THE TULLY-FISHER RELATION AND ITS APPLICATION TO THE DISTANCE SCALE

M. Aaronson
Steward Observatory, University of Arizona

ABSTRACT

The Tully-Fisher relation applied in the infrared appears to be the best global distance indicator presently available for determining the expansion rate and deviations from uniform Hubble flow. In this article recent results obtained using the IR/H I method are reviewed. A Virgo-directed Local Group velocity of about 300 km s^{-1} is indicated (implying a local value for the deceleration parameter $q_o \sim 0.05 - 0.1$) along with a "best guess" value for the Hubble Constant of 85 km s^{-1} Mpc^{-1}.

1. INTRODUCTION

Only five years ago, a paper by Tully and Fisher (1977) appeared announcing a new method of determining distances to galaxies. Perhaps the most surprising aspect of their simple and beautiful idea was that it was not thought of earlier. Although H I measurements had been previously incorporated into distance estimates (e.g. Balkowski 1973), Tully and Fisher were first to realize that the rotation of a galaxy itself, as measured by the velocity width of the 21 cm line, could be used as a standard candle. This is because rotation, a distance independent quantity, is proportional to mass; while luminosity, a distance dependent quantity, is also proportional to mass. Relative distances therefore follow in a straightforward manner, as two galaxies with the same velocity width but having a ratio of two in luminosity should have a ratio of four in distance.

Unfortunately, the method proposed by Tully and Fisher (1977) contained a serious drawback involving the use of blue magnitudes. Ideally, one would like to observe edge-on galaxies because the correction from observed velocity width to maximum rotation velocity grows as the sign of the inclination angle, becoming very uncertain for inclinations less than 45°. However, for inclinations greater than 45° correction to the optical magnitude for internal absorption due to dust becomes both large and uncertain. In hopes of avoiding this conundrum, Aaronson, Huchra, and

Richard M. West (ed.), Highlights of Astronomy, Vol. 6, 269–282.

https://doi.org/10.1017/S1539299600005232 Published online by Cambridge University Press

Mould (1979, hereafter AHM) turned to the infrared. These authors also felt that by going to longer wavelengths, where the underlying old red stellar population common to all galaxies dominates, the observed luminosities would better reflect galaxian mass, being less subject to the stochastic effects of young blue stars.

AHM did indeed find a tighter empirical correlation than had been seen with optical magnitude using H(1.6μm) photometry uncorrected for inclination effect. The H-band was selected because it offered several advantages over the other near infrared windows: better signal-to-noise ratio, little contribution from non-stellar emission, low atmospheric extinction, and a negligible "k" correction. A surprising result also turned up when the slope of the infrared magnitude/H I relation was found to be near 10, a value considerably steeper than had been seen optically, but one reminiscent of the well-known fourth-power law which had been found to apply to the luminosity/velocity dispersion relation for ellipticals. The dynamical origin of the relation could then be ed accounted for (see below).

The Tully-Fisher technique has today become one of the most popular distance methods in use. Its application to the distance scale problem using optical magnitudes has been pursued by a number of authors, including Sandage and Tammann (1976), Tully and Fisher (1976), Fisher and Tully (1977), Shostak (1978), de Vaucouleurs et al. (1981), and de Vaucouleurs (1982). A second class of papers has been concerned more with delineating the empirical properties of the Tully-Fisher relation, for example Roberts (1978); Rubin, Ford, and Thonnard (1978, 1980); Rubin, Burstein, and Thonnard (1980); Rubin et al. (1982); Bottinelli et al. (1980); Burstein et al. (1982); de Vaucouleurs et al. (1982); and Huchtmeier (1982). Work in the infrared has been primarily conducted by Aaronson, Mould, and their collaborators (Aaronson, Mould, and Huchra 1980; Mould, Aaronson, and Huchra 1980; Aaronson et al. 1980, 1981, 1982a, b; and Aaronson and Mould 1983). Two recent H I catalogs are also noteworthy: the literature compilation of Bottinelli, Gouguenheim, and Paturel (1982) and the Local Supercluster survey of Fisher and Tully (1981). There are of course many additional H I studies of galaxy groups and clusters (e.g. Bothun 1981 and references therein), but these will not be touched upon except as they relate to determination of the expansion rate.

In what follows emphasis will be placed on recent distance scale results found using the IR/H I relation, as the superiority of infrared magnitudes over those obtained optically has now been clearly demonstrated (Aaronson and Mould 1983 and Section 2 below). While various "alternative Tully-Fisher relations have been discussed involving isophotal diameter, color, surface brightness, etc. (e.g. Tully 1982), these shall again be largely ignored in favor of what the author considers the most eloquent and accurate approach. After some further discussion of the methods underpinnings in Section 2, the Virgocentric infall is discussed in Section 3, and the problem of the Hubble constant itself is addressed in Section 4.

2. THE IR/H I RELATION -- THE BEST GLOBAL INDICATOR?

There are five reasons for believing the infrared Tully-Fisher method to be the best global distance indicator we have at present. First, there is a well determined physical basis underlying the relation. Second, all the measurables involved are quantitative; subjective estimates involving such things as luminosity classes do not enter in. Third, the method can be calibrated using nearby galaxies having Cepheid distances (and in this sense is a secondary and not tertiary indicator), but at the same time applied in a straightforward manner to galaxies with redshifts upwards of 10,000 km s^{-1}. Hence there is no "twilight zone" (cf. Sandage and Tammann 1974) and furthermore several traditional and suspect rungs in the distance ladder are avoided. Fourth and perhaps most important, the method exhibits a small scatter, typically $\sigma \sim 0.45$ mag. Finally, because one works in the IR the problem of galactic extinction becomes irrelevant, and an arbitrary galactic latitude cut-off in the sample need not be adopted.

Points 1 and 4 above require some additional comment. In regard to the physical basis, AHM showed that a fourth power law relating luminosity to velocity width (e.g. $L \propto \Delta V^4$) follows from the virial theorem plus three simple assumptions. These are that (a) all galaxies have the same mass profiles and rotation curves as a function of some dimensionless scale-length; (b) all galaxies have the same central mass surface density; and (c) all galaxies have the same mass-to-light ratio.

Recently, however, Burstein (1982) has argued that the existence of a surface brightness/velocity width relation in the AHM data <u>may</u> invalidate one or more of the AHM assumptions. The reason for the "may" is that one does not really know precisely how to hook up the observed luminosity distribution with the underlying mass distribution, given the strong evidence that at least at large radii the two become somewhat decoupled. Nevertheless, Burstein concludes that "there exists no physical interpretation" of the Tully-Fisher relation and furthermore that the slope and zero point of the relation may not be universal.

Several comments can be made in regard to Burstein's quite valid concerns. First, the Tully-Fisher method is applied in a strictly empirical fashion and so the validity of the AHM assumptions is in some sense irrelevant. Even so, there is now considerable evidence which suggests that both the slope and zero point are independent of environment. The zero-point question will be considered further in Section 4, but with regard to slope, Figure 6 from Aaronson and Mould (1983) illustrates that the expected value of 10 fits data from a wide variety of locations, ranging from local conglomerations like the M81 group to dense clusters such as Virgo. Hence it would seem that at least in some average, perhaps crude sense the AHM assumptions are probably okay. In any event, to proclaim that the Tully-Fisher method has no physical basis seems a much too pessimistic attitude, and is tantamount to arguing that there exists no connection whatsoever between galaxian luminosity and mass. However, the very existence of the relation seems to tell us that such a connection does indeed occur.

Turning to scatter in the method, Table 1 presents the magnitude scatter for several samples of data. The results there are based on a catalog of IR photometry and 21 cm line widths for 308 Local Supercluster glaxies, i.e. having a redshift $V < 3000$ km s^{-1} (Aaronson et al. 1982b). We can see that the one sigma estimate of 0.45 mag quoted above is consistent with the scatter seen for nearby calibrating galaxies, for the Virgo and Ursa Major clusters, and for the entire Local sample after correction for infall. No other global indicator has yet been convincingly shown to have such small scatter. For a magnitude limited sample, an upper limit on the distance error obtained using the IR/H I method is then only $\sim 1.38\ \sigma^2 \sim 0.28$ mag. This is an upper limit because the intrinsic scatter in the technique is certainly less than the indications in Table 1, as no account has been made for observational errors, depth effects, group velocity dispersion, deviations from the infall model, etc. One further point seen from Table 1 is that the scatter in the IR is less than in the blue; the opposite claim of Bottinelli et al. (1980) is incorrect.

TABLE 1

SCATTER IN THE TULLY-FISHER RELATION

Sample	N	σ (B magnitudes)	σ (H magnitudes)
Sandage-Tammann Calibrators	16	----	0.42
de Vaucouleurs Calibrators	13	----	0.36
Virgo Cluster	15, 16	0.51	0.45
Ursa Major Cluster	24	0.42	0.40
Infall Sample	221	0.59	0.52

Aside from the potential problem with environmental influences that was already touched upon, the other major concern about the Tully-Fisher method expressed in the literature is possible dependence on morphological type. Roberts (1978) found a very strong type dependence in the blue, a result that has recently received support from the work of Burstein et al. (1982) and Rubin et al. (1982). All of these authors find a clear separation in the blue magnitude/H I plane between early and late type galaxies at fixed luminosity. There are two effects which contribute to, but may not explain completely, this type dependence. First, the

B-H color, morphological type relation (see Aaronson 1978) will lead to larger type dependence in the Tully-Fisher plane as one goes to the blue. Second, the treatment of magnitude as the independent variable will also increase the size of the effect; this point is discussed further by Aaronson and Mould (1983).

In any event, there does not appear to be a significant type dependence in the infrared Tully-Fisher relation, a point illustrated by Figure 1. This figure was constructed from a subset of the 308 Local Supercluster galaxies previously mentioned by treating magnitude and

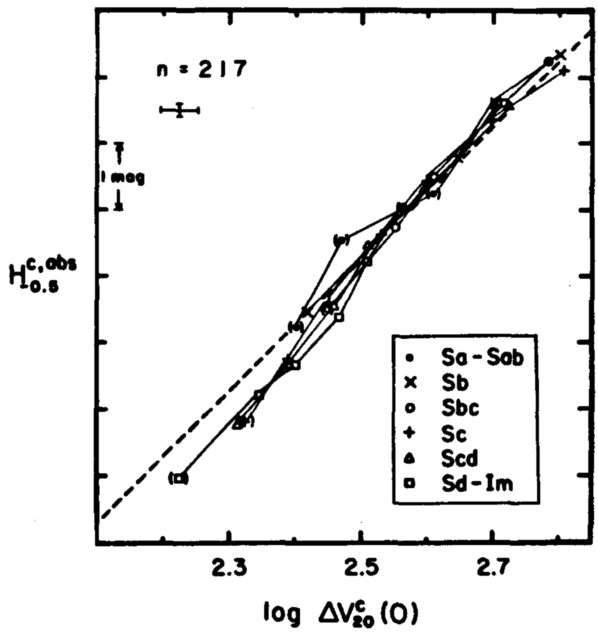

Figure 1. Absolute magnitude/velocity width relation binned by type. The dashed line has a slope of 10. Symbols surrounded by parentheses have less than three galaxies in the bin.

velocity width as equal regression variables and by applying the infall model discussed below. However, other possible constructions of the diagram do not change the basic result that at fixed velocity width, the spread in infrared magnitude for differing types is negligible.

One final recent point of controversy has concerned the slope of the Tully-Fisher relation. Rubin and collaborators have found the slope obtained from their samples of Sb and Sc galaxies to be as steep

https://doi.org/10.1017/S1539299600005232 Published online by Cambridge University Press

or steeper than that seen in the infrared. On the other hand, Aaronson and Mould (1983) find a significant increase in slope with decreasing wavelength for the considerably larger sample in Figure 1. The reasons for this disagreement have not yet been fully resolved, but it is worth noting that wavelength dependence in the slope of the Tully-Fisher relation necessarily follows from the existence of the spiral galaxy color-magnitude effect (see Tully, Mould, and Aaronson 1982).

3. THE LOCAL VELOCITY FIELD

In the last few years considerable effort has been devoted to mapping the velocity field in the Local Supercluster, with some half-dozen major, independent studies of the problem having been conducted. This flurry of activity has been prompted in part by measurement of the microwave background anisotropy and in part by the constraint that the Virgocentric infall provides on the local value of the deceleration parameter q_o. The infall problem is also attractive because it is independent of absolute distance scale.

The remarks here will be primarily confined to the work done by Aaronson et al. (1980a, hereafter AHMST) using the IR/H I relation. This is in fact the only study which has attempted to separate out the infall at the position of the Local Group due to Virgo's gravity from the peculiar motion of the Local Group itself.

The analysis of AHMST employed the sample of 300 nearby spirals referred to above, and adopted a Virgocentric flow model of the type discussed by Silk (1974) and Peebles (1976). This model assumes a spherically symmetric Supercluster (see Figure 2) with a power law density enhancement centered on Virgo. From this simplifying assumption the distortion of the velocity field as viewed from the Local Group can then be worked out in a straightforward manner (Schechter 1980). AHMST considered two methods for fitting the data. In the redshift residual scheme magnitudes and velocity widths were used to predict redshifts, while in the width residual scheme magnitudes and redshifts were used to predict velocity widths. The former approach suffers from possible bias arising from Malmquist effect; the latter approach avoids this problem but introduces another type of subtle, systematic bias. In the first case AHMST were not able to estimate the amount of bias because their sample selection effects were too poorly known, but in the second case the amount of bias could be determined through Monte Carlo simulations. (It is nevertheless interesting that both schemes yielded results in good agreement, which suggests that the AHMST data sample is probably closer to being volume-limited rather than magnitude-limited in nature.)

Model results using the width residual scheme are presented in the first line of Table 2. The w_x, w_y, and w_z components are the peculiar motions of the Local Group in the three directions z (toward M87), x (90^o away in the Supergalactic plane), and y (towards the Supergalactic

https://doi.org/10.1017/S1539299600005232 Published online by Cambridge University Press

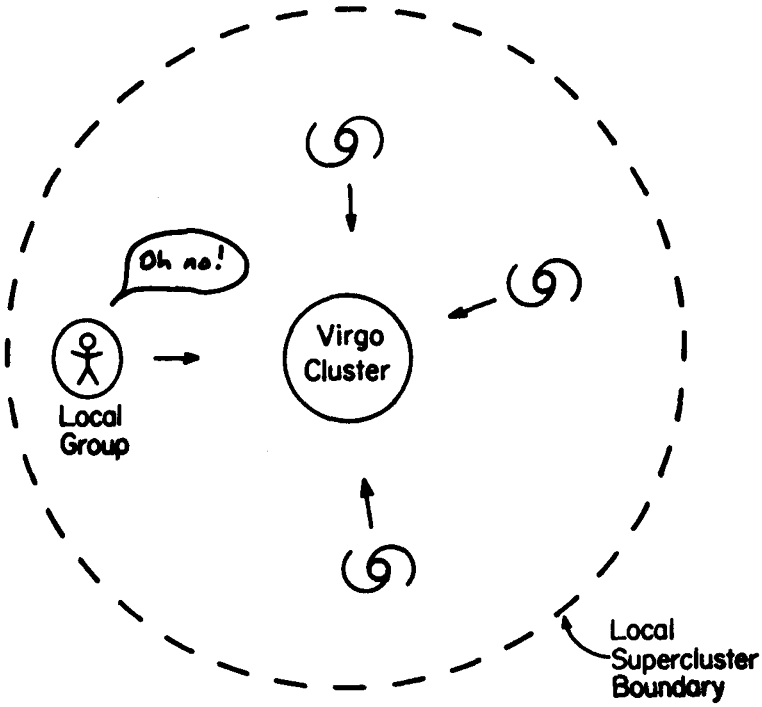

Figure 2. Go with the flow.

pole), while w_i represents the infall velocity towards Virgo at the distance of the Local Group. An infall velocity of ∿ 250 km s^{-1} is indicated, along with a total Virgo-directed motion of ∿ 330 km s^{-1}. AHMST tested the effect of possible "second parameters" in the Tully-Fisher relation (e.g. morphological type, surface brightness, H I flux), but found little difference in their calculations.

An important point to note about the results in Table 2 are their insensitivity to the adopted Virgo redshift (see Figure 5c in AHMST). This indicates that AHMST are not simply measuring a difference in Hubble ratio between Virgo and more distant spirals, but are actually detecting the expected dipole pattern around the sky. Note also that the total motion towards Virgo is better determined than either w_z or w_i; it is this total motion which must be used in estimating the expansion rate.

The second row of Table 2 gives results from AHMST obtained by allowing the Supercluster to rotate, using the empirical rotation curve suggested by de Vaucouleurs (1958). A three sigma effect is found in the data, the explanation of which may however encounter some difficulty (AHMST).

https://doi.org/10.1017/S1539299600005232 Published online by Cambridge University Press

The next lines of Table 2 list two recent measurements of the 3 K microwave background anisotropy. The AHMST motion differs by 3σ from that of Boughn, Cheng, and Wilkinson (1981), and by only 2σ from Smoot and Lubin (1979). While these differences are probably real, it nevertheless appears that a major part of the motion indicated by the microwave experiments arises on a local scale. A recent report by Hart and Davies (1982) is of some interest in this regard. Using a sample of Sbc galaxies having about twice the mean redshift of the AHMST sample, and employing as a standard candle H I flux, these authors find virtually identical agreement with the microwave results. If both the Hart and Davies and AHMST results are correct, the implication is that a bulk Supercluster motion exists amounting to several hundred km s^{-1}. It would clearly be of interest to extend the AHMST analysis to a sample of more distant objects to check on this possibility.

Several other interesting implications follow from the AHMST results. The measured infall implies (for a Friedmann universe) a value of $q_o \sim$ 0.05 - 0.1, the range here reflecting current estimates of the density enhancement interior to the Milky Way (cf. Yahil, Sandage, and Tammann 1980; Davis and Huchra 1982). Also, the apparently significant w_y motion may reflect acceleration toward the supergalactic plane (e.g. White and Silk 1979).

4. THE HUBBLE CONSTANT

Calibration of the IR Tully-Fisher relation must of course rely on the absolute distances to nearby galaxies. There is unfortunately considerable disagreement in the distances to those objects which might be used as calibrators, a point illustrated by Figure 3. Here are shown calibrations based on the two main competing local distances scales, one by Sandage and Tammann and one by de Vaucouleurs, for which we can use 16 and 13 spirals, respectively (see Aaronson and Mould 1983 for details). Either calibration taken by itself is quite respectable, but the zero point difference between the two scales is 0.65 mag!

It seems quite obvious from Figure 3 that present uncertainty in any estimate of the expansion rate rests largely with the distances to the nearby calibrators. There are, however, compelling reasons for believing that on the one hand the de Vaucouleurs distances are too small, and that on the other the Sandage and Tammann distances are too large. In the former case these have to do with the treatment of reddening (see Burstein and Heiles 1982) and in the latter case with the neglect of internal absorption and with the almost certain over-estimate of M101's distance (Humphreys and Strom 1981). For the purposes of discussion, the lead of earlier papers on the IR/H I relation shall be followed and a calibration based solely on the Sandage-Tammann distances to M31 and M33 will be adopted here. As seen below, this in fact represents something of a compromise between the two scales.

https://doi.org/10.1017/S1539299600005232 Published online by Cambridge University Press

TABLE 2

SUPERCLUSTER VELOCITY FIELD

	w_x	w_y	w_z	w_i	w_r	w_z^{TOT}	w_x^{TOT}
Model Results:	- 65	-143	81	250	≡ 0	331	- 65
	± 40	± 48	± 50	± 64	---	± 41	± 40
w/Rotation:	-106	-141	22	281	180	303	74
	± 41	± 47	± 54	± 63	± 58	± 39	± 71
3 K Anisotropy -							
Boughn *et al.*	----	-341	----	----	----	411	318
Smoot and Lubin	----	-311	----	----	----	373	178

$|\underline{w}^{TOT} - \underline{w}_{3K}|$ = 330 ± 107 km s^{-1} (Boughn *et al.*)
208 ± 103 km s^{-1} (Smoot and Lubin)

TABLE 3

THREE ESIMTATES OF THE EXPANSION RATE

Sample	<r> (Mpc)	<v> (km/s)	<v>/<r> (km/s/Mpc)
Virgo	16.4 ± 0.8	1019 ± 51	82 ± 6[1]
10 Distant Clusters	----	4000 - 11000	87[2]
Distant Field Sc's	----	3000 - 13000	84 ± 5[3]

[1] Corrected for Virgocentric velocity of 331 ± 41 km s^{-1}.

[2] Mean value; cluster Hubble ratios range from 78 - 92 with a typical formal error of ± 7 km s^{-1} Mpc^{-1}.

[3] Corrected for Malmquist bias.

https://doi.org/10.1017/S1539299600005232 Published online by Cambridge University Press

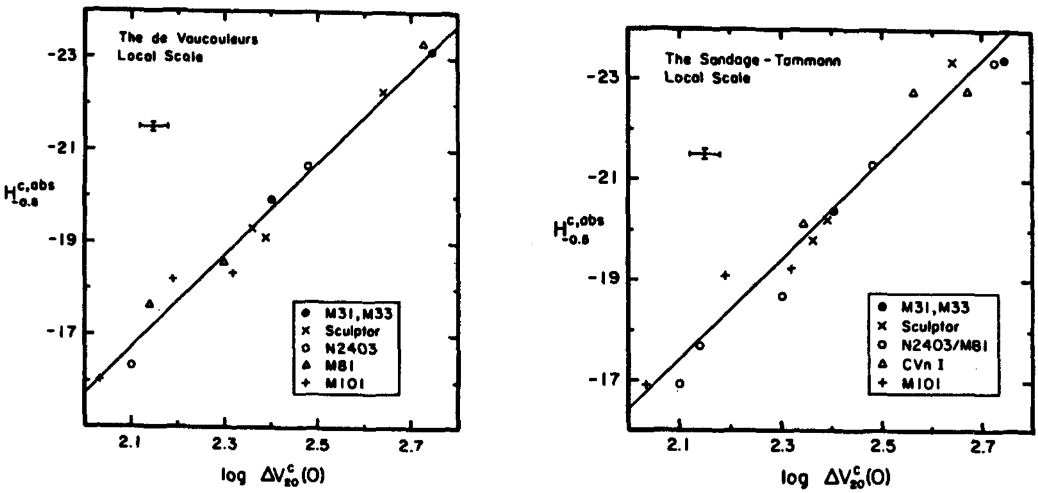

Figure 3. Absolute calibration of the IR/H I relation using two alternative nearby distance scales. The solid lines have slope 10. The zero-point difference between the two scales is 0.65 mag.

Following Aaronson and Mould (1983), a value for the expansion rate can now be estimated from the Virgo distance (16.4 ± 0.8 Mpc), the Virgo velocity (1019 ± 51 km s^{-1}), and the Virgocentric motion (331 ± 41 km s^{-1}), leading to H_o = 82 ± 6 km s^{-1} Mpc^{-1}. The formal one sigma error in this estimate reflects only the scatter of the data, and not the uncertainty in zero point, which may be large. Note that had all 16 Sandage-Tammann calibrators been used, the result would be H_o = 76 km s^{-1} Mpc^{-1}, while with all 13 de Vaucouleurs calibrators, the result would be H_o = 103 km s^{-1} Mpc^{-1}.

It would of course be desirable to determine the expansion rate from galaxies at sufficiently great distances so that any possible perturbations arising from influence of the Local Supercluster can be ruled out. With this goal in mind, the author and collaborators have been conducting H I observations at Arecibo and infrared photometry at Kitt Peak over the last several years for spirals in a number of distant clusters. The preliminary results of this effort, based on only four clusters, have been reported by Aaronson et al. (1980). We have now assembled considerably more data on a total of ten clusters (Aaronson et al. 1983), including Pisces, Abell 400, Abell 539, Cancer, Abell 1367, Coma, Zwicky 74-23, Hercules, Pegasus, and Abell 2634/66. Tully-Fisher diagrams for two of the clusters are illustrated in Figure 4. It should be noted that the 21 cm observations were greatly aided by the introduction of a new, low temperature receiver at Arecibo. Extension of the Tully-Fisher method to clusters with redshifts perhaps as high as 15,000 km s^{-1} appears now to be quite feasible.

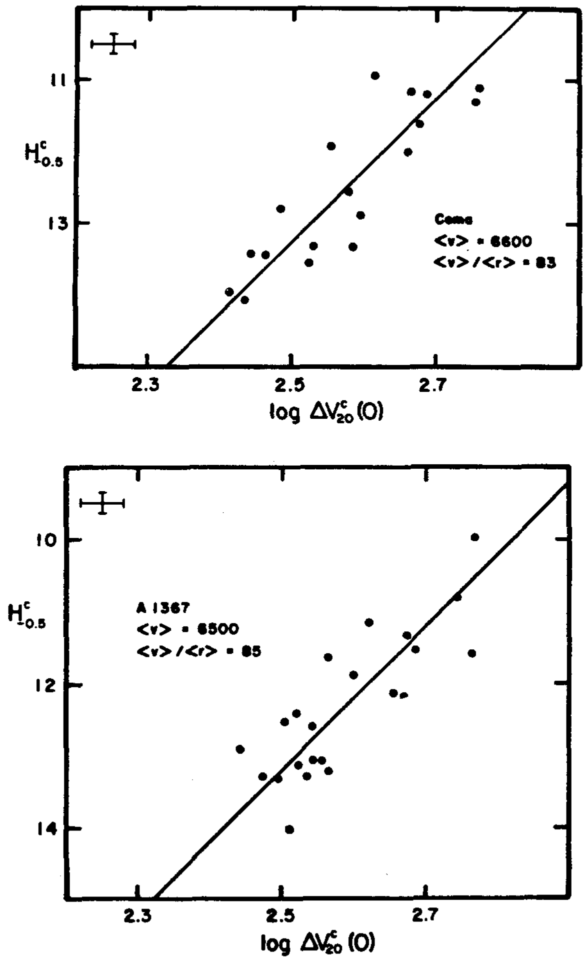

Figure 4. The IR/H I relation for the Coma and Abell 1367 clusters. The solid lines have slope 10. The error bar shown is the typical uncertainty for an individual measurement.

After correction for infall effect, the observed Hubble ratios for the 10 clusters all fall within the range 78 - 92 km s^{-1} Mpc^{-1}, with a typical one sigma error of 7 km s^{-1} Mpc^{-1}. The velocity/distance relation for 11 clusters (including Virgo) is shown in Figure 5. It is important to stress that by working in clusters possible problems with Malmquist effect are avoided, because the objects are all more or less at the same distance. A number of the clusters are in fact sampled as deep in both magnitude and velocity width as is Virgo.

https://doi.org/10.1017/S1539299600005232 Published online by Cambridge University Press

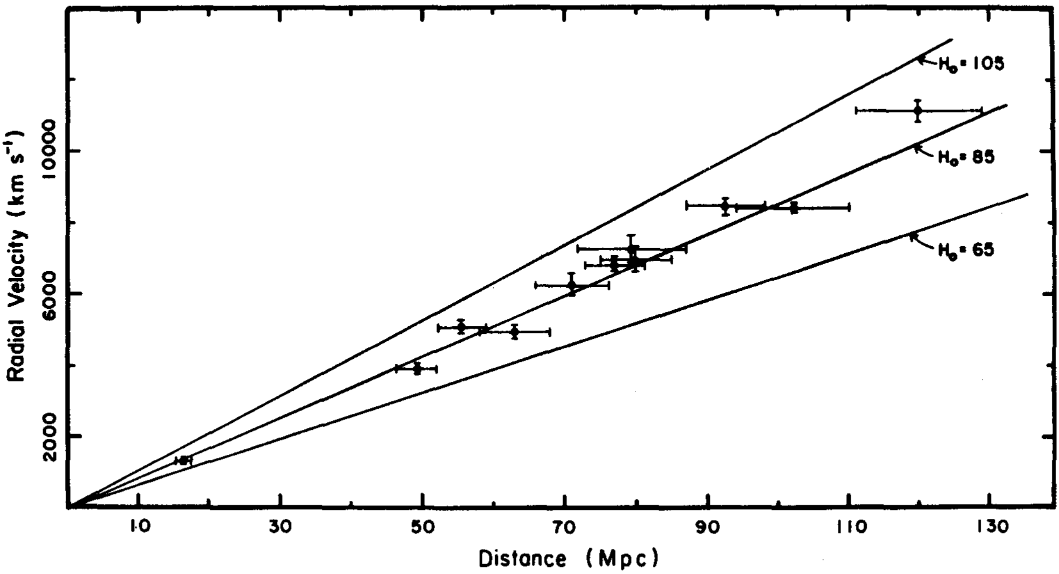

Figure 5. The velocity/distance relation for 11 galaxy clusters.

Following Aaronson et al. (1980), the distant clusters can be used to make an independent estimate of Virgocentric motion. The result obtained indicates a velocity about 100 km s^{-1} greater than that found by AHMST. It is interesting that other studies based at least in part on samples well outside the Local Supercluster also indicate similarly large velocities (e.g. Tonry and Davis 1981; Hart and Davis 1982), but in the present case there is a suspicion that the effect may in part be a consequence of problems with the isophotal diameters for spirals in some of the distant clusters. The reason for this suspicion was first pointed out by van den Berg (1981), who found that the magnitude/infrared surface brightness (H, Σ) relation for the data in Aaronson et al. (1980) deviated from cluster to cluster, and lead in two instances to absurdly large infall values. Variations in the H, Σ relation (which may be real) are also seen in the 10 cluster sample reported on here, but it is important to note that any isophotal diameter error will cause a distance error in the IR/H I method only about half that obtained in the H, Σ method, owing to a slope of ~ 2 for the relation in the latter case. We are currently investigating the diameter problem by collecting CCD photometry for a number of the cluster spirals.

In any event, the good agreement in Hubble ratio found among the distant clusters suggests that the IR/H I relation does not depend on environment, as the sample ranges from high-density spiral-poor objects like Coma to low-density spiral-rich objects like Abell 1367. Further support for this argument comes from a study of the IR/H I properties of distant Sc field galaxies by Bothun et al. (1983), who selected spirals from the field samples used by Sandage and Tammann (1975) and Rubin et al. (1976). Unlike the cluster data, any estimate of distances for these objects is subject to Malmquist bias, but after correction for this effect (according to 1.38 σ^2), an expansion rate of 84 $\pm$ 5 km s^{-1} Mpc^{-1} is obtained. The three estimates of the expansion rate discussed here are summarized in Table 3, and the good agreement provides very convincing evidence that the zero-point of the IR/H I relation is universal.

In summary, the results in Table 3 suggest a "best guess" estimate for the Hubble constant of 85 km s^{-1} Mpc^{-1}, to which the reader is invited to attach his or her own uncertainty. The age of the universe implied by this value, $t_o \sim 12$ billion years, is in only marginal disagreement with current nucleocosmochronology estimates. However, present day ages for galactic globulars indicate $t_o \gtrsim 16 - 18$ billion years (see the talk by Bruce Carney elsewhere in this volumne). One appears to have the option either of dismissing one or the other of these age results, or of marveling at how well the two completely independent methods agree. The author prefers the latter viewpoint.

It is a great pleasure to thank my collaborators on various aspects of the work discussed here: Greg Bothun, John Huchra, Jeremy Mould, Paul Schechter, Bob Schommer, Woody Sullivan, and Brent Tully. Preparation of this article was partially supported with funds from U.S. National Science Foundation grant AST 81-17365.

REFERENCES

Aaronson, M.: 1978, *Ap. J. (Letters)*, 221, L103.
Aaronson, M., Bothun, G. D., Huchra, J., Mould, J., Schommer, R. A., and Sullivan, W. T.: 1983, in preparation.
Aaronson, M., Dawe, J. A., Dickens, R. J., Mould, J. R., and Murray, J. B.: 1981, *M.N.R.A.S.*, 195, 1p.
Aaronson, M., Huchra, J., and Mould, J.: 1979, *Ap. J.*, 229, 1 (AHM).
Aaronson, M., Huchra, J., Mould, J., Schechter, P. L., and Tully, R. B.: 1982a, *Ap. J.*, 258, 64 (AHMST).
Aaronson, M., Huchra, J., Mould, J. R., Tully, R. B., Fisher, J. R., van Woerden, H., Goss, W. M., Chamaraux, P., Mebold, U., Siegman, B., Berriman, G., and Persson, S. E.: 1982b, *Ap. J. Suppl.*, in press.
Aaronson, M., and Mould, J.: 1983, *Ap. J.*, in press.
Aaronson, M., Mould, J., and Huchra, J.: 1980, *Ap. J.*, 237, 655.
Aaronson, M., Mould, J., Huchra, J., Sullivan, W. T., Schommer, R. A., and Bothun, G. D. 1980, *Ap. J.*, 239, 12.
Balkowski, C.: 1973, *Astr. Ap.*, 29, 43.
Bothun, G. D.: 1981, Ph.D. Thesis, University of Washington.
Bothun, G. D., Aaronson, M., Schommer, R. A., Huchra, J., and Mould, J.: 1983, in preparation.
Bottinelli, L., Gouguenheim, L., and Paturel, G.: 1982, *Astr. Ap. Suppl.*, 47, 171.

Bottinelli, L., Gouguenheim, L., Paturel, G., and de Vaucouleurs, G.: 1980, *Ap. J. (Letters)*, 242, L153.
Boughn, S. P., Cheng, E. S., and Wilkinson, D. T.: 1981, *Ap. J. (Letters)*, 243, L113.
Burstein, D.: 1982, *Ap. J.*, 253, 539.
Burstein, D., and Heiles, C.: 1982, *A. J.*, 87, 1165.
Burstein, D., Rubin, V. C., Thonnard, N., and Ford, W. K., Jr.: 1982, *Ap. J.*, 253, 70.
Davis, M., and Huchra, J.: 1982, *Ap. J.*, 254, 437.
Fisher, J. R., and Tully, R. B.: 1977, *Comm. Ap.*, 7, 85.
____________.: 1981, *Ap. J. Suppl.*, 47, 139.
Hart, L., and Davies, R. D.: 1982, *Nature*, 297, 191.
Humphreys, R. M., and Strom, S. E.: 1981, *Bull. A.A.S.*, 13, 892.
Huchtmeier, W. K.: 1982, *Astr. Ap.*, 110, 121.
Mould, J., Aaronson, M., and Huchra, J.: 1980, *Ap. J.*, 238, 458.
Peebles, P. J. E.: 1976, *Ap. J.*, 205, 318.
Roberts, M. S.: 1978, *A. J.*, 83, 1026.
Rubin, V. C., Burstein, D., and Thonnard, N.: 1980, *Ap. J. (Letters)*, 242, L149.
Rubin, V. C., Ford, W. K., Jr., and Thonnard, N.: 1978, *Ap. J. (Letters)*, 225, L107.
____________.: 1980, *Ap. J.*, 238, 471.
Rubin, V. C., Ford, W. K., Jr., Thonnard, N., and Burstein, D.: 1982, preprint.
Rubin, V. C., Ford, W. K., Thonnard, N., Roberts, M. S., and Graham, J. A.: 1976, *A. J.*, 81, 687.
Sandage, A., and Tammann, G. A.: 1974, *Ap. J.*, 190, 525.
____________.: 1975, *Ap. J.*, 197, 265.
____________.: 1976, *Ap. J.*, 210, 7.
Schechter, P. L.: 1980, *A. J.*, 85, 801.
Shostak, G. S.: 1978, *Astr. Ap.*, 68, 321.
Silk, J.: 1974, *Ap. J.*, 193, 525.
Smoot, G. F., and Lubin, P. M.: 1979, *Ap. J. (Letters)*, 234, L83.
Tonry, J. M., and Davis, M.: 1981, *Ap. J.*, 246, 680.
Tully, R. B.: 1982, in preparation.
Tully, R. B., and Fisher, J. R.: 1976, in *Proceedings IAU Colloquium No. 37, Décalages vers le Rouge et Expansion de l'Univers* (Paris: C.N.R.S.), p. 95.
____________.: 1977, *Astr. Ap.*, 54, 661.
Tully, R. B., Mould, J. R., and Aaronson, M.: 1982, *Ap. J.*, 257, 527.
van den Berg, S.: 1981, *Ap. J. (Letters)*, 248, L9.
de Vaucouleurs, G.: 1958, *A. J.*, 63, 253.
____________.: 1982, *Ap. J.*, 253, 520.
de Vaucouleurs, G., Buta, R., Bottinelli, L., Gouguenheim, L., and Paturel, G.: 1982, *Ap. J.*, 254, 8.
de Vaucouleurs, G., Peters, W. L., Bottinelli, L., Gouguenheim, L., and Paturel, G.: 1981, *Ap. J.*, 248, 408.
White, S. D. M., and Silk, J.: 1979, *Ap. J.*, 231, 1.
Yahil, A., Sandage, A., and Tammann, G. A.: 1980, *Ap. J.*, 242, 448.