Canad. J. Math. Vol. **69** (4), 2017 pp. 851–853
http://dx.doi.org/10.4153/CJM-2017-004-3

Erratum: Translation Groupoids and Orbifold Cohomology

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Abstract. We correct an error in the proof of a lemma in *Translation groupoids and orbifold cohomology*. Canad. J. Math **62**(2010), no. 3, 614–645. This error was pointed out to the authors by Li Du of the Georg-August-Universität at Göttingen, who also suggested the outline for the corrected proof.

This note contains a correction of an error in the proof of a lemma in [1]. This error was pointed out to the authors by Li Du of the Georg-August-Universität at Göttingen, who also suggested the outline for the following corrected proof.

The lemma in question reads as follows.

Lemma 8.1 ([1]) *The class of essential equivalences between Lie groupoids satisfies the 3-for-2 property,* i.e., *if we have homomorphisms* $\mathcal{G}\xrightarrow{\varphi}\mathcal{K}\xrightarrow{\psi}\mathcal{H}$ *such that two out of* $\{\varphi,\psi,\varphi\circ\psi\}$ *are essential equivalences, then so is the third.*

The given proof of this lemma is incorrect in the case where $\psi\circ\varphi$ and ψ are essential equivalences. There the following is stated:

> It is a standard property of fibre products that if any two out of (A), (B), and the whole square are fibre products, so is the third.

This is incorrect in general; in particular, when φ and $\psi\circ\varphi$ are merely fully faithful it is not necessary that ψ is also, and counter-examples can be created. Below is a corrected proof of the case in question.

Proof We consider the case where φ and $\psi\circ\varphi$ are essential equivalences. Since $\psi\circ\varphi$ is essentially surjective, the map $G_0\times_{H_0}H_1\to H_0$ is a surjective submersion. This map factors as the top arrow in the diagram

$$\begin{array}{ccccccc}
G_0\times_{H_0}H_1 & \longrightarrow & K_0\times_{H_0}H_1 & \longrightarrow & H_1 & \xrightarrow{t} & H_0\\
\big\downarrow & & \big\downarrow & & \big\downarrow{\scriptstyle s} & & \\
G_0 & \xrightarrow[\varphi_0]{} & K_0 & \xrightarrow[\psi_0]{} & H_0, & &
\end{array}$$

and we see that this implies that the composite of the last two maps, $K_0\times_{H_0}H_1\to H_0$, is a surjective submersion.

Received by the editors January 9, 2017.
Published electronically April 26, 2017.
AMS subject classification: **57S15**.
Keywords: orbifold, equivariant homotopy theory, translation groupoid, bicategory of fractions.

Next we consider the diagram

$$\begin{array}{ccccc} G_1 & \xrightarrow{\varphi_1} & K_1 & \xrightarrow{\psi_1} & H_1 \\ {\scriptstyle (s,t)}\downarrow & (A) & {\scriptstyle (s,t)}\downarrow & (B) & \downarrow{\scriptstyle (s,t)} \\ G_0 \times G_0 & \xrightarrow[\varphi_0\times\varphi_0]{} & K_0 \times K_0 & \xrightarrow[\psi_0\times\psi_0]{} & H_0 \times H_0. \end{array}$$

Since φ and $\psi\circ\varphi$ are essential equivalences, the left square (A) and the entire rectangle are both pullbacks. We want to show that the right square has to be a pullback as well. As indicated by the discussion above, the fact that φ is essentially surjective is an important ingredient. In fact, we would like to assume that φ_0 is actually surjective.

If φ_0 is not surjective, then consider the weak pullback groupoid

$$\begin{array}{ccc} G' = G \times_K^w K & \xrightarrow{\varphi'} & K \\ {\scriptstyle \pi}\downarrow & \cong & \downarrow{\scriptstyle 1_K} \\ G & \xrightarrow[\varphi]{} & K. \end{array}$$

Since φ is an essential equivalence, so is φ'. In addition, π is also an essential equivalence, because it is a weak pullback of an identity arrow (which is obviously an essential equivalence).

So we replace (A) by a new square (A'), which is again a pullback:

$$\begin{array}{ccccc} G_1' & \xrightarrow{\varphi_1'} & K_1 & \xrightarrow{\psi_1} & H_1 \\ {\scriptstyle (s,t)}\downarrow & (A') & {\scriptstyle (s,t)}\downarrow & (B) & \downarrow{\scriptstyle (s,t)} \\ G_0' \times G_0' & \xrightarrow[\varphi_0'\times\varphi_0']{} & K_0 \times K_0 & \xrightarrow[\psi_0\times\psi_0]{} & H_0 \times H_0. \end{array}$$

Furthermore, the entire rectangle is again a pullback: note that $\psi\circ\varphi' \cong (\psi\circ\varphi)\circ\pi$. The latter is an essential equivalence as a composite of essential equivalences, and hence so is the former, because it is isomorphic to an essential equivalence. We also have that the map $\varphi'\colon G_0' = G_0 \times_{K_0} K_1 \times_{K_0} K_0 \to K_0$, defined by $(x, k, t(k)) \mapsto t(k)$, is surjective, since φ is essentially surjective.

Now consider the pullback

$$\begin{array}{ccc} P & \longrightarrow & H_1 \times_{t,H_0} K_0 \\ \downarrow & & \downarrow{\scriptstyle s\pi_1} \\ K_0 & \xrightarrow[\psi_0]{} & H_0. \end{array}$$

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Since the map $s\pi_1$ is a surjective submersion, this pullback is a smooth manifold, and we get a smooth map $K_1 \to P = K_0 \times_{H_0,s} H_1 \times_{t,H_0} K_0$. Next consider the diagram

$$\begin{array}{ccccc} G_1' & \longrightarrow & P & \longrightarrow & H_1 \\ \downarrow & & \downarrow & & \downarrow \\ G_0' \times G_0' & \longrightarrow & K_0 \times K_0 & \longrightarrow & H_0 \times H_0. \end{array}$$

We know that the right square is a pullback, and therefore the left square is a pullback if and only if the whole rectangle is a pullback. But the whole rectangle is a pullback as we just observed, and so the left square is a pullback.

So now consider

$$\begin{array}{ccc} G_1' & \longrightarrow & K_1 \\ \| & & \downarrow \\ G_1' & \longrightarrow & P \\ \downarrow & & \downarrow \\ G_0' \times G_0' & \longrightarrow & K_0 \times K_0. \end{array}$$

The bottom square is a pullback according to the previous argument, and we know that the whole rectangle is a pullback, since $\varphi' : G' \to K$ is fully faithful. Therefore, the top square is also a pullback.

Now the bottom map is a surjective submersion (it is surjective as argued above and it is a submersion because the groupoids are étale), and therefore the pullback map $G_1' \to P$ is also a surjective submersion. Then looking at the top square, we see that the pullback of the map $K_1 \to P$ is the identity map, and hence a diffeomorphism. Therefore the original map must also have been a diffeomorphism, so $K_1 \cong P$ and the original square (B) is a pullback, as required. ∎

References

[1] D. Pronk and L. Scull, *Translation groupoids and orbifold cohomology*. Canad. J. Math **62**(2010), no. 3, 614–645. http://dx.doi.org/10.4153/CJM-2010-024-1

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https://doi.org/10.4153/CJM-2017-004-3 Published online by Cambridge University Press