

PART 5:
POSTER PAPERS

Section A:
Planet Formation

Characterisation of SPH noise in simulations of protoplanetary discs

Serena E. Arena¹, Jean-François Gonzalez² and Elisabeth Crespe³

¹Université de Lyon, Lyon, F-69003, France; Université Lyon 1, Villeurbanne, F-69622, France;

²CNRS, UMR 5574, Centre de Recherche Astrophysique de Lyon;

³École normale supérieure de Lyon, 46, allée d'Italie, F-69364 Lyon cedex 07, France

email: {Serena.Arena, Jean-Francois.Gonzalez, Elisabeth.Crespe}@ens-lyon.fr

Abstract. The effects of *turbulence* on the dynamics of dust grains in protoplanetary discs is of relevant importance in the study of pre-planetary formation. The complex interplay between gas and dust and the modelling of turbulence require *numerical simulations*.

A statistical study of the noise in SPH simulations of gas-only protoplanetary accretion discs is performed in order to determine if it could mimic turbulence and to what extent.

Keywords. planetary systems: protoplanetary disks, turbulence

We performed four SPH simulations (see Fig. 1) of the same gas disc changing the two parameters (α, β) in the artificial viscosity prescription of Monaghan & Gingold (1983).

To each realisation of the disc we applied turbulence diagnostics measuring both its magnitude (turbulent viscosity and diffusion) and its structure (power spectrum, structure function and density probability distribution function).

Results: MAGNITUDE of the noise.

Turbulent viscosity and accretion. Turbulent viscosity in accretion discs is often quantified by the parameter α_{ss} (Shakura & Sunyaev 1973) as $\nu_T = \alpha_{\text{ss}} c_s H$ (H : scale height of the disc) and it is proportional to the mass accretion rate $\dot{M} \propto \nu_T$. We estimated α_{ss} from: (1) the mass flux: $\alpha_{\text{MF}} = -2\sqrt{r}\langle\rho v_r\rangle/(3\langle\rho\rangle)$, (2) the Reynolds stress: $\alpha_{\text{RS}} = \langle u_r u_\theta \rangle / \langle P/\rho \rangle$ (u : velocity fluctuations, P : gas pressure), (3) the SPH artificial viscosity: $\alpha_{\text{SPH}} = ah/(5H)$, (Megglicki *et al.* 1983, h : SPH smoothing length).

We found that in the simulations the mass accreted onto the star \dot{M}_{sim} increases with both α and β from $7.5 \cdot 10^{-9}$ to $1.3 \cdot 10^{-8} \text{ M}_\odot \text{ yr}^{-1}$ remaining consistent with observations ($\dot{M} \approx 10^{-8} \text{ M}_\odot \text{ yr}^{-1}$ with $\alpha_{\text{ss}} \approx 10^{-2}$, Hartmann *et al.* 1998) for all (α, β) considered.

α_{SPH} increases with α , but it is ten times larger than the expected α_{ss} for high α . In the inner region α_{MF} , related to *mean quantities*, also increases with α and in addition with β but with a value closer to the expected α_{ss} for all (α, β). In contrast, α_{RS} , related to *fluctuations*, has an opposite behaviour with respect to the other two estimates: we conclude that here fluctuations are not responsible for accretion.

Turbulent diffusion. We found that the diffusive mechanism is correctly represented by large artificial viscosity parameters, for which the turbulent diffusion coefficient $D_T \approx 5 \cdot 10^{-3} c_s H$, computed as in Fromang & Papaloizou (2006), is comparable to their value.

Results: STRUCTURE of the noise

Power spectrum. The power spectrum of velocity fluctuations (top right plot in Fig. 1) for scales below the smoothing length h presents a cascade with a slope very close to the Kolmogorov one. For larger scales, $k \in (3-16)$, the cascade is steeper than or close to the Kolmogorov one ($P \approx k^{-5/3}$). In particular, for fluctuations of v_r and v_θ the cascade is: (a) more extended, $k \in (3 : 10)$, and less steep ($P \approx k^{-2.5} \cdot k^{-1.8}$) for large (α, β); (b) less

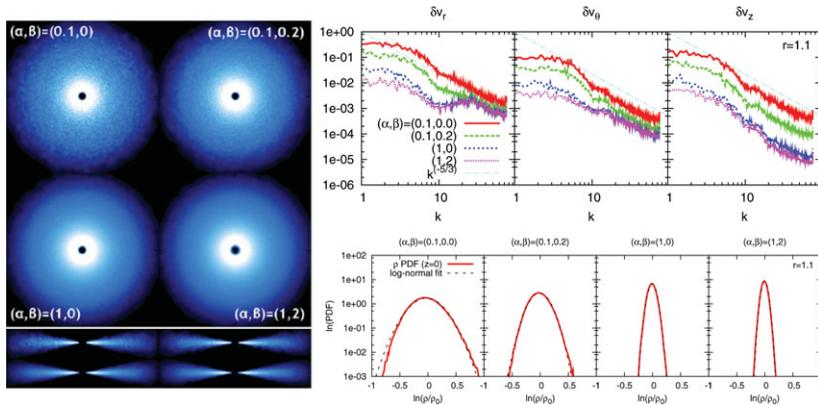


Figure 1. **Left:** face-on and edge-on view of the four SPH discs around $1M_{\odot}$ star, sampled by $2 \cdot 10^5$ particles and evolved with the code presented in Barrière-Fouchet *et al.* (2005). Units: $1 M_{\odot}$, 100AU, $10^3/2\pi$ yr, in order to have $G = 1$. Initial conditions: $M_{\text{disc}} = 0.02M_{\text{star}}$, radial extension from 20AU to 400AU, projected density and sound speed profiles: $\Sigma \propto r^{-3/2}$, $c_s \propto r^{-3/8}$. The disc is let to reach a stationary state and then it is followed for 9 orbits (at 100AU). **Top right:** power spectrum of the three velocity component for the four artificial viscosity combinations. **Bottom right:** density (ρ) probability distribution function (PDF).

extended, $k \in (6 : 10)$, and steeper ($P \approx k^{-3.5}-k^{-3}$) for small (α, β) . For fluctuations of v_z all combinations of (α, β) lead to a similar slope ($P \approx k^{-3.5}$).

Slope of the structure function. The slope ζ_p of the structure function $S(r, s) = \langle |v_i(r) - v_i(r+s)|^p \rangle$ gives information concerning the presence of intermittency. For scales larger than the smoothing length h there is no evidence of intermittency ($\zeta_p \approx 0$). However, for scales close to or smaller than h : low values of (α, β) lead to slopes similar to or slightly larger than the Kolmogorov case ($p/3$); increasing (α, β) significantly decreases ζ_p (similarly to the intermittent case) except for the vertical component of velocity.

Density probability distribution function. It is well described by a log-normal distribution (bottom right plot in Fig. 1), as expected in compressible turbulence. The 3rd and the 4th order moment show deviation from gaussian distribution for small values of (α, β) .

Conclusions. Artificial viscosity guarantees accretion rates onto the star in the observed range, even for (α, β) as large as (1, 2). The properties of the SPH noise depends both qualitatively and quantitatively on (α, β) parameters, in any case no clear sign of intermittency has been observed: (1) Low values lead to a cascade of strong fluctuations, however diffusion is not correctly described. (2) High values lead to a cascade and to a correct description of diffusion, however fluctuations are weak. Since a single (α, β) pair can partially mimic the desired properties of turbulence, additional input/model is necessary. *Acknowledgements.* This research was supported by the Agence Nationale de la Recherche (ANR) of France through contract ANR-07-BLAN-0221.

References

- Barrière-Fouchet, L., Gonzalez, J.-F., Murray, J. R., Humble, R. J., & Maddison, S. T. 2005, *A&A*, 443, 185
- Fromang, S. & Papaloizou, J. 2006, *A&A*, 452, 751
- Hartmann, L., Calvet, N., Gullbring, E., & D'Alessio, P. 1998, *ApJ*, 495, 385
- Meglicki, Z., Wickramasinghe, D., & Bicknell, G. V. 1983, *MNRAS*, 264, 691
- Monaghan, J. J. & Gingold, R. A. 1983, *Journal of Computational Physics*, 52, 374
- Shakura, N. I. & Sunyaev, R. A. 1973, *Proc. IAU Symp.*, 55, 155