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Entropy in universes evolving from initial to final de Sitter eras

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Abstract. This work studies the behavior of entropy in recent cosmological models that start with an initial de Sitter expansion phase, go through the conventional radiation and matter dominated eras to be followed by a final de Sitter epoch. In spite of their seemingly similarities (observationally they are close to the Λ-CDM model), different models deeply differ in their physics. The second law of thermodynamics encapsulates the underlying microscopic, statistical description, and hence we investigate it in the present work. Our study reveals that the entropy of the apparent horizon plus that of matter and radiation inside it, increases and is a concave function of the scale factor. Thus thermodynamic equilibrium is approached in the last de Sitter era, and this class of models is thermodynamically correct. Cosmological models that do not approach equilibrium appear in conflict with the second law of thermodynamics. (Based on Mimoso & Pavon 2013)

Keywords. equation of state, cosmology: theory, miscellaneous

1. Introduction

Macroscopic systems tend spontaneously to thermodynamic equilibrium. This constitutes the empirical basis of the second law of thermodynamics: The entropy, S, of isolated systems never decreases, $S' \geqslant 0$, and it is concave, $S'' < 0$, at least in the last leg of approaching of the equilibrium (see, e.g. Callen 1960). The second law of thermodynamics encapsulates the underlying microscopic, and statistical description, and hence is an important tool to investigate the consistency of cosmological models.

We have compelling reasons to believe that there was a primordial stage of inflation, and observations revealed another late stage of accelerated expansion. Models interpolating between two end stages dominated by a cosmological constant are therefore of manifest interest. The idea that the universe may have started from an instability of a de Sitter stage can be traced back to Barrow (1986)'s deflationary universe, to Prigogine *et al.* (1989), as well as to Carvalho *et al.* (1992) and Lima & Maia (1994) phenomenological models of irreversible particle production, or equivalently of dissipative effects as discussed by Lima & Germano (1992). The initial de Sitter space exists for the decay time of its constituents. Subsequently, the universe transits its normal radiation and matter dominated eras, to become finally attracted to a new, late de Sitter stage. The details of these Λ decaying models and transitions have been the endeavor of various proposals, e.g. Carneiro (2006), Carneiro and Tavakol (2009), Basilakos *et al.* (2012) and references therein. Recently two models starting and ending in de Sitter eras were put forward in Lima *et al.* (2012) and in Lima *et al.* (2009). We denote these latter models I and II, respectively. Both assume a spatially flat Friedmann-Robertson-Walker metric, and from the observational viewpoint they are very close to the conventional ΛCDM model. The physics behind the models is deeply different. While model I rests on the production of particles induced by the gravitational field (and dispenses altogether with dark energy), model II assumes dark energy in the form of a cosmological constant that in reality varies with the Hubble factor in a manner prescribed by quantum field theory.

Here we report on our findings of Mimoso & Pavon (2013), where have addressed the consistency from the perspective of the second-law of thermodynamics of the latter models promoting a cosmological transition from de Sitter initial and final stages. The laws of thermodynamics set macroscopic criteria that should be met by sound cosmological models.

2. The entropy

The entropy S of the universe is the entropy of the apparent horizon, $S_h = k_B\,\mathcal{A}/(4\,\ell_{pl}^2)$, plus the entropy of the radiation, S_γ, and/or pressureless matter, S_m, inside it, where $\mathcal{A}$ and ℓ_{pl} denote the area of the horizon and Planck's length, respectively (see Radicella & Pavon 2012 and references therein). The area of the apparent horizon

$$\mathcal{A} = 4\pi\tilde{r}_{\mathcal{A}}^2\,, \tag{2.1}$$

where $\tilde{r}_{\mathcal{A}} = (\sqrt{H^2 + ka^{-2}})^{-1}$ is the radius of the horizon. Accordingly, for a flat model, the entropy of the apparent horizon is $S_h = k_B\,\pi/(\ell_{pl}\,H)^2$, as the universe transits from de Sitter, $H = H_I$, to a radiation dominated expansion. In turn, the evolution of the entropy of the radiation fluid inside the horizon can be determined with the help of Gibbs equation (Callen 1960)

$$T_\gamma\, dS_\gamma = d\left(\rho_\gamma\,\frac{4\pi}{3}\tilde{r}_{\mathcal{A}}^3\right) + p_\gamma\, d\left(\frac{4\pi}{3}\,\tilde{r}_{\mathcal{A}}^3\right)\,, \tag{2.2}$$

On the other hand, for the entropy of dust matter, it suffices to realize that every single particle contributes to the entropy inside the horizon by a constant bit, say k_B. Then,

$$S_m = k_B\,\frac{4\pi}{3}\tilde{r}_{\mathcal{A}}^3\, n\,, \tag{2.3}$$

where the number density of dust particles obeys the conservation equation

$$n' = (n/(aH))[\Gamma_{dm} - 3H] < 0 \tag{2.4}$$

with the decay into matter characterized by the rate $\Gamma_{dm} = 3H_0^2\,\tilde{\Omega}_\Lambda/H > 0$.

In model I the creation rate of massless, radiation particles is given by $p_c = -(1 + w)\rho\Gamma_r/(3H)$. One derives $H = H(a)$, and subsequently $S(a)$, $S'(a)$, $S''(a)$, and $T(a)$ from the above expressions for S_h, S_γ and S_m We find that in the phenomenological model of Lima *et al.* (2012) the universe behaves as an ordinary macroscopic system (Radicella & Pavon 2012); i.e., it eventually tends to thermodynamic equilibrium characterized by a never ending de Sitter expansion era with $H_\infty = H_0\,\sqrt{\tilde{\Omega}_\Lambda} < H_0$.

In the model II (Lima *et al.* 2009) it is assumed that in quantum field theory in curved spacetime the cosmological constant is a parameter that runs with the Hubble rate in a specified manner (see Parker & Toms 2009 and Solá 2011):

$$\Lambda(H) = c_0 + 3\nu H^2 + 3\alpha\frac{H^4}{H_I^2}\,, \tag{2.5}$$

where c_0, α and ν are are constant parameters of the model. The absolute value of the latter is constrained by observation as $|\nu| \sim 10^{-3}$. At early times the last term dominates, and at late times ($H \ll H_I$) it becomes negligible whereby (2.5) reduces to $\Lambda(H) = \Lambda_0 + 3\nu(H^2 - H_0^2)$ with $\Lambda_0 = c_0 + 3\nu H_0^2$.

Proceeding to equate the relevant quantities in terms of the scale factor, we find that, as in the previous model, $S'_h > 0$ and $S''_h < 0$. However at variance with it, the matter entropy, $S_m = k_B \frac{4\pi}{3} \tilde{r}_{\mathcal{A}}^3 n \propto H^{-3} n$, decreases with expansion and is convex. This is so because, in this case, the rate of particle production, Γ_{dm}, goes down and cannot compensate for the rate of dilution caused by cosmic expansion. Nevertheless, S'_h and S''_h dominate over S'_m and S''_m, respectively, as $a \to \infty$. Thus, as in model I, the total entropy results a growing and concave function of the scale factor, in the far future stage. Hence, the universe gets asymptotically closer and closer to thermodynamic equilibrium.

3. Conclusions

In both models considered, the entropy, as a function of the scale factor, never decreases and is concave at least at the last stage of evolution, signaling that the universe is finally approaching thermodynamic equilibrium. So, we conclude that models I and II show consistency with thermodynamics, and that their overall behavior can be most easily understood from the thermodynamic perspective. Further, these results remain valid also if quantum corrections to Bekenstein-Hawking entropy law are incorporated.

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