LONG-TERM AMPLITUDE AND PERIOD VARIATIONS OF δ SCUTI STARS: A SIGN OF CHAOS?

M. BREGER
Institut für Astronomie, University of Vienna,
Türkenschanzstr. 17, A-1180, Wien, Austria

Abstract. On short time-scales of under a year, the vast majority of δ Scuti stars studied in detail show completely regular multiperiodic pulsation. Nonradial pulsation is characterized by the excitation of a large number of modes with small amplitudes. Reports of short-term irregularity or nonperiodicity in the literature need to be examined carefully, since insufficient observational data can lead to an incorrect impression of irregularity. Some interesting cases of reported irregularities are examined.

A few δ Scuti stars, such as 21 Mon, have shown stable variations with sudden mode switching to a new frequency spectrum. This situation might be an indication of deterministic chaos. However, the observational evidence for mode switching is still weak.

One the other hand, the case for the existence of long-term amplitude and period changes is becoming quite convincing. Recently found examples of nonradial pulsators with long-term changes are 4 CVn, 44 Tau, τ Peg and HD 2724. (We note that other δ Scuti pulsators such as X Cae and θ^2 Tau, have shown no evidence for amplitude variations over the years.) Neither the amplitude nor the period changes are periodic, although irregular cycles with time scales between a few and twenty years can be seen. While the amplitude changes can be very large, the period changes are quite small. This property is common in nonlinear systems which lead to chaotic behavior. There exists observational evidence for relatively sudden period jumps changing the period by about 10^{-5} and/or slow period changes near $dP/dt \leq 10^{-9}$. These period changes are an order of magnitude larger than those expected from stellar evolution.

The nonperiodic long-term changes are interpreted in terms of resonances between different nonradial modes. It is shown that a large number of the nonradial acoustic modes can be in resonance with other modes once the mode interaction terms, different radial orders and rotational m-mode splitting are considered. These resonances are illustrated numerically by the use of pulsation model. Observational evidence is presented that these interaction modes exist in the low-frequency domain.

1. Are the Variations of δ Scuti Stars Really Periodic?

The majority of the small-amplitude variables show nonrepetitive light curves and the periods reported by early investigators were cycle-count periods. The important question arises whether this seeming irregularity is really caused by irregular (possibly chaotic) variability or simply the combination of the light curves of several strictly periodic multiple pulsation modes.

The difficulty of distinguishing between the two hypotheses for each star is caused by insufficient observational data in both quantity and quality. Insufficient data may favor either of the two hypotheses in an unpredictable manner. Irregular variability might be deduced if limited data does not permit the detection of the multiple periods present. One the other hand, even for variability which is not strictly periodic, short data sets might be fit by a set of periods regarded as real, constant pulsation periods. The multiple

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periods seen in this example would fit only this data set and never be seen again. This observational dilemma can be avoid not only by the collection of long data sets (possibly from multiple site to avoid aliasing), but also by carefully examining power spectra of independent data sets.

For most δ Scuti stars discovered so far too little is known in order to discuss the regularity or stability of the periods of variation. For about thirty small-amplitude variables some conclusions are possible and the case for irregular pulsation looks very weak. This can already be seen in the power spectra, where the widths of the individual peaks due to the different pulsation modes generally correspond to the widths expected from the length and spacing of the observational data. This is strong evidence in favor of stable and nondrifting periods, at least over time scales of several months, possibly even years. A good example of such a nonradial pulsator is the star θ^2 Tau (Breger et al. 1989).

In a series of papers (e.g. Morguleff, Rutily and Terzan 1976a) deduced irregular variability for some δ Scuti stars. For 14 Aur they state 'no periodicities are present in the pulsation'. In our view, they should have considered multiple modes as a more serious alternative for all their stars. Fitch and Wisniewski (1979) presented new photometry and an independent analysis of 14 Aur. They could describe the pulsation in this close binary system by nonradial modes. We have analyzed seven nights of unpublished photometry of 14 Aur obtained at McDonald Observatory and confirm the regularity reported by Fitch and Wisniewski.

Another star studied by Morguleff, Rutily and Terzan (1976b) was 44 Tau for which they also found no periodicities. Again, with this star we are dealing with a complex nonradial pulsator. Poretti, Mantegazza and Riboni (1991) present 25 new nights of photometry and present seven frequencies of pulsation. Since their set of nonradial pulsations can also fit the previous data, this star also provides no evidence for the existence of irregular pulsation in δ Scuti stars.

2. Mode Switching

One could also imagine a situation, in which the pulsation appears regular for time spans of several years before it becomes unstable and period (or mode) switching occurs. More than ten years ago, Kurtz (1980) examined the stability of the observed periods in δ Scuti stars. Except for the star 21 Mon, he found the previously reported period switching to be unconvincing.

Another example for possible mode switching is the star HN CMa (Breger, Balona and Grothues 1991). Between 1981 and 1990 the star became almost constant with a 'new' dominant frequency. In their paper, the authors emphasize that a conclusion of actual mode switching is not necessary, if one considers the long-term variability of nonradial pulsation amplitudes (see next section) in a star with small amplitudes near the limit of detectability.

A further reported example of possible variable frequency spectra is FM Com. While Antonello et al. (1985) could fit their observed light variations with three frequencies, on the basis of new photometry Paparo and Kovacs (1984) argue in favor of variable frequency spectra. This star might also be explained by variable amplitudes leading to an appearance of mode switching and we refer to another paper in this journal.

The previous discussions dealt with stars for which 'interesting' irregular variations had been reported. The discussion should not create the impression that suspected irregular behavior is the rule. One the contrary, the majority of the δ Scuti stars studied in detail show multiperiodic variations which are remarkably constant from cycle to cycle, or even year to year. The small observed variations in the periods ($dP/dt \sim 10^{-9}$) are within a one or more orders of magnitudes of the expected evolutionary effects.

3. Long-Term Amplitude and Period Variations

The recent years have seen a large improvement in the observational data of δ Scuti stars. The change was motivated by the realization of the complexity of the nonradially pulsating stars, which led to a concentration on selected stars and long observations (often multisite in order to decrease the aliasing problem) over a single observing season. Among the nonradial pulsators the frequency spectra of about two dozen stars can be regarded as understood.

The determination of the long-term constancy or variability of the individual amplitudes associated with individual pulsation modes requires another quantum jump in the quantity of data required: sufficient data for a multiple frequency solution has to be available for at least a second observing season. Over 100 nights of observation would not be an unrealistic requirement. The verification of constant amplitudes in a multiperiodic pulsator is probably one of the most difficult tasks in this kind of work. We note here that the appearance of variable amplitudes may be a sign of insufficient data as well as true amplitude variability. Table I lists δ Scuti stars whose reported amplitude variability we regard as reasonably reliable; the selection is probably neither complete nor perfect.

We note that the typical time scales of amplitude and period variations are years. However, even a single star can show a wide variety in behavior for its different pulsation modes. This is demonstrated by the star 4 CVn (see Breger 1990), but other stars have shown this variety as well:

(i) the amplitudes of some pulsation modes appear essentially constant for ten or even twenty years,

(ii) some amplitude variations, such as decreases in size, are steady over twenty years,

(iii) a steady increase in amplitude can be followed by a very rapid col-

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TABLE I
Some δ Scuti star whose pulsation modes have variable amplitudes with long time scales

Star	Frequency (cycles per day)	Amplitude range (mag)	Time scale	Ref.
GN And	single	0.007 to 0.022	~15 years	1
τ Peg	single	$\leq$0.005 to 0.012 irregular	decade(s)	2
4 CVn	f_1, 8.60	0.011 to 0.023	decade(s)	3
	f_2, 5.85	0.08 to 0.018	decade(s)	
	f_3, 5.05	0.004 to 0.025	decade(s)	
	f_5, 7.38	0 to 0.014	a few years	
44 Tau	f_2, 7.01	0.003 to 0.021	decade(s)	4
	f_7, 9.56	0.006 to 0.021	decade(s)	
BK Cet	f_1, 11.1	0.046 to 0.01 steady decline	decade(s)	5,6,7
HN CMa	f_1, 4.5	$\leq$0.001 to 0.005		8,9
HD 2724	7.38	0.012 to 0.036	a few years	10

References:
1: Garrido et al. (1985), 2: Breger (1991), 3: Breger (1990), 4: Poretti et al. (1991), 5: Lampens and Rufener (1990), 6: Poretti (1989), 7: Kurtz (1990), 8: Breger et al. (1991), 9: Baade and Stahl (1982), 10: Lampens (1991)

lapse and decrease to near zero in less than two years. However, the oscillation does not disappear and slowly starts up again. As an example, for 4 CVn the amplitude of the 7.37 cycles per day oscillation (with a possible identification as P_2, $l = 2$, $m = -1$) decreased from a reliably determined 0.014 mag in 1974 to near zero during 1976 and 1977 with a subsequent increase to an intermediate value of 0.007 mag.

This variety is typical of chaotic behavior. Nevertheless, the question also arises whether or not the observed amplitude and period variability could be the result of beating between two close frequencies. We regard this explanation as unlikely for the following reason: the observed variations can be modelled by two close frequencies only for stars where little data from different years are available. For the stars with extensive data, three or more close frequencies beating with each other would be required to provide a reasonable fit to the observations. This suggests 'true' amplitude and period variability.

4. The Resonance Hypothesis as an Explanation for the Irregularities

An explanation for the amplitude variability might be found in the resonance condition between different nonradial pulsation modes. On the theoretical side, it was shown by Däppen (1985) that nonradial modes can contribute to irregular behavior more than radial modes. Moskalik (1985) extended the calculations of Dziembowski (1980, 1982) to resonant mode coupling. Takeuti (1990) showed that nonlinearly coupled oscillations can show apparent period switching and amplitude variations, while a radial resonance condition ($P_o/P_3 = 2.0065$) was examined by Takeuti and Zalewski (1991).

There exists a variety of different resonance conditions which can lead to amplitude and period variations. We note that resonances can occur from interaction modes originating from the different pulsations. Nonradial pulsation naturally produces a large number of possible resonances through the interaction terms ($\nu_k - \nu_{k-1}$). Let us demonstrate this with an example using a model of a typical δ Scuti star of 2 solar masses, 7340 K, 3.53 solar radii, a rotation period of 5 days and $l = 2$ modes. For the pulsational frequencies we use the models of Fitch (1981), while the rotational effects including the second order were taken from Saio (1981). Despite criticism of the Fitch modes (e.g. see Dziembowski and Krolikowska 1990) we use these models because the $m = 0$ frequencies agree with the observed values for the star 4 CVn, and because the demonstration of the interaction terms is insensitive to the model used. In fact, for radial and nonradial pulsation, at high radial orders the frequency spacing, ($\nu_k - \nu_{k-1}$), becomes equidistant.

Figure 1 shows the computed frequency spectrum in both the nonrotating frame (observer) and the rotating stellar frame. We note that in the rotating frame the rotational frequency splitting is, of course, quite small and the second-order effect becomes relatively large.

Now consider the interaction modes, ($\nu_k - \nu_{k-1}$). Even for different values of m, the frequencies of these interaction terms lie in a narrow range. Due to the large number of possible interaction terms (consider the range in k and m values even at a constant l), a number of different resonances can occur. These may fulfill the conditions of deterministic chaos. We note in this regard that the variations in amplitude can be very large, while the effects on the periods are relatively small (less than 10^{-4}).

The present explanation predicts the existence of interaction terms with frequencies ($\nu_k - \nu_{k-1}$). We would expect that even if the amplitude of the individual interaction terms were too small to be observed, the resonances would kick some interaction term towards detectability.

These interaction terms may already have been observed. We refer to the star 1 Mon (Shobbrook and Stobie 1974), the large-amplitude pulsator Al Vel (Walraven, Walraven and Balona 1991), and 4 CVn. For the star X

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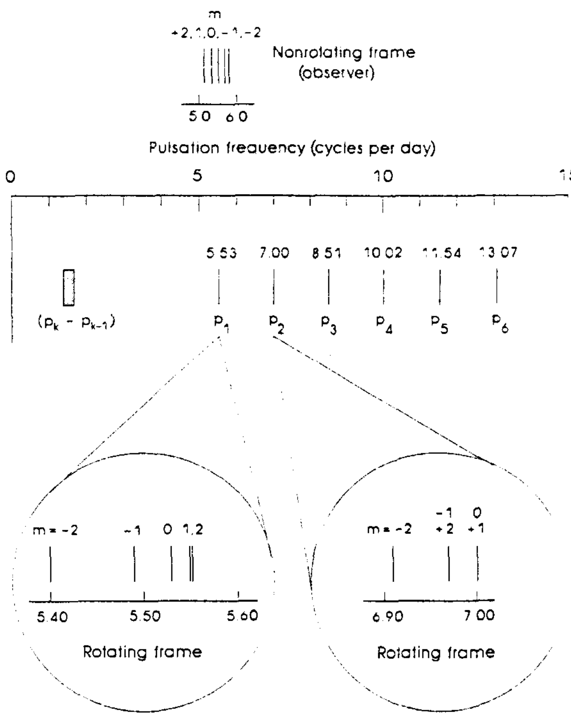

Fig. 1. Frequency spectrum for the nonradial acoustic $l = 2$ pulsation modes computed from the Fitch (1981) model 2.0M48 with first and second-order rotational effects by Saio (1981). A rotational frequency of $\Omega = 0.2$ revs per day was assumed. The results are shown for the frames of reference of both the star and the observer.

Cae, Mantegazza and Poretti (1991) present evidence in favor of coupled oscillations and excitation by resonance (though no difference terms and no amplitude variations). Furthermore, there are a number of papers in which small frequencies outside the p-mode range have been reported. Whether these frequencies are caused by observational errors, g-modes, rational effects, or interaction terms is still an open question. Examples are HR 8210 (0.64 and 0.34 cycles per day, Kurtz 1979); HD 93044 (2.146 cycles per day, Li Zhi-ping et at. 1991). Finally, we would like to present some evidence for a mode interaction term in 4 CVn for the year 1974, for which 27 nights of unpublished photometry by Fitch is available. If the seven-frequency solution in the 5 to 9 cycles per day range applicable for 1966 to 1984 (Breger 1990) is subtracted, very strong power near 1.4 cycles per day remains (Figure 2). This value (or one of its aliases) can be fit by several differences of observed p-modes. For the other years the amplitude of the interaction mode is weak.

Further work should therefore concentrate more on reliable observations in the 0 to 3 cycles per day frequency domain in order to establish the

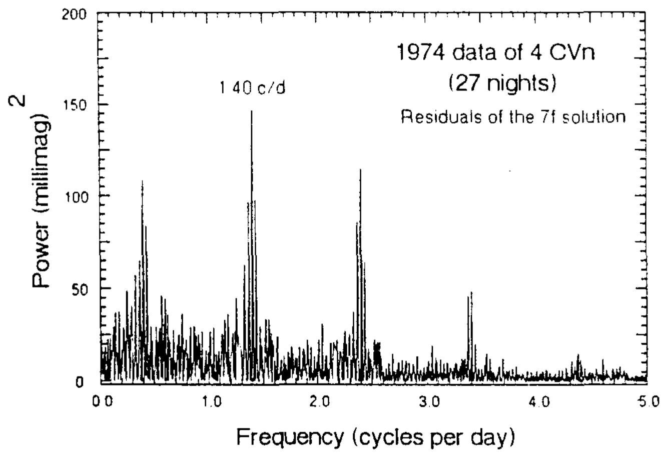

Fig. 2. Observational evidence for low-frequency power in unpublished data of 4 CVn by Fitch. The overall solution has already been subtracted. We note that the power near 1.40 cycles per day is also present in the raw data. This frequency (or one of its aliases) is interpreted here as an interaction mode with a large amplitude temporarily pushed up by resonance.

link between amplitude variability and interaction-term resonances. It might even be possible to observe one of the 'kicks' which lead to transfer of power to different pulsation mode.

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