# DETERMINATIONS OF PRECESSION

*(Invited Review Paper)*

WALTER FRICKE

*Astronomisches Rechen-Institut, Heidelberg, D.B.R.*

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**Abstract.** Recent determinations of lunisolar precession and of the motion of the equinox are reviewed. Methods of determination are discussed which are based on proper motions referred to fundamental systems, on planetary motions, and on proper motions referred to galaxies. It is concluded that a new fundamental catalogue, which will replace the FK4 at some future date, should be based on revised values of precession and freed from errors in the motion of the equinox.

## 1. Introduction

The value of the general precession in longitude adopted by the Conférence Internationale des Etoiles Fondamentales de 1896 in Paris (1896) is the value $p$ resulting from Newcomb's (1897) investigation of the proper motions of Auwers-Bradley stars ($p = 5025''.64$ per tropical century, for 1900). By international convention Newcomb's precession is still in use at the present time.

A revision requires the knowledge of the following 'precessional corrections'

$\Delta p$, correction to Newcomb's general precession,

$\Delta p_1$, correction to lunisolar precession,

$\Delta\lambda$, correction to planetary precession,

$\Delta e$, correction to all right ascensions as a consequence of an error of Newcomb's motion of the equinox.

Fictitious proper motions caused by these four errors are

$$\begin{aligned} \mu_\alpha &= \Delta k + \Delta n \sin\alpha \tan\delta\,, \\ \mu_\delta &= \Delta n \cos\alpha \end{aligned} \tag{1}$$

where, in the usual notation, with $\varepsilon$, obliquity of the ecliptic,

$$\begin{aligned} \Delta k &= \Delta m - \Delta e \\ \Delta m &= \Delta p_1 \cos\varepsilon - \Delta\lambda \\ \Delta n &= \Delta p_1 \sin\varepsilon \end{aligned} \tag{2}$$

and from these

$$\Delta k = \Delta n \cot\varepsilon - (\Delta\lambda + \Delta e)\,. \tag{3}$$

Therefore, from proper motions of stars the two quantities $\Delta p_1$ and $\Delta\lambda + \Delta e$ can be derived. The correction $\Delta\lambda$ to planetary precession can be determined on the basis of

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a set of improved values for the masses of the planets. With the knowledge of $\Delta p_1$ and $\Delta\lambda$, the correction to Newcomb's general precession is determined by

$$\Delta p = \Delta p_1 - \Delta\lambda \cos\varepsilon . \tag{4}$$

Experience has shown that $\Delta\lambda$ is small compared with $\Delta p_1$ and with $\Delta e$. Three methods are available for the determination of the precessional corrections: (a) Analysis of proper motions of stars given in reference systems which have adopted Newcomb's precession; (b) Analysis of planetary motions; (c) Comparison of proper motions of stars referred to galaxies with proper motions of the same stars determined in systems with Newcomb's precession.

## 2. A Survey of Older Determinations

Determinations of precession carried out from 1925 to about 1960 were reviewed by Böhme and Fricke (1965) at *IAU Symp.* **21** held in Paris in 1963. Most of these determinations are based on proper motions, and only a few on investigations of planetary motions.

The results are values for the correction $\Delta p_1$ to lunisolar precession in the interval $+0\overset{''}{.}65$ to $+1\overset{''}{.}50$ per century and corrections $\Delta\lambda + \Delta e$ in about the same range. Some of these determinations may be excluded from further consideration due to obvious deficiencies; among these are investigations whose primary purpose was not an improvement of Newcomb's values. Concerning the rest, Fricke (1965) was of the opinion that it is not possible to make a selection of these determinations with which a reliable decision could be made concerning the best possible values of the corrections for reasons given below.

Other authors made attempts to estimate the most probable values on the basis of older determinations. Gordon (1952) assigned weights to the results obtained by 13 authors on the basis of proper motions in different systems (Auwers', PGC, GC and FK3); the mean weighted values given in his paper are

$$\Delta p_1 = +\, 1\overset{''}{.}09 \pm 0\overset{''}{.}03 \quad \text{per century},$$
$$\Delta\lambda + \Delta e = +\, 1\overset{''}{.}24 \pm 0\overset{''}{.}03 \quad \text{per century}.$$

Morgan and Oort (1951) made a rediscussion of various determinations by reducing previous results to the average of the systems of FK3 and N30. The reduction was made by the differential method first suggested by Schilt (1928). This method gives differential corrections $\Delta(\Delta p_1)$ and $\Delta(\Delta\lambda + \Delta e)$, etc. from catalogue differences directly. It is applicable on the condition that important characteristics of primary solutions, such as the distribution of stars in the sky and weights, be taken into account. Morgan and Oort, whose method deviates from rigorous procedure, give

$$\Delta p_1 = +\, 0\overset{''}{.}75, \qquad \Delta\lambda + \Delta e = +\, 1\overset{''}{.}09 \quad \text{per century}$$

(without stating errors) as the most likely values on the basis of the averaged FK3/N30 system; they recommended these values for application to proper motions in investigations of galactic kinematics, but not for a revision of the conventional values of Newcomb's precession.

Since 1964 the system of FK4 has been in use as the internationally adopted reference frame for observations. The completion of the FK4 suggested new investigations and an attack on the following open problems:

(1) The basic observational data used in previous determinations were either obtained in one of the older reference systems GC, FK3 and N30, or were reduced to one of these, or to an average system formed from two of them. A new determination on the basis of the latest system, the FK4, appeared to be necessary together with an investigation of the effects of the various systems on the solutions.

(2) Some of the older solutions were derived by application of the differential method without taking into account the distribution of the stars in the sky in the primary solution, different weights assigned to proper motions in different parts of the sky, etc. The effects of neglecting these important characteristics may be considerable and must be investigated.

(3) Solutions based on observations in limited regions of the sky (e.g. selected declination zones and the region of the galactic plane) may be more affected by regional systematic errors of the reference frame than solutions based on material from the whole sky. The effects of systematic errors have to be considered.

(4) There may exist other rotational effects in the motions of the stars than those assumed in the current model of galactic rotation, and there may be other unidentified rotations in the reference frame of observations.

(5) An attempt to trace the origin of the errors in Newcomb's determination appeared to be justified.

(6) The solutions obtained from investigations of planetary motions require confirmation by more detailed determinations.

(7) Forthcoming proper motions of stars measured with respect to galaxies may permit the derivation of precessional corrections from the comparison with motions determined in the fundamental system which are affected by precessional errors.

Recent investigations, which are reviewed in the following sections, have given contributions which solve some of these problems satisfactorily and others which indicate the direction for further research.

## 3. Recent Determinations on the Basis of Proper Motions Referred to Fundamental Systems

### 3.1. Results of a Reinvestigation of the McCormick and Cape Proper Motions of Faint Stars

Basic material for this investigation carried out by Fricke (1967a) were the McCormick and Cape proper motions of stars of average photovisual magnitude 11.1 presented and analysed by Williams and Vyssotsky (1947) in the FK3 system. The McCormick material consists of 265 group means in $\mu_\alpha$ and $\mu_\delta$ formed in groups of regions with many stars. The regions are fairly uniformly distributed from the north pole to declination $-25°$. The Cape material consists of 48 mean motions formed in the declination zone $-40°$ to $-52°$ at intervals of half an hour. For the purpose of the reinvestigation, all 313 mean proper motions were reduced individually to the systems of N30 and FK4.

Following the same procedure adopted by Williams and Vyssotsky, that is using the same weights and parallax factors, solutions were carried out in each of the systems FK3, N30, and FK4. It was found that the correction $\Delta p_1$ to Newcomb's lunisolar precession derived from combined solutions including both $\mu_\alpha$ and $\mu_\delta$ has the same value in all three systems. The numerical value of the correction, however, varies sharply with the relative weight assigned to the McCormick and Cape proper motions. Williams and Vyssotsky had assigned to the Cape proper motions twice the weight of the McCormick motions.

Since there is strong indication that the reduction of the Cape motions to the FK3 is affected by serious uncertainties (assumptions and magnitude equations), and in view of the inferior accuracy of fundamental proper motions in the southern sky, one may consider the assignment of lower weight to the Cape motions as more realistic. New solutions were therefore made with equal weight for the McCormick and Cape motions and with the McCormick motions alone. In the first case, the values for $\Delta n$ per century are: $+0\overset{''}{.}55$ (FK3), $+0\overset{''}{.}55$ (N30), $+0\overset{''}{.}56$ (FK4) with equal probable error $\pm 0\overset{''}{.}03$. With McCormick proper motions alone, the values for $\Delta n$ are slightly different in the three systems: $+0\overset{''}{.}43$ (FK3), $+0\overset{''}{.}40$ (N30), $+0\overset{''}{.}47$ (FK4), probable error $\pm 0\overset{''}{.}04$. From these two sets of solutions the conclusion was drawn that the correction $\Delta p_1$ to lunisolar precession lies in the interval

$$+1\overset{''}{.}08 \pm 0\overset{''}{.}10 \leq \Delta p_1 \leq +1\overset{''}{.}38 \pm 0\overset{''}{.}08 \quad \text{per century},$$

with the actual value more likely closer to the lower than to the upper limit. This conclusion is supported by the solution from fundamental proper motions (see the next section). The interval for the most probable values for $\Delta\lambda + \Delta e$ is

$$+1\overset{''}{.}25 \pm 0.12 \leqslant \Delta\lambda + \Delta e \leqslant +1\overset{''}{.}52 \pm 0\overset{''}{.}10 \quad \text{per century},$$

with the actual value more likely closer to the lower limit for the same reasons.

This investigation has also shown that the solution on the basis of N30 differs noticeably from that of Morgan and Oort (1951) and of Morgan (1952, *Introduction to* N30, p. XXIII), who had applied the differential method in reductions between GC, FK3, and N30. The divergences are fully explained by the improper disregard of the peculiarities in the distribution of stars over the sky, and of the weights of proper motions in the primary solution. The same objection holds for an investigation by Boshniakovich (1965).

Finally, the analysis of the proper motions $\mu_\alpha$ and $\mu_\delta$, each separately and both combined, has pointed out a serious error in some previous investigations which averaged the separate results from $\mu_\alpha$ and $\mu_\delta$. This procedure violates the least-squares method and cannot give results in agreement with solutions based on $\mu_\alpha$ and $\mu_\delta$ combined.

### 3.2. Results from fundamental proper motions of distant stars

The following investigation was made in order to derive precessional corrections on the basis of the FK4 directly, without utilizing proper motions from other sources reduced to the FK4, and to determine the effects of different systems on the solutions. The fundamental catalogue as a whole is not well suited for this purpose because of its high percentage of nearby stars. This investigation carried out by Fricke (1967b) was therefore based on the proper motions of 512 FK4/FK4 Sup stars with distances greater than about 100 parsec and distributed over the whole sky. The advantages of this material are: the fundamental proper motions are the most accurate known; they are less affected by errors depending on magnitude than the motions of faint stars measured on photographic plates; the motions may be taken directly from the catalogues FK4, N30, FK3, and GC for comparisons, thus avoiding errors of reduction; for most of these stars, MK spectral types (giving approximate distances) and radial velocities are known, thus permitting an examination of peculiarities of the stellar motions and of the photometric distance scale.

Solutions for the solar motion (taking into account parallax factors), for the precessional corrections, and for the parameters of galactic rotation were performed with the proper motions given for these stars in GC, FK3, N30, and FK4, following strictly the same procedure.

Most surprising is the result that the correction to Newcomb's lunisolar precession turned out to have the same value, within 0"01 per century, in the systems FK3, N30, and FK4, namely

$$\Delta p_1 = +1''\!.10 \pm 0''\!.10 \quad \text{(p.e.) per century}.$$

The correction

$$\Delta\lambda + \Delta e = +1''\!.20 \pm 0''\!.11 \quad \text{(p.e.) per century}$$

is the same in FK4 and N30, while the value is slightly different in the older systems (FK3 and GC).

The results for Oort's constants of galactic rotation obtained from this investigation deserve mention. The values and their probable errors are (in units of km $s^{-1}$ $kpc^{-1}$)

$$\text{FK4: } A = +14.2 \pm 1.9, \qquad B = -11.8 \pm 1.9,$$
$$\text{N30: } A = +17.1 \pm 1.9, \qquad B = -10.0 \pm 1.9.$$

The solutions for GC and FK3 reveal deviations not greater than may be expected from the lower accuracy of these systems.

By dividing the material into stars nearer than 250 parsec (351 stars) and those farther away, it was found that the local group shows no indication of a rotation deviating from differential galactic rotation. We conclude that no serious objections against the model of the velocity distribution of the stars underlying the solutions are indicated.

### 3.3. Preliminary results from AGK3 proper motions

Dieckvoss (1968) made a statistical study of about 160000 preliminary AGK3 proper motions. The material was divided into groups according to spectral type and magnitude, and solutions for solar motion, galactic rotation and precessional corrections were performed in 42 groups. The values of the precessional corrections found by weighted averaging of the results are

$$\Delta p_1 = +\,1''\!.27 \pm 0''\!.03 \quad \text{per century, and}$$
$$\Delta\lambda + \Delta e = +\,1''\!.49 \pm 0''\!.03 \quad \text{per century},$$

where the errors are mean errors of the weighted means. The surprisingly large dispersion of the results for the solar apex, secular parallax, and Oort's constants $A$ and $B$ in the 42 groups casts, however, strong doubt on the applicability of the averaging procedure and suggests a rediscussion of the material. Until this has been performed, it appears premature to make a final statement about the precessional corrections on the basis of the AGK3.

### 3.4. A method of B. Thüring and E. Thüring for the determination of precession

Thüring and Thüring (1967) have developed their own method for the determination of precessional corrections. They have applied their method, which they claim is free of hypotheses, to the FK4 with the result that no significant correction to Newcomb's value of lunisolar precession was found.

The claim of freedom from hypotheses concerns, as far as can be seen from their analysis, the assumption of a Gaussian distribution (on which the method of least-squares is based) for the peculiar proper motion components. They propose, therefore, a method which avoids least-squares solutions and excludes the 'tail' of large proper motions in the distribution function. The determination of the precessional corrections $\Delta m$ (which may include $\Delta e$) and $\Delta n$ is accomplished by searching for the point of maximum density in the two dimensional distribution of $\mu_\alpha$, $\mu_\delta$, for a given catalogue that has adopted Newcomb's precession. Sought are the values $\Delta m$ and $\Delta n$ that provide the maximum number A of stars with proper motions about $\mu = 0$. In other words, $\Delta m$ and $\Delta n$ are determined (for instance by a graphical method) such that

$$\Delta m, \Delta n\,[\mathrm{A}\,(\mu = 0) = \max]\,.$$

In their application to the stars of the FK4, they deliberately disregarded the existence of parallactic components in the proper motions (which is acceptable in principle, but uneconomical in practice), and they did not consider contributions to the rotational components other than those due to incorrect precession. Nevertheless, the method of Thüring and Thüring may yield the determination of the rotational components without application of the hypothesis of a Gaussian distribution of peculiar proper motions. The determination of precessional corrections from the rotational components necessitates, however, the application of hypotheses; and the assumption that the rotational components are the precessional corrections is unrealistic. On the other hand, the neglect of galactic rotation by the Thürings can produce only a small effect as can be seen from the following equations due to Fricke (1967b), relating the precessional corrections $\Delta n$, $\Delta\lambda + \Delta e$, and Oort's constant $Q$ with the rotational components $\omega_1$, $\omega_2$, $\omega_3$ (in the usual equatorial coordinate system) for 1950 :

$$\left.\begin{aligned} \omega_1 &= -\,0.8676\,Q\,, \\ \omega_2 &= -\,\Delta n - 0.1884\,Q, \\ \omega_3 &= \Delta n \cot\varepsilon - (\Delta\lambda + \Delta e) + 0.4602\,Q, \end{aligned}\right\} \quad \text{for} \quad 1950.0 \tag{5}$$

where $Q$ is of the order of $-0\rlap{.}''20$ per century. (According to Fricke (1967b), $\omega_1 = +0\rlap{.}''22$, $\omega_2 = -0\rlap{.}''39$, and $\omega_3 = -0\rlap{.}''34$ in the FK4. Note the change of the sign of $\omega_1$ and $\omega_2$ compared with the original paper. This is due to the adoption of a right-hand coordinate system for the equations above).

The large difference between Thüring and Thüring's results and those of Fricke and of other authors cannot be explained by the non-Gaussian distribution of peculiar proper motions which is principally due to star streaming and regional systematic errors. While no method can eliminate regional errors, the effects of star streaming were investigated by Fricke (1967b), and found to be negligibly small. There remains the explanation, that Thüring and Thüring's result may, however, be affected by the disregard of parallactic motions. The disregard considerably diminishes the number of stars, called by Thüring *Träger des Fundamentalsystems* (stars with proper motions smaller than $0\rlap{.}''1$ per century). The method has been discussed here in detail because its underlying principals are of interest, although the method is less powerful than the conventional one.

### 3.5. Aoki's Hypothesis of an Excess Secular Motion of the Equator

As has been pointed out in the previous section, the interpretation of the rotational components $\omega_1$, $\omega_2$, $\omega_3$ demands the application of hypotheses. While it is certain that the components represent the cumulative effect of various physically real and some spurious rotations

$$\omega_i = \sum_k \omega_{ik}\,, \tag{6}$$

we do not know how many contributing rotational vectors exist. There is general agreement on the contributions given by the precessional errors, by the error $\Delta e$ in the motion of the equinox, and by galactic rotation. The Equations (5) contain only these effects, and they do not permit a solution for more than the unknowns $\Delta n$, $\Delta\lambda + \Delta e$ and $Q$. If another rotational vector would exist, it would have to be determined from other sources, and the Equations (5) must be extended to include the additional rotation.

Aoki (1967, 1969) has called attention to the possible existence of a rotational motion of the Earth due to a frictional coupling between the mantle and core of the Earth. He has given arguments in favor of the assumption that an 'observed' and unexplained excess secular change, $\Delta\dot{\varepsilon}$, in the obliquity of the ecliptic is due to a disregarded motion of the equatorial pole. In fact, the discussions of observations of the

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Sun, Moon and some planets have indicated deviations from the value of the variation of the obliquity determined on the basis of Newcomb's theory of the motion of the Earth. Lieske's (1967) work has shown that Newcomb's theory is correct. Thus, the origin of a significant deviation $\Delta\dot{\varepsilon}$ has to be explained either by an erroneous treatment of the observations, by unrecognized systematic errors in the observations, or by a motion of the equator.

Aoki makes the assumption that the correction $\Delta\dot{\varepsilon}$ has to be interpreted as a rotation of the equatorial plane about an axis passing through the equinox with an angular velocity

$$\Delta\omega_1 = -0\overset{''}{.}32 \quad \text{per century,} \qquad \Delta\omega_2 = \Delta\omega_3 = 0,$$

so that the first of the Equations (5) changes into

$$\omega_1 = -0.8676\,Q + \Delta\omega_1\,. \tag{7}$$

If we adopt from Fricke's (1967b) investigation of fundamental proper motions the values (FK4, N30)

$$\omega_1 = +0\overset{''}{.}20, \qquad \omega_2 = -0\overset{''}{.}40, \qquad \omega_3 = -0\overset{''}{.}31,$$

then the solutions for $\Delta p_1$, $\Delta\lambda + \Delta e$ and $Q$ are, adopting first $\Delta\omega_1 = 0$, and then $\Delta\omega_1 = -0\overset{''}{.}32$, the following:

| | $\Delta p_1$ | $\Delta\lambda + \Delta e$ | $Q$ |
|---|---|---|---|
| $\Delta\omega_1 = 0$: | $+1\overset{''}{.}10$ | $+1\overset{''}{.}21$ | $-0\overset{''}{.}23$ |
| $\Delta\omega_1 = -0\overset{''}{.}32$: | $+1\overset{''}{.}28$ | $+1\overset{''}{.}20$ | $-0\overset{''}{.}60$. |

While the effect on the precessional corrections is small, Oort's constant is considerably affected, and, hence, doubt was cast on the present views of galactic kinematics.

Doubt about the validity of Aoki's hypothesis has, however, arisen from the following sources:

(a) In investigations of the motions of bodies in the planetary system, the ecliptic was assumed to be in motion and not the equator. The results for $\Delta\dot{\varepsilon}$ may be different in both cases, and there is no basis for adopting $\Delta\dot{\varepsilon} = -0\overset{''}{.}32 = \Delta\omega_1$.

(b) In a re-examination of the observations of Eros, Lieske (1970) found that the data do not indicate the existence of a secular error in the obliquity which is significantly different from zero.

(c) In a search for systematic errors in the observations, Fricke (paper in preparation) found that the declination systems established before 1900 differ from modern systems in the sense that a correction

$$\Delta\omega_1 = -0\overset{''}{.}2$$

should be expected in solutions from observations of the sun and planets.

(d) Analyses of proper motions of AGK3 stars measured with respect to galaxies do not confirm the effect (see Section 5).

In conclusion one may say that, although a real motion of the equator due to friction between the mantle and core of the Earth may exist, it is probably very small and near the limit of accuracy of the observations.

### 3.6. A REINVESTIGATION OF NEWCOMB'S DETERMINATION

Newcomb's (1897) famous determination of lunisolar precession was based on the proper motions, which Auwers derived for Bradley's stars. Newcomb applied some corrections which, however, were of negligibly small influence on precession. In a repetition of this determination of precession, Fricke (1971) selected from Newcomb's basic material the stars which are among the distant fundamental stars discussed in Section 3.2. The use of all of Newcomb's material would have been of no advantage because of its great percentage of nearby stars. With 265 distant stars, solutions were carried out on the basis of Newcomb's data and, for comparison, with the FK4 proper motions. The results for $\Delta p_1$ are

| | |
|---|---|
| Newcomb's data; galactic rotation neglected: | $+0\overset{''}{.}25 \pm 0\overset{''}{.}25$ (p.e.) per century, |
| Newcomb's data; galactic rotation included: | $+0\overset{''}{.}43 \pm 0\overset{''}{.}27$ (p.e.) per century, |
| FK4 proper motions; galactic rotation included: | $+1\overset{''}{.}10 \pm 0\overset{''}{.}18$ (p.e.) per century. |

As can be seen from the errors, the resulting correction $\Delta p_1 = +0\overset{''}{.}25$ to Newcomb's adopted value is not significant, and Newcomb's neglect of galactic rotation has only a small effect. The correction $+1\overset{''}{.}10$ derived from the FK4 proper motions is precisely the same as that derived from the FK4 motions of 512 distant stars distributed over the whole sky (see Section 3.2). The increase of the probable error from $\pm 0\overset{''}{.}10$ to $\pm 0\overset{''}{.}18$ results mainly from the fact that the Bradley-Auwers stars cover only part of the sky ($\delta > -25°$) and that the number is smaller.

The difference between $\Delta p_1 = +0\overset{''}{.}43$ (Newcomb corrected for galactic rotation) and $\Delta p_1 = +1\overset{''}{.}10$ (FK4) is the effect of the difference between the declination system compiled by Auwers and the FK4.

From this investigation, it appears apt to conclude that Newcomb's determination is free of serious errors and that his result is largely affected by systematic errors in the $\mu_\delta$ system compiled by Auwers (1879), even though Auwers' system was the best available at that time.

Occasionally, the question is raised as to why the constant of precession derived by Struve-Peters, and widely used in the 19th century, was nearer the truth than Newcomb's. From a study of Struve's material and method it is obvious that this fact is merely fortuitous.

## 4. Determinations on the Basis of the Dynamical Method

No satisfactory solution has so far been obtained by this method. Results obtained by

Clemence (1966) and Lieske (1970) from planetary observations indicate that precession is only weakly determined by these observations. This is best demonstrated by solutions for $\Delta p$ (the correction to Newcomb's general precession) which were carried out by Lieske from the observations of Eros. With reasonably varied conditions for the solutions (addition of two or three unknowns) the results for $\Delta p$ vary from $-2''.1$ to $+1''.6$ per century. In other investigations, the failure of the method may partly be seen in the insufficient reduction of the observations to a common or to a fundamental system. In most reductions of planetary observations, periodic errors in right ascension, which cannot be eliminated by the accumulation of observations, are neglected.

## 5. Determinations on the Basis of Proper Motions Referred to Galaxies

Although the utility of galaxies as a reference frame was recognized by Herschel (1785) and Laplace (1797), programs for measuring proper motions of stars with respect to galaxies were initiated only about twenty years ago. Following the proposals of Wright (1950) and of Deutsch (1954), programs have been carried out with the Lick astrograph by Vasilevskis (1954, 1957 and 1963) and by the Pulkovo astronomers with the normal astrograph.

Proper motions of stars determined by differential measurements with respect to galaxies, in the following denoted by $\mu_{gal}$, are free from errors due to incorrect precession and other errors which may arise from the inaccurate definition of the fundamental equatorial system. In principle, therefore, the proper motions $\mu_{gal}$ permit the determination of the velocity distribution of stars including galactic rotation free from errors of the local reference frame. If proper motions of the same stars are known with respect to galaxies and with respect to the fundamental system, the precessional errors can be determined. In the current programs, stars of the AGK3 have been included whose proper motions $\mu_{AGK3}$ are known in the system of the FK4. From the differences

$$\Delta\mu = \mu_{AGK3} - \mu_{gal} \tag{8}$$

the precessional corrections can be derived.

Preliminary results of the measurements at Pulkovo were reported by Fatchikhin (1968, 1970a). A refined analysis was made after the elimination of errors depending on magnitude and after adding a number of AGK3 stars (resulting in a total of 779 AGK3 stars in 85 areas of the northern sky). The latest results were recently published by Fatchikhin (1970b).

Precessional corrections were derived from the proper motion differences $\Delta\mu$ which, according to Equations (1), yield $\Delta k$ and $\Delta n$ from

$$\begin{aligned} \Delta\mu_x &= \Delta k \cos\delta + \Delta n \sin\alpha \sin\delta , \\ \Delta\mu_y &= \Delta n \cos\alpha . \end{aligned} \tag{9}$$

(In Fatchikhin's notation $\Delta m$ is written instead of $\Delta k$ by inclusion of $\Delta e$ in $\Delta m$).

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The results are

$$\Delta n = +\,0\overset{''}{.}41 \pm 0\overset{''}{.}08 \quad \text{(p.e.) per century},$$
$$\Delta k = +\,0\overset{''}{.}43 \pm 0\overset{''}{.}08 \quad \text{(p.e.) per century}.$$

From these the following values of the precessional corrections are given

$$\Delta p_1 = +\,1\overset{''}{.}04 \quad \text{per century},$$
$$\Delta\lambda + \Delta e = +\,0\overset{''}{.}53 \quad \text{per century},$$

where the cited probable error is $\pm 0\overset{''}{.}08$, which is obviously a misprint, and must read $\pm 0\overset{''}{.}20$ per century due to the cited errors in $\Delta n$ and $\Delta k$.

The agreement between the correction $\Delta p_1$ given here and that derived from proper motions of FK4 (see Section 3.2) is very satisfactory. There is, however, a discrepancy between the corresponding values of the correction $\Delta\lambda + \Delta e$ ($+0\overset{''}{.}53 \pm 0\overset{''}{.}20$ compared with $+1\overset{''}{.}20 \pm 0\overset{''}{.}11$ in FK4 and N30), which is unbelievably large and requires explanation. Before commenting on this question, the preliminary results of the Lick program presented by Vasilevskis and Klemola (1971) shall be considered.

From the Lick program, the proper motion differences $\Delta\mu$ were analysed for about 860 AGK3 stars ($\bar{m}_{pg} = 11.0$) measured on long-exposure plates, and for about 1700 AGK3 stars ($\bar{m}_{pg} = 10.1$) on short exposure plates. The results for $\Delta n$ and $\Delta k$ (per century) from the two sets of material are

| | long-exposure plates | short-exposure plates |
|---|---|---|
| $\Delta n$: | $+\,0\overset{''}{.}28 \pm 0\overset{''}{.}08$ p.e. | $+\,0\overset{''}{.}32 \pm 0\overset{''}{.}07$ p.e. |
| $\Delta k$: | $-\,0\overset{''}{.}45 \pm 0\overset{''}{.}08$ p.e. | $-\,0\overset{''}{.}59 \pm 0\overset{''}{.}08$ p.e. |

From these quantities one may estimate (the authors haven't done so) the following values for the corrections $\Delta p_1$ and $\Delta\lambda + \Delta e$

$$\Delta p_1 = +\,0\overset{''}{.}75 \pm 0\overset{''}{.}20 \quad \text{(p.e.) per century},$$
$$\Delta\lambda + \Delta e = +\,1\overset{''}{.}21 \pm 0\overset{''}{.}20 \quad \text{(p.e.) per century}.$$

While the agreement between the correction $\Delta\lambda + \Delta e$ given here and that derived from FK4 (see Section 3.2) is very satisfactory, the agreement between the corresponding values for $\Delta p_1$ ($+0\overset{''}{.}75 \pm 0\overset{''}{.}20$ compared with $+1\overset{''}{.}10 \pm 0\overset{''}{.}10$ in FK4 and N30) appears to be at the limit of the range of the errors.

The comparison between the results of Fatchikhin and of Vasilevskis and Klemola reveals an obvious discrepancy in the values of $\Delta k$; while the absolute values are about the same, the sign is different. The difference, if the signs would be correct, cannot be real, since we are dealing in both cases with AGK3 proper motions in about the same regions of the sky. A sign error appears to be indicated in the solution made by Fatchikhin; this would explain the low value of $\Delta\lambda + \Delta e$ derived from the Pulkovo data.

Concerning the accuracy achieved in both determinations – which may be regarded as low at first sight – one has to take into account that the measurements were made in about two regions of the northern sky separated by the zone of avoidance. The accuracy cannot be expected to be much higher in view of (a) possible regional systematic errors in the FK4 on which the AGK3 is based, (b) the errors of measurement in the AGK3 proper motions including large spurious proper motions to which attention is drawn by Vasilevskis and Klemola, and (c) the errors of measurement on plates with galaxies.

Despite these limitations in the accuracy, one may conclude from the results that the reference of the proper motions of AGK3 stars to galaxies has revealed no alarming new effects in the fundamental proper motion system.

Fatchikhin (1970b) reports the value $Q = -0\rlap{.}''22 \pm 0\rlap{.}''06$ per century for Oort's constant in almost unbelievably good agreement with the value $-0\rlap{.}''23 \pm 0\rlap{.}''04$ derived by Fricke (1967b) on the basis of FK4 and N30. This means that Fricke's value cannot be affected by a rotation of the equator as suggested by Aoki (see Section 3.5). Moreover, Vasilevskis and Klemola (1971) sought a rotation, which corresponds to a secular change of the obliquity, in the proper motion differences $\Delta\mu$. They found no results significantly different from zero. This means that the effect suggested by Aoki must be very small.

## 6. Conclusions and Recommendations

(1) The fundamental proper motions have provided precessional corrections which are suitable for improving the conventional precessional values by one order of magnitude. The remaining errors are about 15% of the corrections, that is $\pm 0\rlap{.}''15$ (m.e.) per century in general precession.

(2) The origin of the error in Newcomb's determination of lunisolar precession has been traced. The overwhelming part of the error is due to systematic errors in the declination system of the FC compiled by Auwers (1879). (The FC is the first of the fundamental catalogues in the series of which the FK4 is the latest). Boss (1910) already recognized this error and the erroneous motion of the equinox, and derived precessional corrections which are near to recent values. From the fact that the systems of FK3, N30 and FK4 have yielded identical values $\Delta p_1 = +1\rlap{.}''10$ and nearly the same values $\Delta\lambda + \Delta e = +1\rlap{.}''20$, although all three systems were compiled from partly different observations and by different procedures, it must be concluded that (a) the addition of new fundamental observations from about 1930 onwards has not resulted in appreciable changes in the systems, and that (b) no further improvement of the precessional corrections may be expected in the near future from fundamental observations.

(3) Proper motions of stars in the northern sky measured with respect to galaxies have revealed no significant deviations in the precessional corrections from those based on fundamental proper motions, although the results of measurement with respect to galaxies must be considered as preliminary ones. Reliable results would necessitate the inclusion of the southern sky by initiating a project which cannot be expected to yield results quickly.

(4) An improvement and a drastic enlargement of the FK4 will be urgently required by galactic research and radio astrometry. In view of the fact that the FK3 was in-

troduced into the national and international almanacs in 1940, that the introduction of the FK4 followed in 1964, and that the need for more accurate ephemerides is rapidly growing, one should expect the need for the next fundamental catalogue in about 1980. Any change in the precessional constants should be made with the next fundamental catalogue. The necessity for a change arises from the large values of the precessional errors and from their disturbing effect on ephemerides and proper motions of stars as basic data for galactic research.

(5) It is recommended that a Working Group of the International Astronomical Union consider suggestions for a set of corrections and work out a proposal for consideration at the IAU General Assembly in 1973.

## References

Aoki, S.: 1967, *Publ. Astron. Soc. Japan* **19**, 585.
Aoki, S.: 1969, *Astron. J.* **74**, 284.
Auwers, A.: 1879, *Publ. Astron. Gesellschaft* **XIV**.
Böhme, S. and Fricke, W.: 1965, *Bull. Astron. Paris.* **25**, 269 (in J. Kovalevsky (ed.), 'The System of Astronomical Constants', *IAU Symp.* **21**).
Boshniakovich, N. A.: 1965, *Bull. Astron. Paris.* **25**, 119 (in J. Kovalevsky (ed.), 'The System of Astronomical Constants', *IAU Symp.* **21**).
Boss, L.: 1910, *Astron. J.* **24**, 111.
Clemence, G. M.: 1966, *Quart. J. Roy. Astron. Soc.* **7**, 10.
*Conférence Internationale des Étoiles Fondamentales de 1896. Procès-Verbaux*, Gauthier-Villars, Paris, p. 54.
Deutsch, A. N.: 1954, *Trans. IAU* **8**, 789.
Dieckvoss, W.: 1968, *Astron. Nachr.* **290**, 141.
Fatchikhin, N. V.: 1968, *Highlights of Astronomy*, Reidel Publ. Comp., Dordrecht-Holland, p. 297.
Fatchikhin, N. V.: 1970a, *Izv. Glav. Astron. Obs. Pulkovo*, No. 185, 93.
Fatchikhin, N. V.: 1970b, *Astron. Zhurn. Akad. Nauk SSR* **47**, 619 (English translation in *Soviet Astron. AJ*, Vol. **14**, 495).
Fricke, W.: 1965, *Bull. Astron. Paris* **25**, 5 (in J. Kovalevsky (ed.), 'The System of Astronomical Constants', *IAU Symp.* **21**).
Fricke, W.: 1967a, *Astron. J.* **72**, 642.
Fricke, W.: 1967b, *Astron. J.* **72**, 1368.
Fricke, W.: 1971, *Astron. Astrophys.* **13**, 298.
Gordon, J. E.: 1952, *Izv. Glav. Astron. Obs. Pulkovo* **19**, No. 148, 72.
Herschel, W.: 1785, *Phil. Trans. Roy. Soc.* **75**, 213.
Laplace, S.: 1797, *Exposition du Système du Monde* (German Edition by F. K. Hauff), p. 335.
Lieske, J. H.: 1967, *Newtonian Planetary Ephemerides 1800–2000. Development Ephem.* No. 28, Tech. Report 32–1206, Jet Propulsion Laboratory.
Lieske, J. H.: 1970, *Astron. Astrophys.* **5**, 90.
Morgan, H. R.: 1952, *Astron. Papers Am. Ephem. Naut. Alm.* **13**, part 3.
Morgan, H. R. and Oort, J. H.: 1951, *Bull. Astron. Inst. Neth.* **11**, 379.
Newcomb, S.: 1897, *Astron. Papers Am. Ephem. Naut. Alm.* Vol. **8**, part 1.
Schilt, J.: 1928, *Astron. J.* **39**, 17.
Thüring, B. and Thüring, E.: 1967, *Astron. Nachr.* **290**, 145.
Vasilevskis, S.: 1954, *Astron. J.* **59**, 40.
Vasilevskis, S.: 1957, *Astron. J.* **62**, 126.
Vasilevskis, S.: 1963, *Stars and Stellar Systems*, Vol. **III**, *Basic Astronomical Data*, p. 30.
Vasilevskis, S. and Klemola, A. R.: 1971, *Celest. Mech.*, this issue, p. 163.
Williams, E. and Vyssotsky, A. N.: 1947, *Astron. J.* **53**, 63.
Wright, W. H.: 1950, *Proc. Am. Phil. Soc.* **94**, 1.