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CONVERGENCE FORMULAS FOR SEQUENCES OF SETS

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This paper is concerned with the convergence of sequences of subsets of a topological space, as defined by F. Hausdorff [**6**]. Such a sequence converges if and only if its limit inferior equals its limit superior, where its limit inferior (respectively, superior) is that set each of whose elements satisfies the condition that each of its neighborhoods has nonempty intersection with all but finitely (respectively, with infinitely) many terms of the sequence.

F. Hausdorff [**6**] showed that the limit superior of a sequence (A_n) of subsets of a topological space X equals

$$\bigcap_{m=1}^{\infty}\left(\bigcup_{n>m} A_n\right)^{-X}.$$

K. Kuratowski [Problems, Colloq. Math. *1* (1947), p. 29] first formally posed the natural question: Can a "similar" formula be found for the limit inferior? R. Engelking [**3**] answered this question negatively, for "similar" formula characterized as a finite sequence of finite and countable Boolean operations and the Kuratowski closure operator. Indeed analysis of Engelking's proof reveals a stronger result than necessary, viz., such a formula cannot be found in a certain subspace of any metric space possessing two nonisolated points. More exactly the result in [**3**] shows that there is no formula for the limit inferior that is valid, i.e., yields the limit inferior of every sequence of subsets of the space, in all spaces. Such a formula is necessarily hereditary, since the limit inferior is hereditary. Moreover discrete spaces do admit a formula for the limit inferior, viz.,

$$\bigcup_{m\geqq 1}\bigcap_{n>m} A_n.$$

What follows shows that not all formulas are hereditary and among those that are, none of these is a formula for the limit inferior in a nondiscrete first countable Hausdorff space. This result improves the result in [**3**] in that it identifies a class of spaces and a class of formulas for which the result in [**3**] is valid.

The result mentioned above (see Corollary 3.11) is obtained by considering a more general question, i.e., when can the convergence of sequences of subsets

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be determined by a "method" based on formulas? Abstracting from the situation in discrete spaces, where the formulas for the limit inferior and superior could be equated for a particular sequence and their equality would guarantee the convergence of this sequence (indeed it would exhibit its limit), we define a "method" based on formulas as a system of equations drawn from the class of formulas. A complete solution for first countable Hausdorff spaces is obtained (see Theorem 3.10): A countable system of hereditary formulaic equations determines the convergence of sequences of subsets of a first countable Hausdorff space if and only if the space is discrete. Also a more general but incomplete result is obtained (see Theorem 3.9): No countable system of hereditary formulaic equations determines the convergence of sequences of subsets of a space possessing the one-point compactification, X, of a countably infinite discrete space as a closed subspace. 3.9 depends on Theorem 3.4: No countable system of formulaic equations determines the convergence of sequences of nonempty closed subsets in X.

The contents of Sections 1 and 2 are necessary for Section 3. Section 1 contains some convergence properties of sequences of sets. In Section 2 the formulas of [**3**] are developed in some detail and a few properties of formulas are presented.

1. We denote the limit inferior (respectively, superior) of a sequence (A_n) of subsets of X by $\mathrm{Li}_X\ (A_n)$ (respectively, $\mathrm{Ls}_X\ (A_n)$) and the limit (when it exists) by $\mathrm{Lim}_X\ (A_n)$. Where no confusion arises we will write these symbols without their subscript. If $\mathrm{Lim}_X\ (A_n) = A$, then (A_n) is said to converge in X to A. The following facts are direct consequences of the results in [**5**] and [**7**, pp. 335–344].

1.1 If Y is a closed subspace of X, then $\mathrm{Ls}_X\ (A_n) = \mathrm{Ls}_Y\ (A_n)$, for every sequence (A_n) of subsets of Y.

1.2 If Y is a closed subspace of X, then a sequence (A_n) of subsets of Y converges in Y if and only if (A_n) converges in X.

1.3 If h is a homeomorphism from X onto Y, then a sequence (A_n) of subsets of X converges in X if and only if $(h[A_n])$ converges in Y.

2. Suppose $N = \{1, 2, 3, \ldots\}$, f is a function from N^k into N, where $k \in N$, and $(n_1, \ldots, n_k)$ (respectively, $(a_1, \ldots, a_k)$) is a variable (respectively, fixed) element of N^k. A' designates a variable sign and cp, cl, $\cup_m$ (for $m \in N$) and $\cup$ are all constant signs.

Definition. (i) A'_f is an expression with k-many parameters;

(ii) if W is an expression with k-many parameters, then cp W and cl W are also expressions with k-many parameters and $\cup_{n_j} W$, where $j \in \{1, \ldots, k\}$ is an expression with $(k-1)$-many parameters; if W^* is an expression with k^*-many parameters, then $W \cup W^*$ is an expression with $(k + k^*)$-many parameters.

When writing the tuple of parameters of length $k + k^*$ on which $W \cup W^*$ dpends, we will use the convention of writing the k-many parameters on which W depends in positions 1 through k followed in positions $k + 1$ through $k + k^*$ by the k^*-many parameters on which W^* depends.

We remark that (i) and (ii) define all expressions and that an expression without parameters, i.e., $k = 0$, will be called a formula. We observe that a formula can be written as a finite sequence of constant and variable signs, where the first sign in the sequence is, necessarily, a variable sign. The length of this sequence is r if $r + 1$ signs have been used to compose W. W_0 is the first expression in the sequence, i.e., $W_0 = A'_f$; W_i is the expression that results at the ith-step in the sequence and $W_r = W$ is the formula of length r.

The following definition applies these expressions to sequences of sets. Suppose (A_n) is a sequence of subsets of a topological space X and W is an expression with k-many parameters. exp (X) denotes the power set of X.

Definition. $V(W)$ maps $N^k \times \exp(X)^N$ into $\exp(X)$ such that $V(W)(((a_1, \ldots, a_k), (A_n)))$ depends upon the signs forming W in the following manner:

(1) $V(A'_f)((a_1, \ldots a_k), (A_n)) = A_{f(a_1,\ldots,a_k)}$;

(2) $V(\text{cp } W')((a_1, \ldots, a_k), (A_n)) = X \backslash V(W')((a_1, \ldots, a_k), (A_n))$;

(3) $V(\text{cl } W')((a_1, \ldots, a_k), (A_n)) = (V(W')((a_1, \ldots, a_k), (A_n)))^{-X}$;

(4) $$V\left(\bigcup_{n_j} W'\right)((a_1, \ldots, a_k), (A_n)) = \bigcup_{m=1}^{\infty} V(W')((b_1, \ldots b_{j-1}, m, b_{j+1}, \ldots, b_{k+1}), (A_n)),$$

where $a_i = b_i$, for $i < j$ and $a_i = b_{i+1}$, for $i \geqq j$;

(5) $$V(W' \cup W^*)((a_1, \ldots, a_k), (A_n)) = V(W')((a_1, \ldots, a_{k'}), (A_n)) \cup V(W^*)((a_{k'+1}, \ldots, a_{k'+k^*}), (A_n)).$$

$V(W)$ is always a function, since it is always either the composition of functions or the composition or sum of Boolean functions, and $V(W)((n_1, \ldots n_k), (A_n))$ represents a k-tuple sequence of sets. If $k = 0$, then $N^0 = \{\emptyset\}$, where $\emptyset$ is the empty function and $V(W)((n_1, \ldots, n_k), (A_n)) = V(W)(\emptyset, (A_n)) = V(W)((A_n))$ represents a 0-tuple sequence of sets, i.e., a set.

We note that value of the function $V(W)$ for a fixed argument depends upon the space X with respect to which $V(W)$ is computed. Thus we adopt the notation $V_X(W)$ rather that $V(W)$, whenever necessary.

We remark that Hausdorff's formula for the limit superior is a formula of the type described above of length $r = 6$, i.e., the limit superior equals $V(\text{cp} \bigcup_{n_1} \text{cp cl} \bigcup_{n_2} A_f)$, where f maps N^2 onto N via $f(n_1, n_2) = n_1 + n_2$.

2.1 *If h is a homeomorphism from X to Y, (A_n) is a sequence in* exp (X) *and W is a formula, then* $h[V_X(W)((A_n))] = V_Y(W)((h[A_n]))$.

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Proof. If $W_m = A'_f$ (and it does for $m = 0$), then $h[V_X(W_m)((a_1, \ldots, a_{km}), (A_n))] = h[A_{f(a_1,\ldots,a_{km})}] = V_Y(W_m)((a_1, \ldots, a_{km}), (h[A_n]))$. Suppose for $m > 0$, $h[V_X(W_i)((a_1, \ldots, a_{k_i}), (A_n))] = V_Y(W_i)((a_1, \ldots, a_{k_i}), (h[A_n]))$, for $i = 0, 1, \ldots, m - 1$. Now the remaining possibilities for W_m are $\bigcup_{ns} W_i$, cp W_i, cl W_i or $W_i \cup W_j$, for some $i, j \in \{0, 1, \ldots m - 1\}$. Since the pattern of proof in each of these four cases is the same, we pause to examine only the third case.

Thus $W_m =$ cl W_i, for some $i \in \{0, 1, \ldots, m - 1\}$ implies $k_m = k_i$ and

$$\begin{aligned} h[V_X(W_m)((a_1, \ldots, a_{km}), (A_n))] &= h[(V_X(W_i)((a_1, \ldots, a_{k_i}), (A_n)))^{-X}] \\ &= (h[V_X(W_i)((a_1, \ldots, a_{k_i}), (A_n))])^{-Y} \\ &= (V_Y(W_i)((a_1, \ldots, a_{k_i}), (h[A_n])))^{-Y} = V_Y(W_m)((a_1, \ldots, a_{km}), (h[A_n])). \end{aligned}$$

Finally if $m = r$, then

$$W_m = W_r = W \text{ and } h[V_X(W)((A_n))] = V_Y(W)((h[A_n])).$$

If W is a formula of length r and (A_n) is a sequence in exp (X), define $R_i(X, W_i, (A_n))$, for $i = 0, 1, \ldots, r$, to be

$$\{A \subset X: A = V_X(W_i)((a_1, \ldots, a_{k_i}), (A_n)), \quad \text{for some } (a_1, \ldots, a_{k_i}) \in N^{k_i}\}.$$

For each distinct triple $(X, W_i, (A_n))$, R_i is countable, as N^{k_i} is countable and $V_X(W_i)$ is a function. Defining $R(X, W, (A_n)) = \bigcup \{R_i: i = 0, \ldots, r\}$, we note that R is a countable class of subsets of X.

2.2 *If W is a formula and $\emptyset_n = \emptyset$, for every $n \in N$, then* $R(X, W, (\emptyset_n)) \subset \{\emptyset, X\}$.

Proof. If $W_m = A'_f$ (and it does for $m = 0$), then $\emptyset = \emptyset_{f(a_1,\ldots,a_{km})} = V_X(W_m)((a_1, \ldots, a_{km}), (\emptyset_n))$. For $m > 0$, suppose $V_X(W_i)((a_1, \ldots, a_{k_i}), (\emptyset_n))$ belongs to $\{\emptyset, X\}$, for $i = 0, \ldots, m - 1$. We now proceed by distinguishing three cases for W_m, viz., $W_m = \bigcup_{ns} W_i$ or $W_m = W_i \cup W_j$, $W_m =$ cp W_i, and $W_m =$ cl W_i, for some $i, j \in \{0, \ldots, m - 1\}$ and for some $s \in \{1, \ldots, k_i\}$. In each of these three cases $V_X(W_m)((a_1, \ldots, a_{km}), (\emptyset_n))$ belongs to $\{\emptyset, X\}$, as $\{\emptyset, X\}$ is closed under unions, complementation and closures, respectively. Thus $R_m(X, W_m, (\emptyset)_n))$ is a subset of $\{\emptyset, X\}$ and hence by finite induction on the length of the formula W, $R(X, W, (\emptyset_n)) \subset \{\emptyset, X\}$.

To say that a subspace Y of a space X is R-open means $(A)^{-X} \cap Y = (A \cap Y)^{-Y}$, for every $A \in R \subset$ exp (X). We note that Y is open in X implies that Y is R-open, for every $R \in$ exp (exp (X)), since Y is open in X if and only if Y is exp (X)-open (see [**2**]). The proof of 2.3 below is patterned after the proof of 2.1 and is contained in [**1**].

We remark that every subspace Y of a space X is $\{\emptyset, X\}$-open, hence is $R(X, W, (\emptyset_n))$-open, for every formula W.

2.3 *If W is a formula, (A_n) is a sequence in* exp (X) *and Y is a subspace of X that is $R(X, W, (A_n))$-open, then $V_X(W)((A_n)) \cap Y = V_Y(W)((A_n \cap Y))$.*

We remark that even if (A_n) is a sequence of subsets of Y, 2.3 need not be true if Y is not $R(X, W, (A_n))$-open. For let $Y = \{1/n: n \in N\} \cup \{0\}$, $X = Y \cup \{2^{-(n+1/2)}: n \in N\}$, $W = \text{cl cp} \bigcup_n A_f$, where f is the identity on N, and $A_n = \{2/n\}$, if n is even and $A_n = \{0\}$, if n is odd, then $\{0\} = V_X(W)((A_n)) \cap Y$, but $\emptyset = V_Y(W)((A_n))$. Clearly $V_X(W)((A_n))$ is the member of $R(X, W, (A_n))$ causing Y to not be $R(X, W, (A_n))$-open.

We note that if X is arbitrary then the equation in 2.3 is valid for every formula and for every sequence in exp (X) if and only if Y is open in X. To verify this observe that if Y is open then it is $R(X, W, (A_n))$-open, for every formula W and every sequence (A_n) in exp (X), so the equation is valid by 2.3 and if the equation is valid for every formula then it is valid for the limit superior, so Y is open (see [**2**]).

We are led by this discussion to make the following useful

Definition. W is a hereditary formula means $V_X(W)((A_n)) \cap Y = V_Y(W)((A_n))$, for every sequence (A_n) in exp (Y) and for each subspace Y of an arbitrary space X.

We note that the formula for the limit superior is valid in all spaces and the limit superior is hereditary, thus its formula is also.

3. A formulaic equation S is a pair (W, W^*), where W and W^* are formulas. A collection F of formulaic equations will be called a system. To say a sequence (A_n) of subsets of a space X satisfies a system F in X means $V_X(W)((A_n)) = V_X(W^*)((A_n))$ for every pair $S = (W, W^*)$ belonging to F. Moreover (A_n) satisfies S in X, if (A_n) satisfies $F = \{S\}$ in X. A system F is countable means its cardinality is at most countably infinite.

Definition. A system determines the convergence of sequences of subsets of a space means a sequence of subsets of the space converges in the space if and only if it satisfies the system in the space.

3.1 LEMMA. *If two spaces are homeomorphic and a system determines convergence for one of them, then it also determines convergence for the other.*

Proof. Suppose $F = \{S_i = (W_i, W_i^*): i \in I\}$ determines convergence for Y, h is a homeomorphism from X to Y and (A_n) is a sequence in exp (X). Now (A_n) converges in $X \Leftrightarrow (h[A_n])$ converges in $Y \Leftrightarrow (h[A_n])$ satisfies F in $Y \Leftrightarrow V_Y(W_i)((h[A_n])) = V_Y(W_i^*)((h[A_n]))$, for every $i \in I \Leftrightarrow$ for every $i \in I$, $h[V_X(W_i)((A_n))] = h[V_X(W_i^*)((A_n))] \Leftrightarrow V_X(W_i)((A_n)) = V_X(W_i^*)((A_n))$, for every $i \in I \Leftrightarrow (A_n)$ satisfies F in X. The first, fourth and fifth equivalences are valid due to 1.3, 2.1 and h is invertible, respectively.

3.2 LEMMA. *If Y is a subspace of X, $S = (W, W^*)$, (A_n) is a sequence in* exp (X) *and* Y *is* $R(X, S, (A_n)) = R(X, W, (A_n)) \cup R(X, W^*, (A_n))$*-open, then* (A_n) *satisfies* S *in* X *implies* $(A_n \cap Y)$ *satisfies* S *in* Y.

Proof. (A_n) satisfies S in X means $V_X(W)((A_n)) = V_X(W^*)((A_n))$ implies $V_Y(W)((A_n \cap Y)) = V_X(W)((A_n)) \cap Y = V_X(W^*)((A_n)) \cap Y = V_Y(W^*)((A_n \cap Y))$ means $(A_n \cap Y)$ satisfies S in Y.

3.3 LEMMA (Engelking [**3**]). *If* X *is a countably infinite set and* R *is a countable class of subsets of* X*, then there is a countably infinite subset* E *of* X*, satisfying* $A\backslash E$ *is countably infinite, for every countably infinite* $A \in R$.

3.4 THEOREM. *No countable system determines the convergence of sequences of closed subsets in the one-point compactification,* X *of a countably infinite discrete space.*

Proof. Suppose $\Omega = \{x_n: n \in N\} \cup \{p\}$, where (x_n) is a sequence of distinct terms different from p and convergent in Ω to p and $\Omega\backslash\{p\}$ is a discrete subspace of Ω. Let $F = \{S_m: m \in N\}$ be a countable system and set $A_n = \{x_n\}$, for every $n \in N$. Define

$$R(\Omega, F, (A_n)) = \bigcup \{R(\Omega, S_m, (A_n)): m \in N\} = R.$$

Now R is a countable subset of exp (Ω), since it is the countable union of countable subsets of exp (Ω).

Let E be the countably infinite subset of Ω satisfying 3.3 for R. We note that $\Omega\backslash E$ is countably infinite, since for any countably infinite $A \in R$, $A\backslash E \subset \Omega\backslash E$ and if every $A \in R$ is finite, choose a suitable E, e.g., $E = \{x_{2n}: n \in N\}$. Thus $Y = (\Omega\backslash E) \cup \{p\}$ is closed in Ω and is homeomorphic to Ω in its relativized topology, since it is also the one-point compactification of a countably infinite discrete space. Moreover Y is R-open, since if A is finite, $(A)^{-\Omega} \cap Y = A \cap Y = (A \cap Y)^{-Y}$, as $A \cap Y$ is finite and Ω (hence Y) is T_1; if A is infinite, $A \cap Y \supset A\backslash E$, so $(A \cap Y)^{-Y} = (A \cap Y) \cup \{p\} = (A \cup \{p\}) \cap Y = (A)^{-\Omega} \cap Y$, as Ω (hence Y) is Fréchet (see [**4**]).

Since $\{p\} = \mathrm{Lim}_\Omega(A_n)$, assuming F determines convergence in Ω yields (A_n) satisfies F in Ω. Now Y is R-open, so it is $R(\Omega, S_m, (A_n))$-open, for every $m \in N$. Thus $(A_n \cap Y)$ satisfies S_m in Y, for every $m \in N$, by 3.2, so $(A_n \cap Y)$ satisfies F in Y. But F determines convergence in Y by 3.1, so $(A_n \cap Y)$ converges in Y.

But $(A_n \cap Y)$ does not converge in Y, since $\mathrm{Ls}_Y(A_n \cap Y) = \{p\}$ and $\mathrm{Li}_Y(A_n \cap Y) = \emptyset$, as $A_n \cap Y = \{x_n\}$, if $x_n \in \Omega\backslash E$ and $A_n \cap Y = \emptyset$, if $x_n \in E$ and both E and $\Omega\backslash E$ are countably infinite. This contradiction implies that F does not determine convergence in X.

We remark that admitting sequences some of whose terms may be the empty set can be avoided in the above proof, by letting $A_n = \{x_1, x_{n+1}\}$, for every $n \in N$ and $Y = (\Omega\backslash E) \cup \{p, x_1\}$, where E is chosen as before.

Suppose L is a function from exp $(X)^N$ to exp (X). To say that W is a formula for L means $L = V_X(W)$.

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3.5 COROLLARY. *There is no formula for the limit inferior over the class of sequences of closed subsets of* Ω.

Proof. If there were such a formula, W_i, then $F = \{(W_i, W_s)\}$, where W_s is the formula for the limit superior, would be a system determining convergence for Ω, contradicting 3.4.

3.6 COROLLARY. *There is no hereditary formula for the limit inferior in a space possessing* Ω *as a subspace.*

Proof. Let Z be the space described above, W a formula for Li_Z and (A_n) a sequence in exp (Ω). The contradiction that W is a formula for Li_Ω results from:

$$\mathrm{Li}_\Omega(A_n) = \mathrm{Li}_Z(A_n) \cap \Omega = V_Z(W)((A_n)) \cap \Omega = V_\Omega(W)((A_n)).$$

To say a system F is hereditary means every formula composing every formulaic equation S belonging to F is hereditary. Moreover, a formulaic equation S is hereditary means $F = \{S\}$ is hereditary.

3.7 LEMMA. *If Y is a subspace of X, $S = (W, W^*)$, (A_n) is a sequence in* exp (Y), $(\emptyset_n)$ *satisfies S in X and at least one of*
(a) *S is hereditary, or*
(b) *Y and $X\backslash Y$ are both $R(X, S, (A_n))$-open*
is true, then (A_n) satisfies S in Y if and only if (A_n) satisfies S in X.

Proof. Now

$$V_X(T)((A_n)) = V_Y(T)((A_n \cap Y)) \cup V_{X\backslash Y}(T)((A_n \cap (X\backslash Y))),$$

by (a) or (b), for $T = W$ or W^*. But $A_n \cap Y = A_n$, for every $n \in N$, thus $V_X(T)((A_n)) = V_Y(T)((A_n)) \cup V_{X\backslash Y}(T)((\emptyset_n))$, for $T = W$ or W^*. Also by 3.2, $(\emptyset_n)$ satisfies S in $X\backslash Y$, as it satisfies S in X and $X\backslash Y$ is always $R(X, S, (\emptyset_n))$-open (see 2.2 and the remark following). Thus $V_X(W)((A_n)) = V_Y(W)((A_n)) \cup V_{X\backslash Y}(W)((\emptyset_n)) = V_Y(W^*)((A_n)) \cup V_{X\backslash Y}(W^*)((\emptyset_n)) = V_X(W^*)((A_n))$. This proves the direct implication, the proof of the reverse implication is a slight modification of the proof of 3.2.

3.8 LEMMA. *If Y is a closed subspace of X, F is a system and at least one of*
(a) *Y is open, or*
(b) *F is hereditary*
is true, then F determines convergence in X implies F determines convergence in Y.

Proof. Let (A_n) be a sequence in exp (Y) and $F = \{S_i \colon i \in I\}$. Now (A_n) converges in $Y \Leftrightarrow (A_n)$ converges in $X \Leftrightarrow (A_n)$ satisfies F in $X \Leftrightarrow (A_n)$ satisfies S_i in X, for every $i \in I \Leftrightarrow (A_n)$ satisfies S_i in Y, for every $i \in I \Leftrightarrow (A_n)$ satisfies F in Y. The first and fourth equivalences follow from 1.2 and 3.7, respectively.

3.9 THEOREM. *No countable hereditary system determines the convergence of sequences of subsets of a space possessing Ω as a closed subspace.*

Proof. Let Z be the space described above. If F is a countable hereditary system which determines convergence in Z, then it determines convergence in Ω by 3.8, which contradicts 3.4.

We remark that in a space possessing unique sequential limits, a sequentially nonisolated point, i.e., the limit point of a sequence, together with the sequence convergent to it constitutes the one-point compactification of a countably infinite discrete space. Thus 3.4 is valid for the class of unique sequential limit spaces possessing at least one sequentially nonisolated point.

3.10 THEOREM. *A countable hereditary system determines the convergence of sequences of subsets of a first countable Hausdorff space if and only if the space is discrete.*

Proof. Suppose Z is a first countable Hausdorff space. If Z is not discrete then it satisfies 3.9. If Z is discrete, then $F = \{(W_i, W_s)\}$ is a countable hereditary system which determines convergence in Z, where W_i and W_s are the formulas for the limits inferior and superior, respectively, in discrete spaces.

3.11 COROLLARY. *There is a hereditary formula for the limit inferior in a first countable Hausdorff space if and only if the space is discrete.*

We note that whenever hereditary formulas were necessary in the above, that it would have been sufficient to assume that these formulas were hereditary only on the one point compactification of a countably infinite discrete space. The counterexample following 2.3 illustrates that a formula need not even satisfy this restricted hereditary condition.

An open question is whether the restriction in 3.6 and 3.10 to hereditary formulas is necessary. Again the counterexample following 2.3 illustrates that not all formulas are hereditary even for closed extensions of the one point compactification of a countably infinite discrete space.

Another open question is whether any of these results can be extended to the case where disjunctions and negations of formulaic equations are permitted, in addition to the conjunctions used here.

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