is surrounded by a gaseous satellite or a gaseous ring having a condensation of matter at one point. This hypothesis is supported by the sign of the variable velocities (negative velocities preceding maximum of intensity of λ4129 and positive velocities following maximum), but the details of the observations still present difficulties which may perhaps be cleared up after more material has been accumulated."

The light curve of α^2 CVn was first measured photoelectrically by Guthnick and Prager (1914). A comparison of their results with a modern light curve (Wolff and Wolff 1971) is shown in Figure 1. The two sets of observations have been phased together according to the period derived by Farnsworth (1933); a slightly better fit could be obtained for a shorter value of the period, but the observations of the Eu variations do not allow such a change.

As Figure 1 shows, Guthnick and Prager not only derived the correct amplitude for the variability but also discovered the asymmetry in the light curve, with the decline from maximum to minimum light occurring more rapidly than the rise from minimum to maximum. The accuracy of the 1914 observations is all the more remarkable since, due to the lack of sensitivity of their equipment, Guthnick and Prager were compelled to use as comparison star δ UMa, which is more than 20° away from α^2 CVn.

Thus by 1914 it was established that α^2 CVn was a spectrum and photometric variable, that the extrema of the light curve coincided in phase with the extrema of the Eu line strength variations, and that the radial

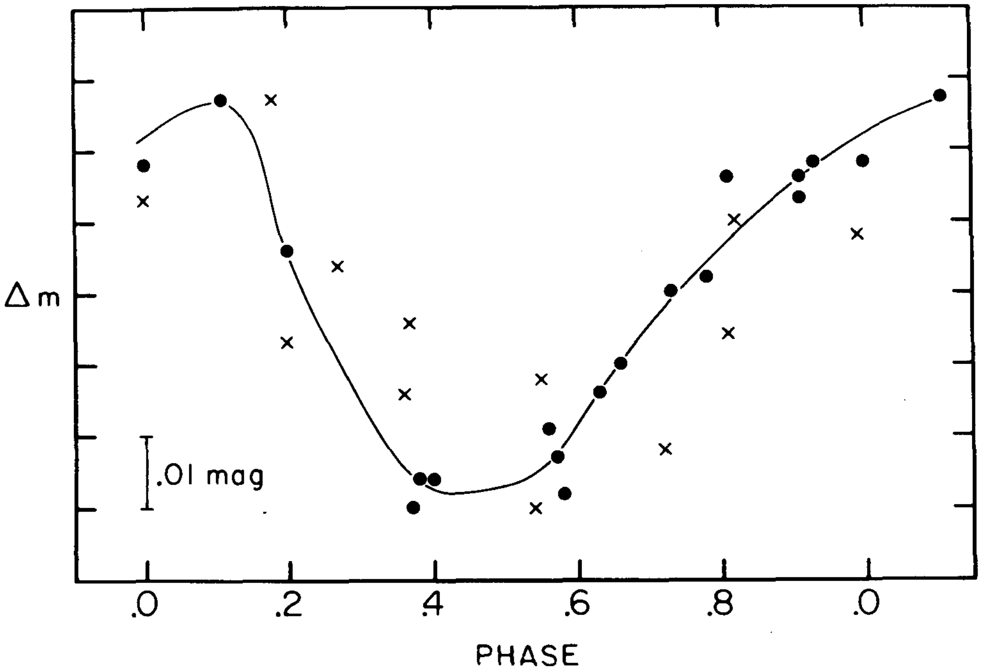

Fig. 1 - Photometric data for α^2 CVn. Crosses represent observations by Guthnick and Prager (1914); filled circles represent observations made with the b filter of the uvby system (Wolff and Wolff 1971).

velocity and spectrum variations were in quadrature. Subsequent studies, particularly by Morgan and Deutsch, showed that these properties are typical of all the Ap variables, although the details of the variations differ from star to star. Modern observations of α^2 CVn itself have been described by Pyper (1969).

2. Rigid Rotator Model

No satisfactory model of the Ap stars was developed until after it was discovered that 78 Vir (Babcock 1947) and most other sharp-lined Ap stars, including α^2 CVn, have variable magnetic fields. The Zeeman observations suggested that a star might possess an axis of symmetry other than the rotation axis, an assumption that is the essential step in formulating the rigid rotator model. This model was first proposed by Babcock (1949) himself:

> "It is true that I have suggested as a revised working hypothesis that intense magnetic activity may be correlated with rapid stellar rotation, but at this stage an equally good case can probably be made for the alternative hypothesis that the spectrum

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> variables of type A are stars in which the magnetic axis is more or less highly inclined to the axis of rotation and that the period of magnetic and spectral variations is merely the period of rotation of the star."

Babcock (1960b), of course, was never one of the major proponents of the rigid rotator model. However, none of the alternatives, including the magnetic oscillator and solar cycle models, has been elaborated to the extent that it can successfully predict the variety of observational phenomena associated with the Ap stars. The rigid rotator model, on the other hand, suggests--and has survived--a number of observational tests.

Among the more notable successes of the rigid rotator model are its explanations of the period vs. line-width relation and of the crossover effect and its correct prediction of the average surface field of β CrB. An excellent review of the properties of the Ap stars, together with a discussion of the applicability of the rigid rotator model, has been presented recently by Preston (1971b). In the paper that follows, I will emphasize those results that have been obtained since Preston's review.

3. Light Variations

For a long time it proved impossible to account for the photometric variations of the Ap stars in terms of the rigid rotator model. Indeed, the fact that the periods of the light and magnetic variations are equal has been cited as evidence against the rigid rotator model, since there are no physical arguments to account for the fact that surface brightness appears to depend on the polarity of the magnetic field. However, the discovery that the magnetic fields of several of the Ap stars can be better represented by decentered, rather than centered, dipoles (Wolff and Wolff 1970; Preston 1970; Huchra 1972), indicates that the two magnetic poles are often not of equal strength and that there can be a basic asymmetry between the two magnetic hemispheres.

The key step in understanding the light variations of the Ap stars was the realization (Peterson 1970) that variations in ultraviolet opacity could produce changes in flux in the visible region of the spectrum. In particular, Peterson suggested that in Si variables the continuous opacity in the ultraviolet would be increased at Si maximum, thereby leading to a reduction in flux in the ultraviolet and, due to backwarming effects, to an increase in flux in the visible.

While the backwarming mechanism suggested by Peterson has been basically confirmed as an important cause of the light variations in Ap stars, it is still not clear whether or not Si itself plays a significant role in this process. At the very least, observations demonstrate that there must be other important factors that influence the photometric variations of the Si stars. Several Si stars, including HD 32633 (Preston and Stepien 1968a) and HD 215441 (Babcock 1960a; Preston 1969a) exhibit large amplitude photometric variations even though no Si variations are evident. In 41 Tau (Wolff 1973) maximum light in u (on the uvby system) coincides with Si minimum, a phase relation exactly opposite to that predicted by Peterson's models. In 56 Ari (e.g. Wolff and Morrison 1975) the light curves exhibit two maxima, only one of which coincides with a Si maximum.

As an alternative hypothesis, Wolff and Wolff (1971) suggested that variations in the line opacity due to rare earth elements might be the dominant factor in producing the photometric variations of the cooler Ap stars. Dieke, Crosswhite, and Dunn (1961) have pointed out that there is a great concentration of doubly ionized rare earth lines in the region $\lambda\lambda$ 2000-3000. In late B and early A-type stars, the rare earths should be predominantly doubly ionized, and furthermore a substantial amount of flux is emitted in the region $\lambda\lambda$ 2000-3000. It therefore seemed plausible that variations in the line strengths of the rare earths could directly cause photometric variations. This suggestion received additional support from the fact that in all the rare earth spectrum variables observed up to that time, V maximum coincided with rare earth maximum (Wolff and Wolff 1971; Preston 1971b).

In contrast with the situation a decade ago, predictions about flux distributions below λ 3000 can no longer be made with impunity. Orbiting satellites have made this region accessible to observers, and α^2 CVn was one of the first objects studied in detail by OAO-2. Figure 2 shows the light curves obtained by Molnar (1973) for α^2 CVn. Maximum light in the visible occurs at about phase 0.10 (Wolff and Wolff 1971), so the light variations shortward of λ 2950 are antiphase to the variations longward of this wavelength. Molnar's data also show that the effective temperature of α^2 CVn remains constant throughout the cycle, as would be expected if backwarming effects were responsible for the light variations. Molnar further suggests that rare earth lines are the primary--but not the only--cause of the light variations.

While the photometric variations of most Ap stars can be successfully accounted for by a combination of backwarming from the ultraviolet plus

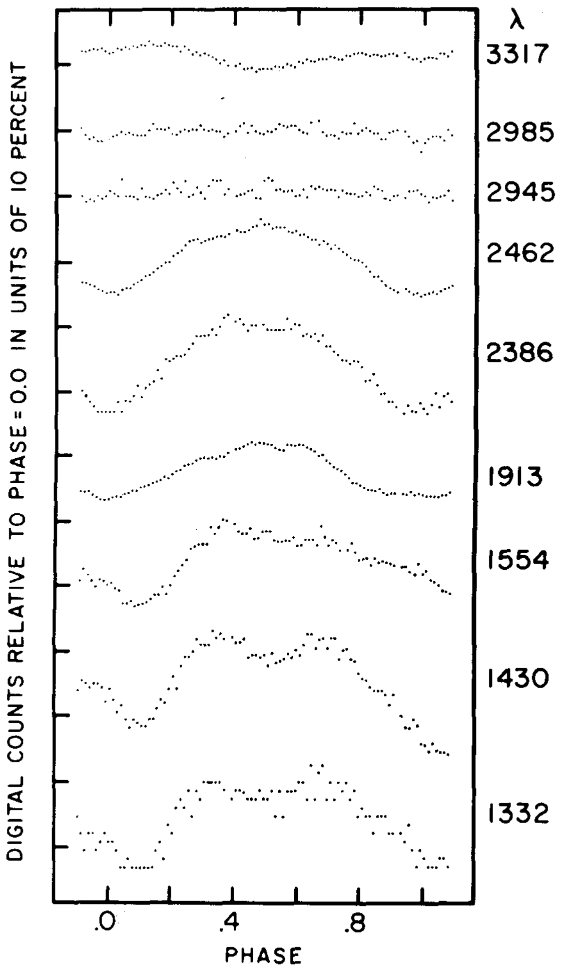

Fig. 2 - Ultraviolet light curves for α^2 CVn (Molnar 1973). Zero phase coincides with Eu maximum.

local line-blocking (e.g. HD 188041; Jones and Wolff 1973), there are several stars whose light curves cannot yet be fully explained. For example, Leckrone (1974) has shown that HD 215441 is similar to α^2 CVn in that the light variations shortward of the null wavelength, which for HD 215441 is at λ2460, are antiphase to the variations longward of this wavelength, and the effective temperature is constant throughout the cycle. The backwarming model thus appears to be applicable to HD 215441, but since no definite spectrum variations have been reported for this star (Babcock 1960a; Preston 1969a), the source of the varying opacity remains unknown. One of the coolest Ap stars, HR 1217, is the only one discovered to date in which V maximum coincides with rare earth minimum (Preston 1972; Wolff and Morrison

1973). The V variation cannot therefore be attributed to backwarming by the rare earths, nor does it appear to be due to changes in line strength within the V passband itself (Bonsack, private communication). In HR 5355 (= HD 125248) the amplitude at λ4100 is substantially larger than the amplitude at λ3600 or λ4600 (Wolff and Wolff 1971; Maitzen and Moffat 1972; Hardorp 1975), and the wavelength variation in the amplitudes cannot be accounted for by backwarming, temperature variations, or local line-blocking (Pilachowski and Bonsack 1975). It may be that there is an unknown source of continuous opacity in the region near λ4100. Some stars, including HD 111133 (Wolff and Wolff 1972; Engin 1974), exhibit large amplitude light variations even though rare earth lines may be weak or absent. Recent work, both theoretical (Leckrone et al. 1974) and observational (Mallama and Molnar 1975) indicate that the lines of the Fe-peak elements may be effective in determining the flux distributions of these objects.

In summary, observational evidence strongly favors the hypothesis that spectrum and photometric variations are causally related. However, several problems must be resolved in order to determine whether or not there are any other causes of photometric variability and before we can explain in detail the light variations of all the Ap stars.

4. Search for Multiple Periods

The model described so far takes into account no periodicities other than the period of rotation, but since this conference is concerned with stars that exhibit multiple periods, I should comment specifically on whether any magnetic Ap stars fall in this category. Before discussing this possibility, I would like to say first that I think recent results indicate that all Ap stars, when observed carefully enough, exhibit cyclic or essentially periodic variability. Periods have now been derived for all but one of the stars that Babcock (1960b) considered to be prototypical irregular variables. Furthermore, the periods of the Ap stars appear to be constant within the present accuracy of measurement (cf. Renson 1972). For example, the observations of α^2 CVn, which span the interval 1913-1970, can all be represented by the period derived by Farnsworth (1933). In some Ap stars (e.g. Bonsack and Wallace 1970), irregular fluctuations may be superposed on cyclic variations. However, the question that concerns us here is whether there are *regular* variations, possibly due to pulsation, in addition to the periodic variations that are a consequence of the rotation of the star. Several observers have attempted to resolve this question, including particularly Rakos (1963) and most recently Percy (1975). The conclusion appears to be that there is one--and only one--Ap star for which the evidence of multiple periodicity is quite convincing. This star is 21 Com, which has a period of about 30 minutes superposed on a longer period of either 1 or 2 days (Bahner and Mawridis 1957; Percy 1973). In addition, Percy (1975) finds that HR 9080 (=HD 224801) has irregular fluctuations of about $0^{m}.02$, while 13 other Ap stars show no evidence of short period variability. The incidence of pulsation among the Ap stars that fall in the instability strip appears therefore to be lower than it is for non-Ap stars of the same temperature (Breger 1969). Of the two stars that do show evidence of variability on a short time scale, one (HR 9080) falls clearly outside of the instability strip and the other (21 Com) probably is slightly hotter than the high temperature boundary of the instability strip (Breger 1969).

5. Evolution of Ap Stars

5.1 Introductory Comments

In discussing the Ap stars so far I have essentially ignored variations in their individual properties. With respect to developing a model for their variations, I believe this approach is justified since the rigid rotator model appears to be applicable to all the SiCrEuSr Ap stars. However, I think that the most important unanswered questions concerning the Ap stars deal with their origin and evolution, and here one must be careful not to oversimplify the problem by ignoring the very real differences among various members of this class of stars. A successful explanation for the abundance anomalies, for example, will have to account for the fact that stars of apparently similar temperature and luminosity have quite different compositions. The stars HD 51418 and HR 465, at the time of rare earth maximum, have much stronger lines of Ho and Dy and other heavy rare earths than do most Ap stars. The Ap stars span a large range in temperature, from HR 7129 with T_e = 20000 K (Wolff and Wolff 1976) to HR 1217 (Preston 1972) at T_e = 7000 K. (New observations (Wolff and Hagen 1976) demonstrate that HD 101065 has a strong magnetic field, but the temperature (Wegner and Petford 1974) of this star and its relation to the Ap stars remain matters of controversy.) A successful model will have to explain why magnetic stars occur within, but apparently not outside, of this temperature range.

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Another important question concerns the time scale for the development of Ap stars. At different times it has been suggested that these stars are in a variety of evolutionary states from the zero-age main sequence (Hyland 1967) to post-red giant (Fowler et al. 1965) phase of evolution. Recently, new observational evidence has been obtained to support the idea that the Ap stars are on the main sequence for the first time, an idea that has been widely accepted for some time, and that their angular momentum, and possibly other properties as well, change in a systematic way as the stars evolve away from the zero-age main sequence. I would now like to describe the evidence in support of this point of view.

5.2 Distribution of Periods for the Ap Stars

In 1970, Preston and Wolff reported that the Ap star HR 465 was a spectrum, photometric, and magnetic variable with a period of 22-24 years. Subsequent observations (Wolff, unpublished) have confirmed this period. In the interval 1967-1972 the field of HR 465 declined from +200 gauss to -1000 gauss; the average field measured by Babcock (1958) in 1948-49 was about -1100 gauss.

The star HR 465 appears to pose serious problems for the rigid rotator model, since it seems unlikely that any star would have a rotation period that exceeds 20 years. Furthermore the long period of HR 465 is not unique; many Ap stars are known to have periods of several hundred days (Wolff 1975a); γ Equ may have a period of 75 years (Bonsack and Pilachowski 1974). Figure 3 shows the distribution of periods (Wolff 1975a) for Ap stars cooler than about 12000 K. (Hotter Ap stars, which all are classified as Si stars, have been excluded since no Si stars are known to have periods greater than 20 days and only a few have periods greater than 5 days). The distribution of periods is continuous, with no distinct separation between stars with P <30 days, for which the rigid rotator model is applicable, and stars with P >30 days. Furthermore, apart from the time scales of variation, the long period stars are in every way similar to the short period stars. For example, in HR 465, the Eu and Cr lines vary in antiphase, as is typical of CrEu stars; the Eu lines are strongest at V maximum; and the extrema of all the variable quantities coincide in phase. The strength of the magnetic field is similar to that found in the short period stars, and the amplitudes of the spectrum, magnetic, and photometric variability are not unprecedented (Jones, Wolff, and Bonsack 1974). The similarities in the variable properties of the long

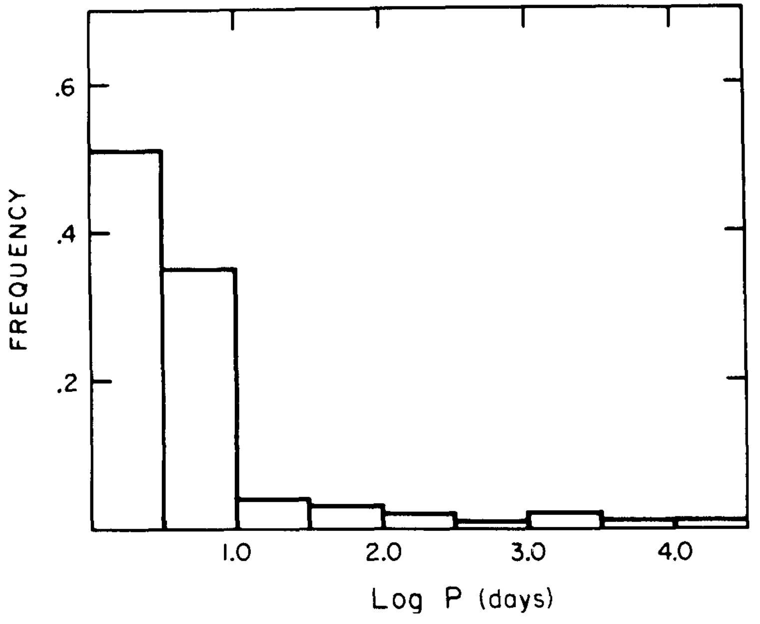

Fig. 3 - Distribution of Ap stars as a function of period. All Si stars have been excluded.

and short period Ap stars, combined with the continuity of the period-frequency distribution, suggest that the same basic mechanism must be responsible for the variations in all Ap stars. Since the rigid rotator model is so successful in accounting for the variations of stars with *P* < 30 days, it is reasonable to ask whether it may also be applicable to the stars of longer period. If it is, of course, then a powerful mechanism for rotational deceleration must be operative in at least some Ap stars.

5.3 Loss of Angular Momentum

The loss of angular momentum of the magnetic Ap stars could occur during either their pre- or post-zero-age main sequence phases. Indeed the fact that Ap stars as a group are slow rotators and few are known with $v \sin i$ > 100 km s^{-1} (Abt *et al*. 1972) suggests that these stars do lose some angular momentum before they reach the main sequence.

Two recent models for the formation of the Ap stars suggest that significant loss of angular momentum can also take place after the magnetic Ap stars reach the zero-age main sequence. One of the models (Havnes and Conti 1971) involves mass accretion from the interstellar medium, the other

(Strittmatter and Norris 1971) postulates mass loss. In each model it is assumed that the magnetic field is strong enough to impose co-rotation out to some limiting radius R_c. In the mass loss model, material crossing this boundary carries angular momentum away from the star. In the mass accretion model, interstellar material crossing the boundary is spun up to the co-rotation velocity, a process that again reduces the angular momentum of the central star. For plausible values of the magnetic field strength and of the density of material at the boundary of the magnetosphere, the *e*-folding time for loss of angular momentum is on the order of 10^7-10^9 years (Strittmatter and Norris 1971). This time is also comparable to the main sequence lifetimes of the Ap stars.

Decrease of angular momentum through mass accretion is simply the inverse of the process of decrease of angular momentum through mass loss, and accordingly the equations governing the loss of angular momentum are the same for both cases. Thus Havnes and Conti (1971) and Norris and Strittmatter (1971) both predict that the amount of angular momentum lost will depend on the strength of the magnetic field, the density of the stellar wind (mass loss) or of the interstellar medium (mass accretion), and on the length of time that the star has undergone magnetic deceleration. Observationally, there is no way to determine whether, or by how much, the density of material in a stellar wind or in the surrounding interstellar medium varies from star to star as a function of time. Furthermore, observations do not support the idea that present rotational velocities of the Ap stars depend strongly on magnetic field strength (Preston 1971*a*). For stars with $P > 5$ days, for which accurate photographic measurements of the Zeeman effect can be made, there is no evidence for a correlation between magnetic field strength and rotational velocity (or, equivalently, period).

The remaining observational test of these two models of angular momentum loss is to determine whether the period of a magnetic Ap star depends on its age. If angular momentum is lost during main sequence evolution, then one might expect more evolved stars to have longer periods. In most cases, of course, there is no way to estimate the age of an individual field star. For Ap stars, however, such an estimate is possible. According to Iben's (1965; 1966) evolutionary tracks, as a star in the temperature range of the Ap stars evolves away from the main sequence, it initially increases in radius by nearly a factor of 2 before undergoing the rapid overall contraction that immediately precedes the disappearance of the convective core. Therefore, for stars of a given mass, the radius of the star is a measure of its age. For Ap stars, if the period of variation is equal to the period of rotation, as the oblique rotator model requires, then

$$R = Pv/50.6, \quad (1)$$

where *R* is the stellar radius in units of the solar radius, *P* is the period in days, and *v* is the rotational velocity in km s^{-1}. Since stars are observed at an unknown angle *i*, only *v* sin *i* can be observed, and this equation takes the form

$$R \sin i = (Pv \sin i)/50.6. \quad (2)$$

Tables 1 and 2 list the Ap stars with known *v* sin *i* (Preston, private communication) and with at least moderately reliable values of *P* < 30 days. (For stars with *P* > 30 days, the maximum value of *v* sin *i* is about 5 km s^{-1}, which is below the resolution of most spectrograms obtained to date. A summary of results for Ap stars with *P* > 30 days has been given (Wolff 1975*a*) elsewhere.) Table 1 is restricted to stars for which the same period has been obtained by more than one observer or for which a single observer has obtained the same period for at least two of the three (spectrum, magnetic, and photometric) kinds of variability. Table 2 includes those stars for which periods have been obtained by only a single observer but for which the amplitudes are large enough that the period is probably correct. Tables 1 and 2 do not include all the stars with published periods. For example, I have excluded all stars for which periods have been derived solely from photometric observations of light curves with amplitudes of 0.02 mag. or less. I have also eliminated a number of stars with large amplitudes for which a reanalysis (Hagen and Wolff, unpublished) of the data indicates that alternate periods cannot be ruled out.

Despite this fairly conservative approach, the periods in Table 2 do need confirmation. An example of the problems that can arise is given by HR 6958. For this star, Winzer (1974) derived a period of 0.9451 days from photometric variations with an amplitude of 0.03 mag. In contradiction to these observations, Wolff and Morrison (in preparation) find that the brightness of this star is constant but that the magnetic field varies in a period of 10-12 days.

For the purpose of searching for a correlation between *R* sin *i* and *P*, I have followed Preston (private communciation) in assigning the stars in Tables 1 and 2 to three different temperature classes according to their *UBV* colors, corrected for reddening. While the colors may be slightly affected by blanketing (Wolff 1967), the assignment of these stars to the various temp-

TABLE 1

Ap Stars with Confirmed Periods

HD	Name	Period (days)	v sin i (km s^{-1})	R sin i	Temperature Class	Source
4778	HR 234	2.5475	42	2.11	2	Winzer (1974)
10783		4.1327	24	1.96	2	Preston and Stepien (1968c)
15089	ι Cas	1.73873	46	1.58	3	van Genderen (1970)
18296	21 Per	2.88422	22	1.25	2	Preston (1969c)
19832	56 Ari	0.7278925	200:	2.88:	1	Hardie and Schroeder (1963)
22374	9 Tau	10.61	7	1.47	3	Wolff (1975a)
24712	HR 1217	12.448	≤ 6	≤ 1.48	3	Preston (1972)
25354		3.9001	18	1.39	2	Rakos (1962)
25823	41 Tau	7.227	21	3.00	1	Wolff (1973)
32633		6.431	23	2.92	2	Preston and Stepien (1968a)
34452	HR 1732	2.4660	58	2.83	1	Rakos (1962)
49976		2.976	38	2.23	3	Pilachowski et al. (1974)
51418		5.4379	≤ 20	≤ 2.15	2	Gulliver and Winzer (1973)
62140	49 Cam	4.285	30	2.54	3	Bonsack et al. (1974).
65339	53 Cam	8.0278	< 20	< 3.17	3	Preston and Stepien (1968b)
71866		6.80001	17	2.28	3	Preston and Pyper (1965)
90569	45 Leo	1.4450	13	.37	2	Winzer (1974)
98088	HR 4369	5.90513	25	2.92	3	Abt et al (1968)
108662	17 Com	5.0808	22	2.21	2	Preston et al. (1969)
111133	HR 4854	16.31	10	3.22	2	Wolff and Wolff (1972)
112185	ε UMa	5.0887	34	3.42	3	Guthnick (1934)
112413	α^2 CVn	5.46939	24	2.59	1	Farnsworth (1933)
118022	78 Vir	3.7220	10	.74	3	Preston (1969b)
119213	HR 5153	2.451	35	1.70	3	Wolff and Morrison (1975)
124224	HR 5313	0.52067	130:	1.34	1	Deutsch (1952a)

Table 1 (Continued)

HD	Name	Period (days)	v sin i (km s^{-1})	R sin i	Temperature Class	Source
125248	HR 5355	9.2954	< 15	< 2.76	2	Hockey (1969)
125823	a Cen	8.814	18	3.14	1	Norris (1971)
133029	HR 5597	2.8881	20	1.14	2	Winzer (1974)
137909	β Cr B	18.487	⩽ 3	⩽ 1.10	3	Preston and Sturch (1967)
140160	χ Ser	1.59584	68	2.14	3	Deutsch (1952b)
140728	HR 5857	1.3049	75	1.93	2	Winzer (1974)
152107	52 Her	3.9:	24	1.85	3	Wolff and Preston (1976)
153882	HR 6326	6.00925	26	3.09	3	Preston and Pyper (1965)
173650	HR 7058	9.9748	16	3.15	2	Burke et al. (1969)
175367	HR 7129	3.670	28	2.03	1	Wolff and Wolff (1976)
184905		1.855	70	2.57	2	Burke et al. (1970)
196502	73 Dra	20.2754	8	3.21	3	Preston (1967b)
203006	θ^1 Mic	2.1219	48	2.01	2	Maitzen et al. (1974)
215038		2.036	36	1.45	1	Stepien (1968)
215441		9.488	⩽ 6	⩽ 1.13	1	Stepien (1968)
220825	κ Psc	0.5853	34	.39	2	van Genderen (1971)
223640	108 Aqr	3.73	35	2.58	1	Morrison and Wolff (1971)
224801	HR 9080	3.73983	38	2.81	1	Stepien (1968)

TABLE 2

Ap Stars With Periods In Need of Confirmation

HD	Name	Period (Days)	v sin i (km s^{-1})	R sin i	Temp. Class	Source
7546	HR 369	5.229	33	3.41	1	Winzer (1974)
10221	43 Cas	3.1848	28	1.76	2	Winzer (1974)
14392	63 And	1.3040	78	2.01	1	Winzer (1974)
24155	HR 1194	2.5352	52	2.61	1	Winzer (1974)
27309	56 Tau	1.5691	66	2.05	1	Winzer (1974)
43819	HR 2258	1.0785	14	0.30	1	Winzer (1974)
72968	3 Hya	5.57	16	1.76	2	Wolff and Wolff (1971)
115708		5.07	13	1.30	3	Wolff (1975a)
137949	33 Lib	23.26	10	4.60	3	Wolff (1975a)
170000	Φ Dra	1.7164	89	3.02	1	Winzer (1974)
171586		2.1436	39	1.65	3	Winzer (1974)
177410	HR 7224	1.1663	110:	2.54	1	Winzer (1974)
192913		16.498	14	4.56	2	Winzer (1974)
193722	HR 7786	1.13254	40	0.90	1	Winzer (1974)
221394	HR 8933	2.8419	53	2.98	3	Winzer (1974)
216533		17.20	7	2.38	3	Wolff and Morrison (1973)

TABLE 3

Average v sin i for Ap Stars

Temperature	< v sin i > (km s^{-1})
12300 K $< T_e$	53
9600 K $< T_e <$ 12300 K	35
$T_e <$ 9600 K	23

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erature classes should be correct in nearly all cases. Temperature class 1 includes Ap stars with T_e > 12300 K, class 2, stars in the range 9600 K < T_e < 12300 K, and class 3, stars with T_e < 9600K; all temperatures are on the Schild, Peterson, and Oke (1971) temperature scale. Temperature class 1 includes essentially all the Si stars, while temperature classes 2 and 3 include SiCr and CrEuSr stars.

The plots of R sin i as a function of period are shown in Figures 4 and 5. In these figures the upper envelope of the points should correspond to the actual values of R for the most massive stars included. For temperature class 1, the hottest stars (a Cen and HR 7129) have masses of about 5 $M_\odot$, provided Ap stars have normal masses, and the radius of a 5 $M_\odot$ star varies from 2.4 $R_\odot$ on the zero-age main sequence to a maximum of 4.25 $R_\odot$ near the end of the main sequence phase of its evolution (Iben 1966). Most stars in Figure 4 have masses in the range 3-4 $M_\odot$ and maximum radii of 3.5-4.0 $R_\odot$. The radii of the Ap stars with T_e > 12300 K are therefore approximately in accord with evolutionary calculations. Apart from a deficiency of stars with R sin i < 2.5 $R_\odot$ and P > 3.5 days, which may be due to angular momentum loss during main sequence evolution, there is no clear correlation of R sin i with P.

The data for the Ap stars with T_e < 12300 K are shown in Figure 5. Here the upper limit on the masses should be about 3 $M_\odot$ and the corresponding variation in radius should be from about 1.7 $R_\odot$ on the zero-age main sequence to about 3 $R_\odot$ near the end of the main sequence phase of evolution (Iben 1965). In Figure 5, although the scatter is significant, as would be expected for stars with a range in masses observed at various inclinations, R sin i does appear to increase with increasing period up to P = 10 days. The upper envelope of the data points corresponds rather well to the variation in R predicted by the evolutionary calculations. The possibility that the correlation between P and R sin i might be due to selection effects has been discussed and discounted elsewhere (Wolff 1975b). The shape of the curve in Figure 5, namely an initial rise followed by a rather sharp turnover, can be accounted for if the rate of angular momentum loss is constant--or nearly constant--during the main sequence lifetime of the Ap stars (Wolff 1975b).

The hypothesis that the Ap stars lose angular momentum during their main sequence lifetimes can explain a number of the properties of these objects in addition to the correlation of radius with period. For example, the variation of radius with age is nearly linear during the main sequence

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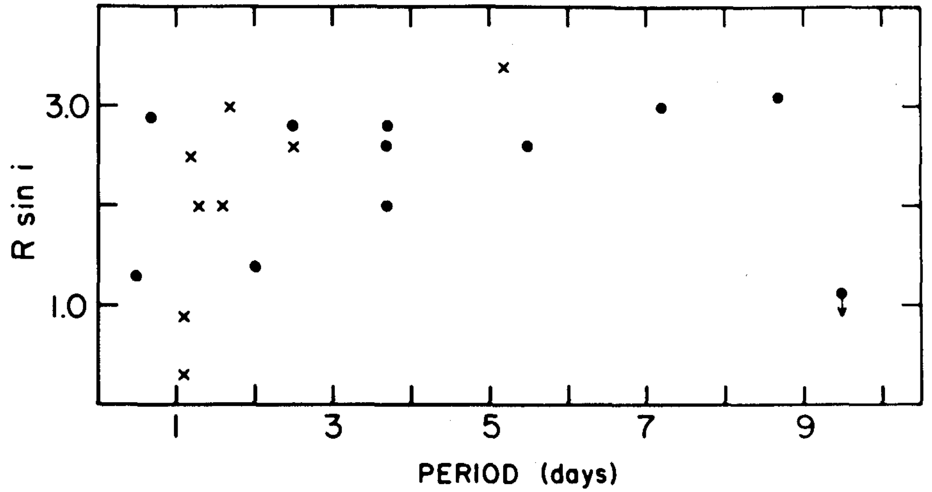

Fig. 4 - Relationship between R sin i, where R is the radius in units of the solar radius, and period for Ap stars with T_e > 12300K. Filled circles represent data from Table 1, crosses represent data from Table 2.

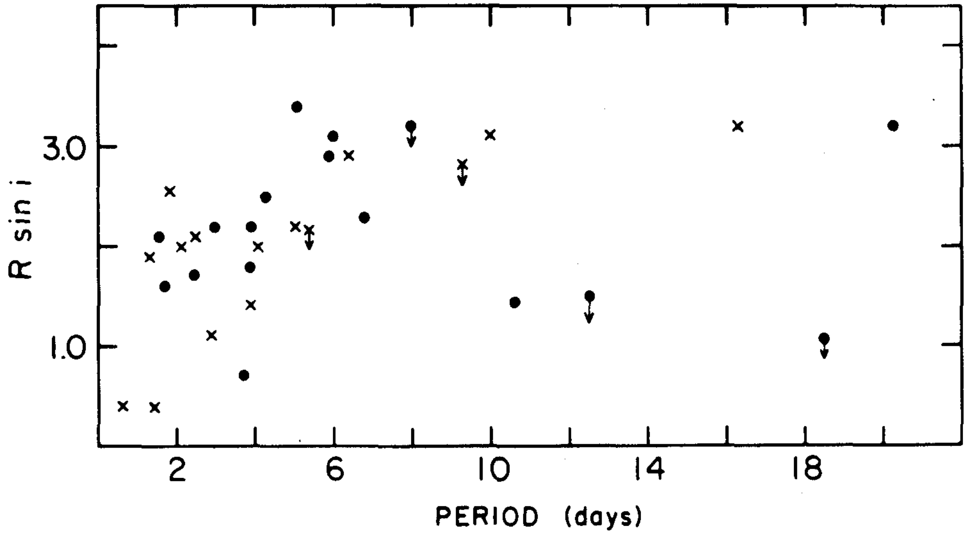

Fig. 5 - Relationship between R sin i and period for Ap stars with T_e < 12300 K. Crosses represent data for stars with T_e in the range 9600 - 12300 K. Filled circles represent data for stars with T_e < 9600 K. All data are from Table 1.

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evolution of a 3 $M_\odot$ star. Therefore, the fact that most stars with P > 6 days have R > 2.7 $R_\odot$ suggests that stars lose enough angular momentum to attain such long periods only after completing about 80 per cent of their main sequence evolution. One might therefore expect only about 20 per cent of the Ap stars to have P > 6 days, a value roughly comparable to the number actually observed. A more detailed comparison between the expected and observed period-frequency distributions has been carried out by Wolff (1975b).

This hypothesis accounts in a natural way for the fact that there are no Si stars with P > 10 days and T_e > 12300 K. The Si stars are hotter than the stars in Figure 5, their main sequence lifetimes are significantly shorter, and there may therefore be insufficient time to reduce angular momentum to extremely low values. Preston (private communication) has also found that the mean rotational velocities of the Ap stars decrease with decreasing temperature. His results, which are summarized in Table 3, can be explained if angular momentum is lost during the main sequence lifetimes of the Ap stars, and if the time scale for the loss of angular momentum is comparable to the main sequence lifetime of a late B- or early A-type star.

It remains to be seen whether mass loss or accretion can account for the very slow rotation of stars like HR 465. However, since P varies inversely with angular momentum, once the angular momentum is fairly low, then additional small reductions in angular momentum can produce extremely large changes in period (Strittmatter and Norris 1971; Wolff 1975b).

5.4 Correlation of Other Properties with Period

If Ap stars do lose angular momentum during their main sequence lifetimes, then the sequence from short to long period stars is an evolutionary sequence, and it is reasonable to ask whether any of the other properties of the Ap stars vary systematically along this sequence.

Recent observations by Landstreet *et al.*(1975) suggest that the strengths of the magnetic fields of Ap stars may be correlated with period. Using photoelectric techniques, Landstreet *et al.* measured magnetic fields in 16 Ap stars with broad lines and found no fields larger than about 1000 gauss; if the distribution of fields for broad-lined stars were like that measured for sharp-lined stars, then Landstreet *et al.* should have detected several stars with fields in excess of 1000 gauss. In interpreting this result Landstreet *et al.* suggest the Ap stars should be divided according to their periods into two groups, those with periods less than 3-5 days

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having systematically smaller fields than the stars of longer period. They suggest that stars of long period may have become slow rotators precisely because they do have larger magnetic fields, while the rapid rotators were left rotating more rapidly because of their relatively weaker fields. In opposition to this interpretation, it could be pointed out that for stars with $\underline{P} > 5$ days, there is no correlation between period and field strength.

There may be an alternative explanation for the observations of Landstreet et al. Several people (e. g. Mestel 1967; Strittmatter and Norris 1971; Maheswaran 1974) have suggested that magnetic field strengths and rotation may be directly correlated in that, if centrifugal forces due to rotation dominate the magnetic forces, rotational circulation currents may tend to drag the field lines beneath the stellar surface, thus reducing the measured magnetic field. For periods in the range 1-3 days, magnetic and rotational forces are comparable, and it seems possible that rotational circulation currents may reduce the measured magnetic field in these stars, either by pulling field lines beneath the surface or by tangling the field in such a way as to reduce the net longitudinal component of the field, which is all that can be detected by the observational techniques currently in use. If the field is strong enough, however, to result in some magnetic braking, then the magnetic field may come to dominate rotation, with a resultant increase in the measured field strength.

The stars observed by Landstreet et al. differ from the majority of the Ap stars in that they exhibit more rapid rotation and smaller magnetic fields. On the present hypothesis, these stars are also less evolved, and one wonders whether they are in some way less peculiar than Ap stars with longer periods. If, for example, mass accretion were responsible for both the abundance anomalies and the loss of angular momentum in Ap stars, then one might expect abundance and rotation to be correlated. While observations are at present inadequate to determine whether such a correlation exists, there is some information on what kind of correlations cannot exist. First of all, lines of Si, Cr, and Sr are conspicuous even in stars with quite broad lines, so it seems unlikely that the abundances of these elements are strongly correlated with rotation. Detailed abundance analyses of broad-lined Ap stars are, however, not available. Based on measurements of 19 stars, Wolff (1967) found that lines of Eu and other rare earths were weak or absent in stars with $\underline{v} \sin \underline{i} > 40$ km s^{-1}, a result that suggests that rare earth abundances may correlate with rotation. However, subsequent observations have shown that rare earth lines are present in some short period stars (e.g. HD 184905; Babcock 1958; Morrison and Wolff 1971). Furthermore, at least some long period stars (e.g. HD 8441; Babcock 1958; Wolff and Morrison 1973) do not exhibit pronounced rare earth lines. Nevertheless, examination of low dispersion spectral types (Cowley et al. 1969) and of photometric amplitudes of cool Ap stars (Wolff 1975b), which are often correlated with rare earth spectrum variations, suggests that on the average sharp-lined stars may have larger over-abundances of rare earths than do broad-lined stars. Additional observations should be made to determine whether there is indeed a difference of this kind between sharp-lined and broad-lined stars. If there is, then available observations suggest that, as is also true for magnetic field strengths (Landstreet et al. 1975), a period of about 3 days serves to separate the more peculiar stars from the less peculiar ones. There are a number of stars (e.g. 78 Vir and HD 51418) with periods only slightly greater than 3 days that have conspicuously strong lines of the rare earths. Therefore, if there is any correlation between rare earth abundances and rotation, it would have to be in the sense that stars rotating more rapidly than some critical value tend not to show large overabundances, presumably because the atmospheres of stars that rotate more rapidly than this threshold value are not sufficiently stable for the process(es) responsible for forming the rare earth overabundances to be effective.

Analyses of Ap stars in clusters are crucial for determining whether angular momentum, or any other property of these stars, varies systematically during main sequence evolution. Surveys made to date (Young and Martin 1973; Hartoog 1975) indicate that SiCrEuSr Ap stars occur in clusters with only about half the frequency that is thought to obtain for field stars. Such a deficiency of Ap stars in clusters is compatible with the hypothesis that Ap stars develop their peculiarities on a long time scale. However, before accepting this hypothesis as the correct interpretation of the observations, we must know a good deal more about precisely what kinds of Ap stars are found in clusters of various ages.

6. Conclusion

Ten years ago, Preston (1967a) presented a summary of what was then known about Ap stars and suggested several directions for future work. Many of the questions posed by Preston have been essentially answered during the past decade. The number of stars with well-established periods has tripled during that time; it now appears that all SiCrEuSr Ap stars are periodic and that the variations are stable over long periods of time; the evidence in favor of spectroscopic patches (concentrations of specific elements) on the stellar surface seems persuasive (Preston and Sturch 1967; Pyper 1969; Wolff 1969); there appears to be a satisfactory explanation for the light variations of most of the Ap stars; the discovery of resolved Zeeman lines in stars other than HD 215441 (Preston 1969d) has led to much better understanding of the magnetic geometry of the Ap stars (Wolff and Wolff 1970; Preston 1970; Huchra 1972). The extensive effort that has been devoted to understanding the variations of the Ap stars has provided a foundation on which we must now try to build a coherent model of the origin and evolution of the magnetic Ap stars. In the next ten years, I expect that research will increasingly be directed toward answering questions about the time scale for the formation of Ap stars; the source(s) of the abundance anomalies (surely one of the most difficult of the remaining problems); and the relationship of the magnetic Ap stars to normal and non-magnetic peculiar stars.

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Discussion to the paper of WOLFF

WALRAVEN: Would you comment on the variability of Eu in Cepheids?

WOLFF: The fact that Eu is variable in Cepheids as well as in Ap stars was noted several decades ago. In the Ap stars, I think the spectrum variations must be explained in terms of a non-uniform distribution of elements over the surface of a rotating star. The variations in Eu in Ap stars cannot be explained in terms of changes in temperature and pressure. Therefore, I think the variations of Eu in the Cepheids and Ap stars must be due to different causes.

SEGGEWISS: You mentioned that the Ap stars in clusters are only half as frequent as in the field. Is the observational material adequate to justify this statement? I found that there are definitely 30% Ap stars among open cluster blue stragglers (Hg-Mn and Si-Cr-Eu-Sr stars). Another 30% are probable Ap stars.

WOLFF: I based my statement on the work of Young and Martin (1973) and Hartoog (1975). I think that in discussing the frequency of Ap stars in clusters, one should distinguish clearly between the Hg-Mn stars and the Si-Cr-Eu-Sr stars. The time scale for the formation of the Hg-Mn stars is probably $< 10^7$ years (Wolff and Wolff 1974), and so these stars might be expected to occur with the same frequency in clusters as among field stars. If, as I have suggested, the magnetic Ap stars lose angular momentum after reaching the zero-age main sequence, then one might expect that Si-Cr-Eu-Sr stars with very low values of v sin i should be deficient in young clusters.