

Preface

Field theory is the framework with which one describes the theory of the standard model of elementary particles and their interactions. The electromagnetic sector (QED) of the standard model is understood extremely well using perturbation theory, but the color interaction (QCD) which is responsible for hadron physics can only be accounted for perturbatively for a limited set of observational data. Due to the fact that at long distances the color interaction is strongly coupled, one cannot reliably apply perturbative methods to extract, for instance, the spectrum of the hadrons. The arsenal of tools to handle strongly coupled systems is obviously much more limited than the one used for weakly coupled ones. Nevertheless, several methods to handle non-perturbative field theories have been developed. The main goal of this book is to expose the reader to those techniques and to describe their applications in two-dimensional and four-dimensional field theories and finally in QCD in four dimensions.

The topic of non-perturbative field theory is by itself very rich and it is clear that one cannot cover it in a non superficial manner in one book. Thus we had to make certain decisions about the flow of the book and about the topics that should be addressed. As for the former issue we have decided to present the book in three parts. In the first part we describe, in detail, the most important non-perturbative techniques of two-dimensional field theory. The reason for this is obvious since physical systems with one space dimension and one time dimension are the simplest and hence it is easier to grasp the non-perturbative tools when applied to these systems. In the second part of the book we study two-dimensional gauge theories with the emphasis on employing the techniques developed in the first part. The third part is devoted to the non-perturbative aspects of gauge dynamics in four dimensions. In this part we elevate the techniques of the first part to four dimensions and we examine to what extent gauge theories in four dimensions behave like their two-dimensional simplified analogs.

There are several books on the shelves discussing non-perturbative methods in general such as [66] and [182], there are books describing one particular method, like conformal field theory in two dimensions for instance [77], there are books that describe two-dimensional QCD, [2] and books that study various aspects of four-dimensional QCD, for example [151] and of course there are books on the basics of field theory, for example [37], [130], [173] and [215]. The aim of this book is three-fold, to review a package of non-perturbative methods, to present a picture which is close to the state-of-the-art in the topics described and to

demonstrate application of the methods in addressing several questions of gauge dynamics.

The particular methods we explore in Part 1 of the book associate with conformal field theory, with affine Lie algebras, with topological properties of fields, solitons and integrable models, with bosonization and with the large N approximation.

In Part 2 we first present the basics of gauge field theories in two dimensions and in particular the bosonized version of them, we then describe the seminal large N solution of 't Hooft of the mesonic spectrum of two-dimensional QCD; we address the mesonic spectrum using current algebra methods, we describe the discrete light-cone quantization of QCD with quarks in the fundamental representation and also adjoint quarks, we compute the spectrum of baryons and their properties in the strong coupling limit, we discuss the issue of confinement versus screening behavior, we analyze QCD_2 using coset model and BRST techniques, and finally we digress and devote a chapter to generalized Yang–Mills theory on Riemann surfaces and their stringy nature.

In Part 3 we demonstrate the applications in four-dimensional gauge dynamics of conformal invariance, techniques of integrable models, of large N expansion and of topology. In particular we devote chapters to Skyrmions, magnetic monopoles and gauge theory instantons.

As we have mentioned above we had to take decisions about what topics related to non-perturbative field theory we should not include. We decided not to address string theories, supersymmetric field theories and the holographic string (gravity)/gauge duality. The main reason for this decision was that to cover each of these topics requires a book in itself, or even more than one book. In fact certain subjects that we do cover in the book, like conformal field theory, magnetic monopoles or instantons would require a full book to cover properly. What we have tried to achieve is to describe the basic ideas of each topic and to demonstrate its application. We have also not treated subjects like anomalies, lattice formulations, sigma models, chiral Lagrangians and other non-perturbative topics.

Some topics described in the book are “fully established topics”, in the sense that presumably the most important developments in those have been already achieved, for instance conformal field theory in two dimensions and bosonization in two dimensions. On the other hand some other topics of the book are under current intensive investigation and are certainly still not fully established. An example of the latter is integrability in four-dimensional gauge dynamics. The reason we have decided to include topics of the latter kind is that we wanted the book to be fairly up to date and useful to researchers investigating “modern” topics.

In the more basic issues we have made an effort to present the material in a pedagogical manner and to be self contained. For instance our discussion started from a free massless scalar field theory in two dimensions and gradually evolved

into general conformal field theories. In dealing with more advanced topics, like for instance instantons in four dimensions, the reader will need to consult with specialized references to obtain a more complete and wider picture of the topic.

Some of the content of the book, mainly in Part 2, is based on the research work of the authors, but most of the material is a review of the work of many researchers in the field.

The book is aimed for advanced Ph.D. students, post-docs and other newcomers to the arena of non-perturbative methods in field theory. The reader should definitely be equipped with a basic knowledge of field theory, group theory and algebra, differential equations, geometry and topology.

Throughout the book we refer to only a limited list of references. The number of scientific contributions to the topics discussed in this book is enormous and since we could not cover all of them we have referred to papers that initiated the various topics, and to review papers and books where a much more exhaustive list of references can be found.

We have made an attempt to keep the same notations throughout the book. However in certain instances we have changed notations during the course of the book, mainly to be in accordance with relevant literature. In these cases we specified explicitly the change in notation made.