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BOOK REVIEWS

WALKER, P. L., *The theory of Fourier series and integrals* (John Wiley & Sons, Chichester 1968), pp. viii + 192, 0 471 90112 1, £14.95.

This book offers an introduction to the theory of Fourier series and integrals which is not based on the Lebesgue integral. The first chapter gives some properties of the real and complex Fourier coefficients of what are called *finitely continuous* (FC) functions (bounded functions, which are continuous on $[0,2\pi]$ except perhaps at a finite number of points), including for example Parseval's theorem and the convolution property; it has sections on the completeness of the trigonometric system and on mean-square approximation. In Chapter 2, which deals with the convergence of Fourier series, it is shown how a Lipschitz condition on an FC function at a point or in an interval leads respectively to convergence or uniform convergence of its Fourier series. There is a good discussion of the behaviour of the series near a discontinuity of f, including the Gibbs phenomenon, and it is shown that the Fourier series of a piecewise monotone FC function converges to $\frac{1}{2}\{f(x+)+f(x-)\}$. Du Bois Reymond's example of a continuous function with Fourier series divergent at a given point is also included.

Chapter 3 deals with harmonic functions and the Poisson integral, leading to the solution of the Dirichlet problem for the disc, which is then extended to other regions by means of analytic mappings; many readers will learn some complex analysis or hydrodynamics, or both! Next come a brief chapter on the conjugate series and a longer one on the Fourier integral. The Dirichlet problem in the upper half-plane is discussed, and the Hilbert transform of a Lipschitz function. The final chapter, on multiple Fourier series and integrals, gives a convergence theorem and an inversion theorem respectively, and also shows how to adapt du Bois Reymond's idea to yield an FC function which is zero in a neighbourhood of the origin in $\mathbb{R}^2$ but whose Fourier series diverges there. An appendix gives the prerequisite elementary analysis, including a good development of the Riemann integral, standard properties of continuous functions and uniform convergence, and a short section on double series and integrals.

The author has chosen to use FC functions for the sake of some applications, and also so that the treatment can correspond as far as possible to one which uses the Lebesgue integral. As a result, I feel that there are easier ways for an engineer to learn the rudiments of Fourier series; on the other hand, it should be possible for one who already knows about Fourier series to dip into the applications in later chapters, which are consistently interesting and informative. The whole book is well written and has good examples, both worked and for the reader. It could hardly be improved on for a mathematics student unless, of course, he is familiar with the Lebesgue integral.

There are many misprints, whose nature suggests that the page proofs were not re-read after correction, and a few mathematical slips, such as the confusion of max and min on page 179, line 12. Some of the misprints will cause difficulty, especially those which confuse the symbols $-$, $=$, $\neq$, $\pm$ or have multiplicity greater than one (e.g. in the displays near the centres of pages 24 and 175), and it hurts to find Riemann spelt as Reimann with probability $\frac{1}{2}$.

PHILIP HEYWOOD

SHIRVANI, M. and WEHRFRITZ, B. A. F. *Skew linear groups* (London Mathematical Society Lecture Note Series 118, Cambridge University Press, 1986), pp. 253, 0 521 33925 1, £15.

The theory of linear groups over commutative rings has been studied over a long period of time and much is now known. In contrast the theory of linear groups over division rings (skew

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linear groups) is a comparatively new area of research. The subject might be considered to have started in the 1950s with the classification of finite subgroups of division rings by Herstein and Amitsur, then foundations of a general theory were laid by Zalesskii in 1967. However, only in the last ten years have many major results on skew linear groups appeared. The aim of the present book is to allow the subject to "come of age" by presenting a systematic development. The authors have succeeded admirably and this book will provide both a vital work for new researchers in the area and act as a standard reference work.

The authors begin by showing how familiar ideas of irreducibility, absolute irreducibility and unipotence extend from the commutative setting to the non-commutative case. These results are fundamental to the rest of the book yet some are from 1983 and 1984 papers showing how new is the subject matter of the book. General methods are given for constructing examples of skew linear groups using Ore domains and Goldie's theorem. The finite subgroups of division rings (both zero and non-zero characteristic) are classified in the second chapter which also deals with finite and locally finite skew linear groups.

The third chapter considers skew linear groups over a division ring D which is a locally finite-dimensional division algebra over a field F. Properties of skew linear groups over D are related to properties of linear groups over F. Chapter 4 studies skew linear groups over division rings D of the following type: $D=F(G)$ where F is a central subfield of D, G is a polycyclic-by-finite subgroup of D and D is generated as a division ring by F and G. This chapter requires the reader to know quite a lot about polycyclic groups and, in particular, about their group algebras. Generalisations are given of Mal'cev's theorem showing that a finitely generated linear group is residually finite and of results of Wehrfritz on residually nilpotent subgroups of finite index in linear groups. Evidence is given to suggest that the Tits alternative for finitely generated linear groups (which does not extend to skew linear groups in general) may extend to the class of skew linear groups considered in this chapter.

The main sections of Chapter 5 deal with locally finite normal subgroups and soluble normal subgroups of skew linear groups. Stronger results are obtained in the absolutely irreducible case. The shortness of the final chapter on applications shows that, as yet, the theory has found few applications but given the recent nature of the work this is not too surprising.

An unavoidable weakness of the book is the wide demand made on the background knowledge of the reader: finite group theory, representation theory, number theory, the theory of soluble and nilpotent linear groups, group algebras over polycyclic groups and results from ring theory. The authors, aware of these difficulties, have provided very full references and this is a valuable feature of this useful book.

E. F. ROBERTSON

PIETSCH, A. *Eigenvalues and s-numbers* (Cambridge studies in advanced mathematics 13, Cambridge University Press, 1987), pp. 360, 0 521 32532 3, £35.

This book is a detailed account of a subject that has developed rapidly in the last ten years. The author is well qualified to write about it, since he has been one of the principal contributors to this development.

The notion of "s-numbers" of an operator is designed to incorporate the approximation, Gelfand and Kolmogorov numbers; further examples are Weyl, Chang and Hilbert numbers. The emphasis here is on quasi-norms formed from the various sequences of s-numbers, and on their relationship with the summing norms. For operators between Hilbert spaces, the s-numbers all coincide, and $(\sum a_n(T)^2)^{1/2}$ equals the 2-summing norm $\pi_2(T)$. Various results of this type have been proved for operators between Banach spaces, often involving mixed summing norms of the type $\pi_{p,2}(T)$. Chapter 2 of the book describes the state of the art in this area, including a number of specific calculations for diagonal operators between sequence spaces.

The starting point for eigenvalue theorems is the theorem of Weyl (1949) that for operators between Hilbert spaces, $\sum|\lambda_k(T)|^r \leq \sum a_k(T)^r$ for any $r>0$. In 1978, König showed that the same

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